Quarks, Partons, and QCD: Lecture 13

Multiparton radiation

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Gluon emission in collinear limit in e^+e^-

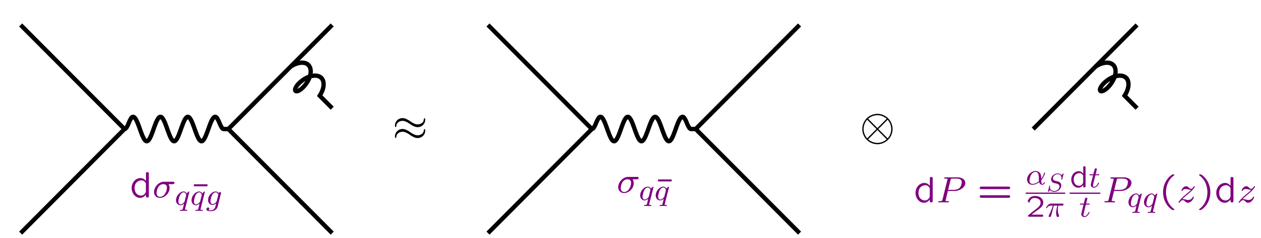
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = C_F\frac{\alpha_S}{2\pi}\sigma_0\frac{x_1^2+x_2^2}{(1-x_2)(1-x_1)} \Rightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}m_{13}^2\mathrm{d}x_1} = \frac{1}{Q^2}\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = C_F\frac{\alpha_S}{2\pi}\sigma_0\frac{1}{\underbrace{Q^2(1-x_2)}_{m_{13}^2\ \ldots\ \text{virtuality of virtual } q}(1-x_1)}x_1^2+x_2^2 = C_F\frac{\alpha_S}{2\pi}\sigma_0\frac{x_1^2+x_2^2}{m_{13}^2(1-x_1)}$$

- dominant contribution from kinematic region $x_2 \to 1$ (and similarly for $x_1 \to 1$)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}m_{13}^2\mathrm{d}x_1} \overset{x_2\to 1}{\to} \sigma_0\frac{\alpha_S}{2\pi}\frac{1}{m_{13}^2}\underbrace{C_F\frac{1+x_1^2}{1-x_1}}_{P_{qq}(x_1)\ \ldots\ \textbf{splitting function}}$$

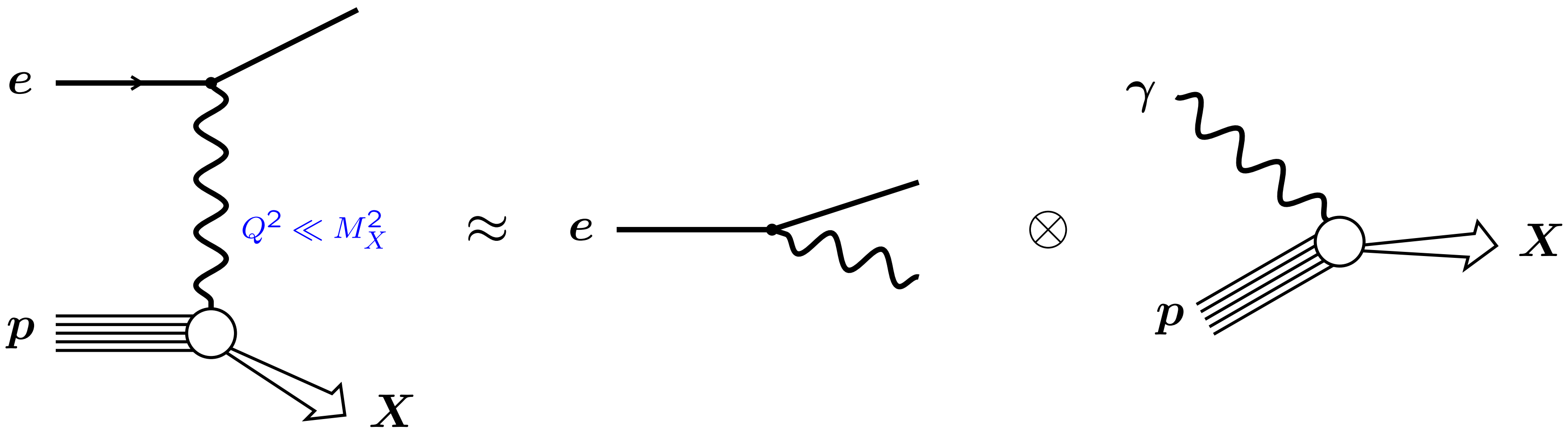
- in this kinematic domain we can say that gluon is radiated from quark
- splitting function $P_{qq}(z)$ describes the distribution of quark fractional energy after radiation of gluon
- universal - the leading (divergent) term does not depend on the origin of quark (ee, DIS, ...)
- factorization - first we create quark ($ee \to q\bar{q}$) and then we let it to radiate with probability

$$\mathrm{d}P = \frac{\alpha_S}{2\pi}\frac{\mathrm{d}t}{t}P_{qq}(z)\mathrm{d}z$$



- z is fractional energy of quark $\Rightarrow$ gluon carries $1-z \Rightarrow$ $P_{gq}(z) = P_{qq}(1-z) = C_F\frac{1+(1-z)^2}{z}$

Equivalent photon approximation



- for small photon virtualities

$$e + p \to e + X$$

can be described as radiation of collinear photon and subsequent interaction of *real* photon with proton

$$\mathrm{d}\sigma_{\mathrm{ep}} = \sigma_{\gamma_\mathrm{p}}(s_{\gamma_\mathrm{p}}) \cdot \frac{\alpha}{2\pi}\left[\frac{1+(1-y)^2}{y} - \frac{2m^2 y}{Q^2}\right]\mathrm{d}y\frac{\mathrm{d}Q^2}{Q^2}$$

- expression

$$\frac{\alpha}{2\pi}\left[\frac{1+(1-y)^2}{y} - \frac{2m^2 y}{Q^2}\right]\mathrm{d}y\frac{\mathrm{d}Q^2}{Q^2}$$

can be interpreted as probability of finding photon inside electron with fractional energy in interval $(y, y+\mathrm{d}y)$ and with virtuality in interval $(Q^2, Q^2+\mathrm{d}Q^2)$

- universality of the mass leading term for process $a \to b + c$:

$$\mathrm{d}P = \frac{\alpha}{2\pi}\frac{\mathrm{d}t}{t}P_{ab}(z)\mathrm{d}z$$

Two gluon emission

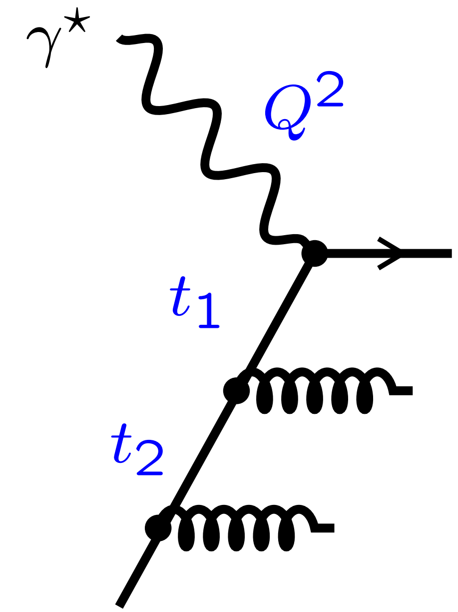
- if one emission is very likely then two (and more) must be happening as well
- Markov chain of independent radiations governed by probability

$$\mathrm{d}P = \frac{\alpha}{2\pi}\frac{\mathrm{d}t}{t}P_{ab}(z)\mathrm{d}z$$

- valid in collinear approx. with strongly ordered virtualities (in WWA, $Q^2 \ll s_{\gamma \mathrm{p}}$)

$$Q^2 \gg |t_1| \gg |t_2| \gg \ldots$$

technically: $\tau_2 \leq \varepsilon\tau_1 \leq \varepsilon^2 Q^2$, with $\tau_i \equiv |t_i|$ and small ε



Phase space of two gluon emission

$$\int \frac{\mathrm{d}x}{x}\ln^n x = \frac{1}{n+1}\ln^{n+1} x \qquad \text{(per partes)}$$

$$\sigma_2 \sim \int_{m^2}^{\varepsilon Q^2} \frac{\alpha_S}{2\pi}\frac{\mathrm{d}\tau_1}{\tau_1}\int_{m^2}^{\varepsilon\tau_1}\frac{\alpha_S}{2\pi}\frac{\mathrm{d}\tau_2}{\tau_2} = \left(\frac{\alpha_S}{2\pi}\right)^2\int_{m^2}^{\varepsilon Q^2}\frac{\mathrm{d}\tau_1}{\tau_1}\ln\frac{\varepsilon\tau_1}{m^2} = \left(\frac{\alpha_S}{2\pi}\right)^2\int_{\varepsilon}^{\varepsilon^2 Q^2/m^2}\frac{\mathrm{d}\xi}{\xi}\ln\xi = \left(\frac{\alpha_S}{2\pi}\right)^2\frac{1}{2}\left[\ln^2\frac{\varepsilon^2 Q^2}{m^2} - \ln^2\varepsilon\right]$$

$$= \left(\frac{\alpha_S}{2\pi}\right)^2\frac{1}{2}\left[\left(2\ln\varepsilon + \ln\frac{Q^2}{m^2}\right)^2 - \ln^2\varepsilon\right] = \left(\frac{\alpha_S}{2\pi}\right)^2\left[\frac{1}{2}\ln^2\frac{Q^2}{m^2} + 2\ln\varepsilon\ln\frac{Q^2}{m^2} + \frac{3}{2}\ln^2\varepsilon\right]$$

- note that the leading divergent term in limit $m \to 0$ does not depend on choice of ε

Multi parton emission

- mass leading term for 3 parton emission

$$\sigma_3 \sim \int_{m^2}^{\varepsilon Q^2} \frac{\alpha_S}{2\pi}\frac{\mathrm{d}\tau_1}{\tau_1} \int_{m^2}^{\varepsilon \tau_1} \frac{\alpha_S}{2\pi}\frac{\mathrm{d}\tau_2}{\tau_2} \int_{m^2}^{\varepsilon \tau_2} \frac{\alpha_S}{2\pi}\frac{\mathrm{d}\tau_3}{\tau_3} \approx \left(\frac{\alpha_S}{2\pi}\right)^3 \int_{m^2}^{\varepsilon Q^2} \frac{\mathrm{d}\tau_1}{\tau_1}\frac{1}{2}\ln^2\frac{\varepsilon\tau_1}{m^2} = \left(\frac{\alpha_S}{2\pi}\right)^3 \frac{1}{2}\int_1^{\varepsilon Q^2/m^2} \frac{\mathrm{d}\xi}{\xi}\ln^2\xi$$

$$= \left(\frac{\alpha_S}{2\pi}\right)^3 \frac{1}{2}\cdot\frac{1}{3}\left[\ln^3\frac{\varepsilon Q^2}{m^2}\right] = \left(\frac{\alpha_S}{2\pi}\right)^3 \left[\frac{1}{3!}\ln^3\frac{Q^2}{m^2} + f_2(\ln\varepsilon)\ln^2\frac{Q^2}{m^2} + \dots\right]$$

- generalization to n emissions is straightforward due to $\int \frac{\mathrm{d}x}{x}\ln^n x = \frac{1}{n+1}\ln^{n+1} x$

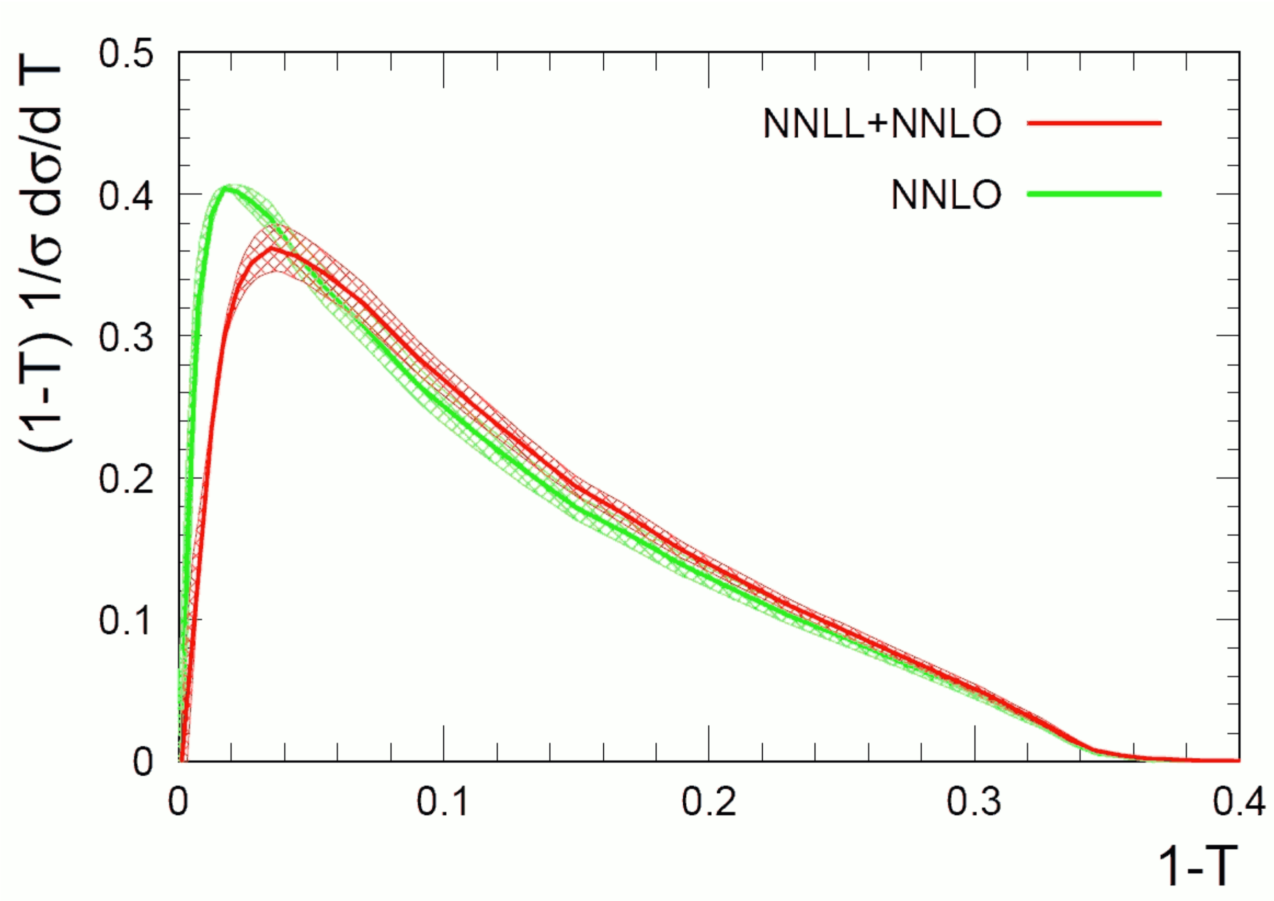
$$\sigma_n \sim \left(\frac{\alpha_S}{2\pi}\right)^n \left[\frac{1}{n!}\ln^n\frac{Q^2}{m^2} + f_{n-1}(\ln\varepsilon)\ln^{n-1}\frac{Q^2}{m^2} + \dots\right]$$

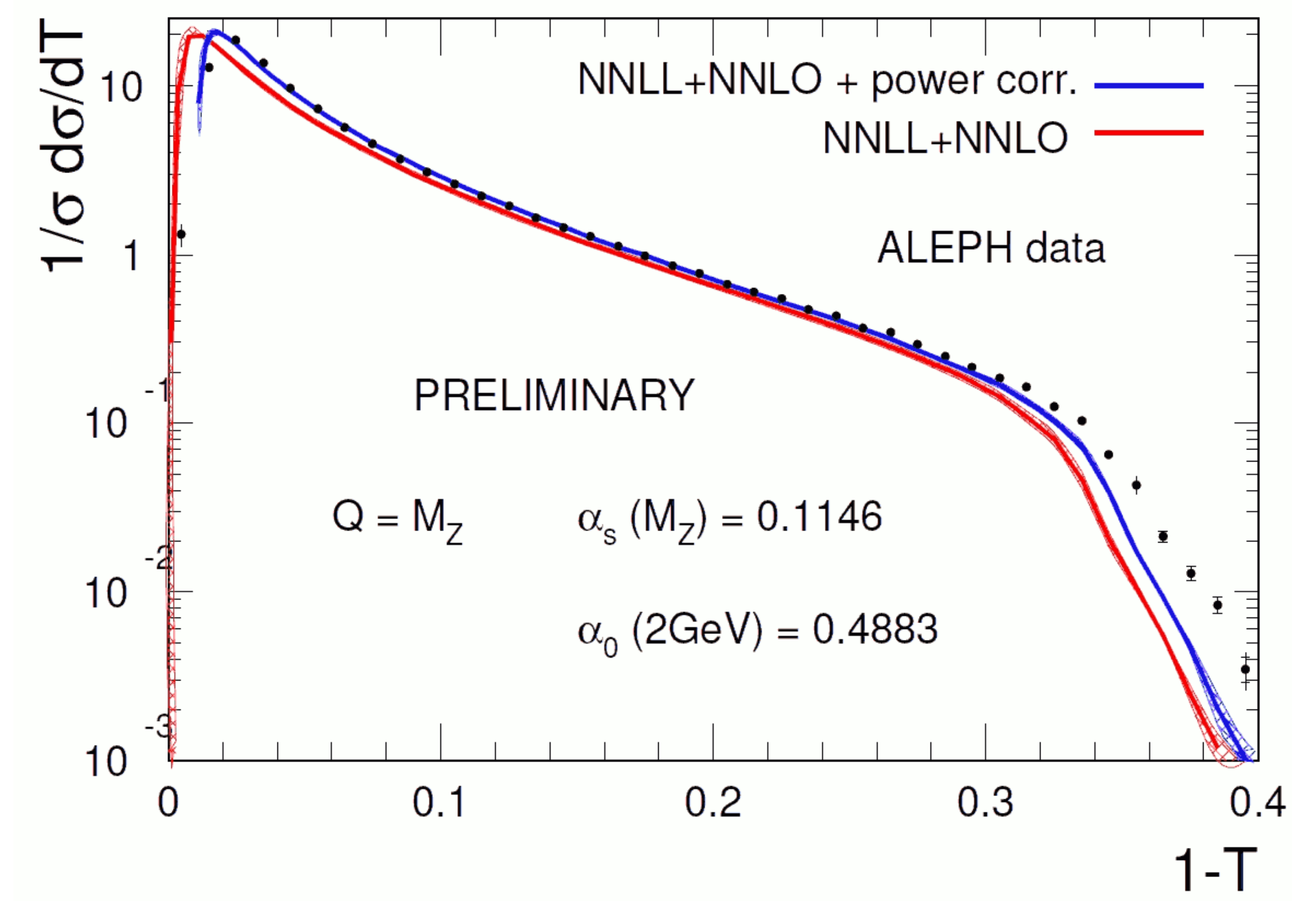
Resummation - summation of multi parton emissions with given precision

- **LL** (leading log) - terms $\alpha^n \ln^n(Q^2/m^2)$
- **NLL** (next-leading log) - terms up to $\alpha^n \ln^{n-1}(Q^2/m^2)$

- either analytical resummation for certain observables
- or numerical approach by modeling parton showers in Monte Carlo generators

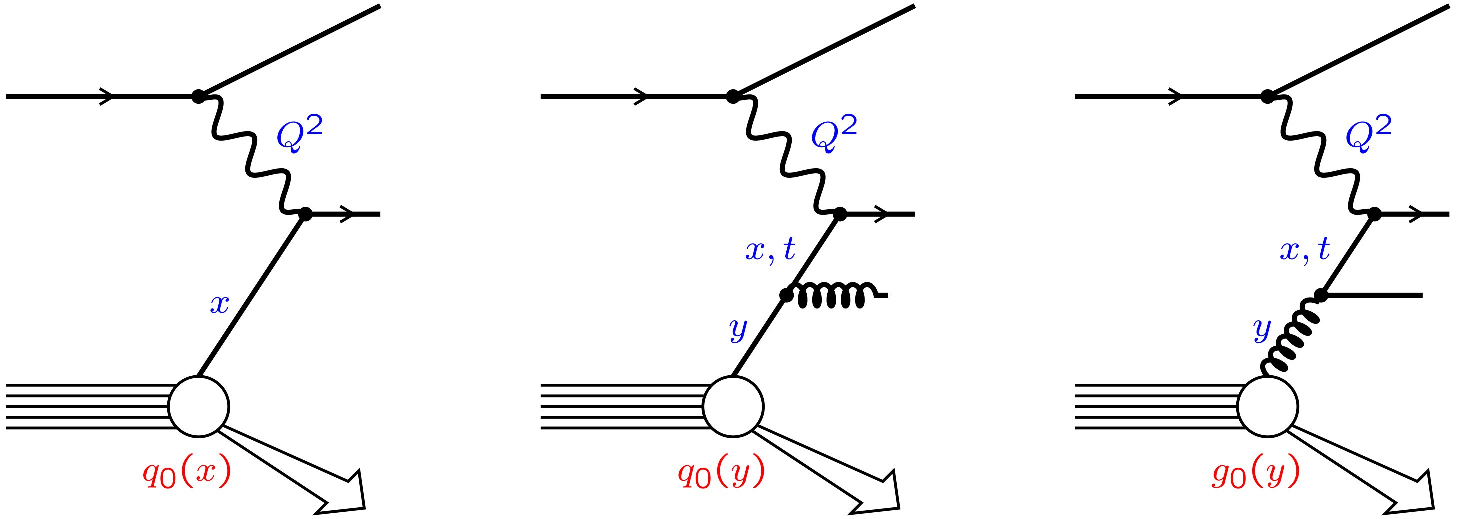
Thrust





P. F. Monni, arXiv:1202.6336 [hep-ph]

- state of the art: NNLL + NNLO
- resummation improves the description in $T \to 1$ region
- matching resummation with fixed higher order calculations not trivial
- hadronization (power corrections) still important to measure α_S precisely



$$\mathrm{d}P = \frac{\alpha_S}{2\pi}\frac{\mathrm{d}t}{t}P_{qq}(z)\mathrm{d}z$$

- QPM - partons are quasi-free
  $\Rightarrow$ on mass shell (no virtuality)
  proton described by **bare** PDF $q_0(x)$ and $g_0(x)$
- let's allow one parton radiation (α_S order)
  dominant contribution from collinear radiation at small parton virtualities $t \ll Q^2$
- proton described by **dressed** PDF $q(x, M^2)$
  number of partons with fractional momentum x and virtuality up to M^2

$$q(x, M^2)\mathrm{d}x = q_0(x)\mathrm{d}x + \int_x^1 \mathrm{d}y\, q_0(y) P_{qq}\left(\frac{x}{y}\right)\frac{\mathrm{d}x}{y}\int_\beta^{M^2}\frac{\alpha_s(t)}{2\pi}\frac{\mathrm{d}t}{t} + \int_x^1 \mathrm{d}y\, g_0(y) P_{qg}\left(\frac{x}{y}\right)\frac{\mathrm{d}x}{y}\int_\beta^{M^2}\frac{\alpha_s(t)}{2\pi}\frac{\mathrm{d}t}{t} + \mathcal{O}(\alpha_S^2)$$

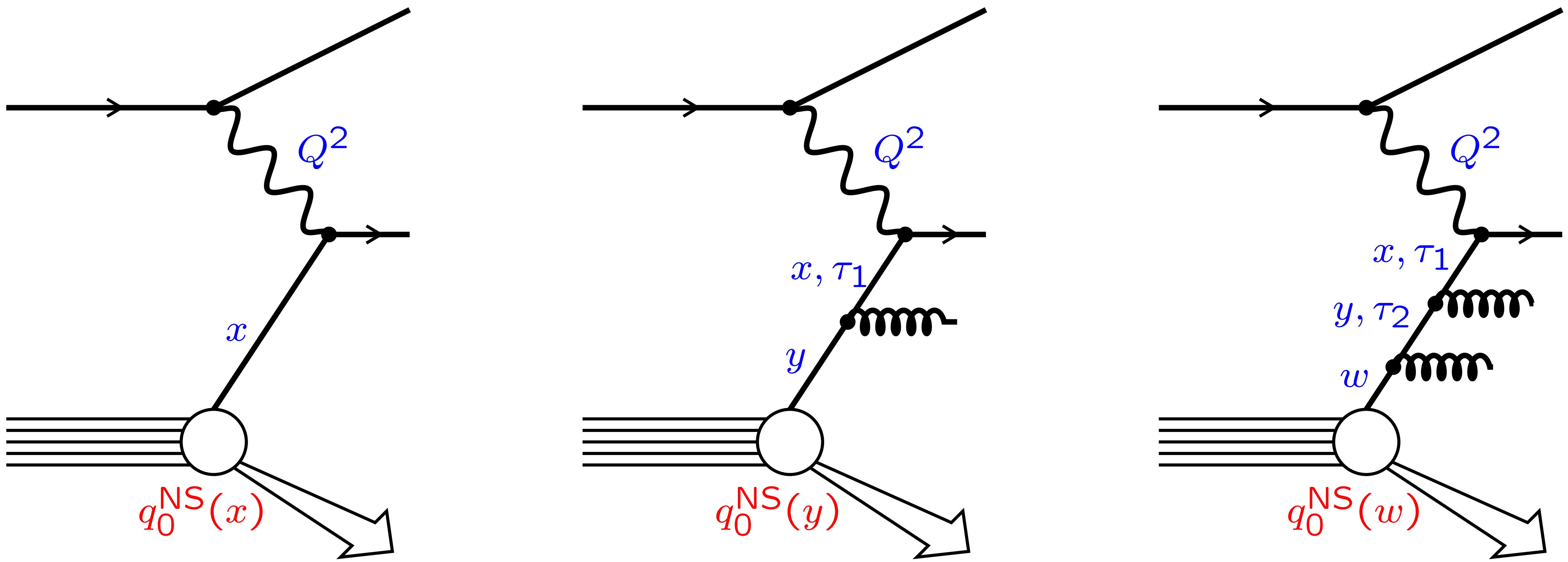
- we can't send the gluon mass (β) to zero, but we can get rid of β by derivation with respect to M^2

$$\frac{\mathrm{d}q(x, M^2)}{\mathrm{d}M^2} = \frac{\alpha_s(M^2)}{2\pi}\frac{1}{M^2}\int_x^1\frac{\mathrm{d}y}{y}\left[P_{qq}\left(\frac{x}{y}\right)q_0(y) + P_{qg}\left(\frac{x}{y}\right)g_0(y)\right] + \mathcal{O}(\alpha_S^2)$$

- $q_0(x)$ and $q(x, M^2)$ differ by quantity proportional to α_S; $M^2/\mathrm{d}M^2 = 1/\mathrm{d}\ln M^2$

$$\frac{\mathrm{d}q(x, M^2)}{\mathrm{d}\ln M^2} = \frac{\alpha_s(M^2)}{2\pi}\int_x^1\frac{\mathrm{d}y}{y}\left[P_{qq}\left(\frac{x}{y}\right)q(y, M^2) + P_{qg}\left(\frac{x}{y}\right)g(y, M^2)\right] + \mathcal{O}(\alpha_S^2)$$

Evolution equations - resummation of multiparton emissions



- consider only nonsinglet PDF $q^{NS}(x) = q(x) - \bar{q}(x)$ (valence quark PDF in QPM)

$\Rightarrow$ only one type of vertex as $g \to q\bar{q}$ does not contribute due to $q - \bar{q}$ symmetry of $P_{qg}(z)$ and $P_{\bar{q}g}(z)$

$$q^{NS}(x, M^2) = q_0^{NS}(x) + \int_x^1 \mathrm{d}y\, q_0^{NS}(x) \frac{1}{y} P_{qq}\left(\frac{x}{y}\right) \int_{m^2}^{M^2} \frac{\mathrm{d}\tau_1}{\tau_1} \frac{\alpha_S(\tau_1)}{2\pi}$$
$$+ \int_x^1 \frac{\mathrm{d}y}{y} P_{qq}\left(\frac{x}{y}\right) \int_y^1 \frac{\mathrm{d}w}{w} P_{qq}\left(\frac{y}{w}\right) q_0^{NS}(w) \int_{m^2}^{M^2} \frac{\mathrm{d}\tau_1}{\tau_1} \frac{\alpha_S(\tau_1)}{2\pi} \int_{m^2}^{\tau_1} \frac{\mathrm{d}\tau_2}{\tau_2} \frac{\alpha_S(\tau_2)}{2\pi} + \ldots$$

$$\frac{\partial q^{NS}(x, M^2)}{\partial \ln M^2} = M^2 \frac{\partial q^{NS}(x, M^2)}{\partial M^2} = \frac{\alpha_S(M^2)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} P_{qq}\left(\frac{x}{y}\right) \left[q_0^{NS}(y) + \int_y^1 \frac{\mathrm{d}w}{w} P_{qq}\left(\frac{y}{w}\right) q_0^{NS}(w) \int_{m^2}^{M^2} \frac{\mathrm{d}\tau_2}{\tau_2} \frac{\alpha_S(\tau_2)}{2\pi} + \ldots \right]$$
$$= \frac{\alpha_S(M^2)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} P_{qq}\left(\frac{x}{y}\right) q^{NS}(y, M^2) = \frac{\alpha_S(M^2)}{2\pi} P_{qq} \otimes q^{NS}$$

strong ordering $Q^2 \gg M^2 \gg \tau_1 \gg \tau_2 \gg \ldots$

LL: $Q^2 > M^2 > \tau_1 > \tau_2 > \ldots$

Moments of PDF and sum rules

- n^{th} moment: $f(n) \equiv \int_0^1 \mathrm{d}x x^n f(x)$

$$\frac{\partial q^{\mathsf{NS}}(n,M^2)}{\partial \ln M^2} = \frac{\alpha_S(M^2)}{2\pi}\int_0^1 \mathrm{d}x x^n \int_x^1 \frac{\mathrm{d}y}{y} P_{qq}\left(\frac{x}{y}\right) q^{\mathsf{NS}}(y,M^2) \stackrel{x=zy}{=} \frac{\alpha_S(M^2)}{2\pi} \overbrace{\int_0^1 \mathrm{d}z\, z^n P_{qq}(z)}^{P_{qq}(n)} \overbrace{\int_0^1 \mathrm{d}y\, y^n q^{\mathsf{NS}}(y,M^2)}^{q^{\mathsf{NS}}(n,M^2)}$$

- integro-differential DGLAP equations transform to a set of ordinary differential equations

$$\frac{\partial q(n,M^2)}{\partial \ln M^2} = \frac{\alpha_S(M^2)}{2\pi}\left[P_{qq}(n)q(n,M^2) + P_{qg}(n)g(n,M^2)\right]$$

$$\frac{\partial \bar{q}(n,M^2)}{\partial \ln M^2} = \frac{\alpha_S(M^2)}{2\pi}\left[P_{qq}(n)\bar{q}(n,M^2) + P_{\bar{q}g}(n)g(n,M^2)\right]$$

$$\frac{\partial g(n,M^2)}{\partial \ln M^2} = \frac{\alpha_S(M^2)}{2\pi}\left[\sum_q^{\text{flav.}} P_{gq}(n)\left(q(n,M^2)+\bar{q}(n,M^2)\right) + P_{gg}(n)g(n,M^2)\right]$$

- 1^{st} moment - total fractional momentum carried by partons $S(1,M^2) = \int_0^1 \mathrm{d}x \left[\sum_q (q(x)+\bar{q}(x)) + g(x)\right] = 1$

$$0 = \frac{\partial S(1,M^2)}{\partial \ln M^2} = \frac{\alpha_S}{2\pi}\left\{ g(1)\left[2n_f P_{qg}(1) + P_{gg}(1)\right] + \left(\sum_q^{\text{flav.}}\left[q(1)+\bar{q}(1)\right]\right)\left[P_{qq}(1)+P_{gq}(1)\right]\right\}$$

$$P_{qq}(1) + P_{gq}(1) = 0 \qquad \text{and} \qquad 2n_f P_{qg}(1) + P_{gg}(1) = 0$$

Virtual correction to splitting function - "+" distribution

- virtual emission at $x = 1$

$$\frac{\mathrm{d}\sigma_{\text{virt}}}{\mathrm{d}x} = \sigma_0 \frac{\alpha_S}{2\pi} C_F \left(-\ln^2\beta - 3\ln\beta + \frac{\pi^2}{3} - \frac{7}{2} \right) \delta(1-x)$$

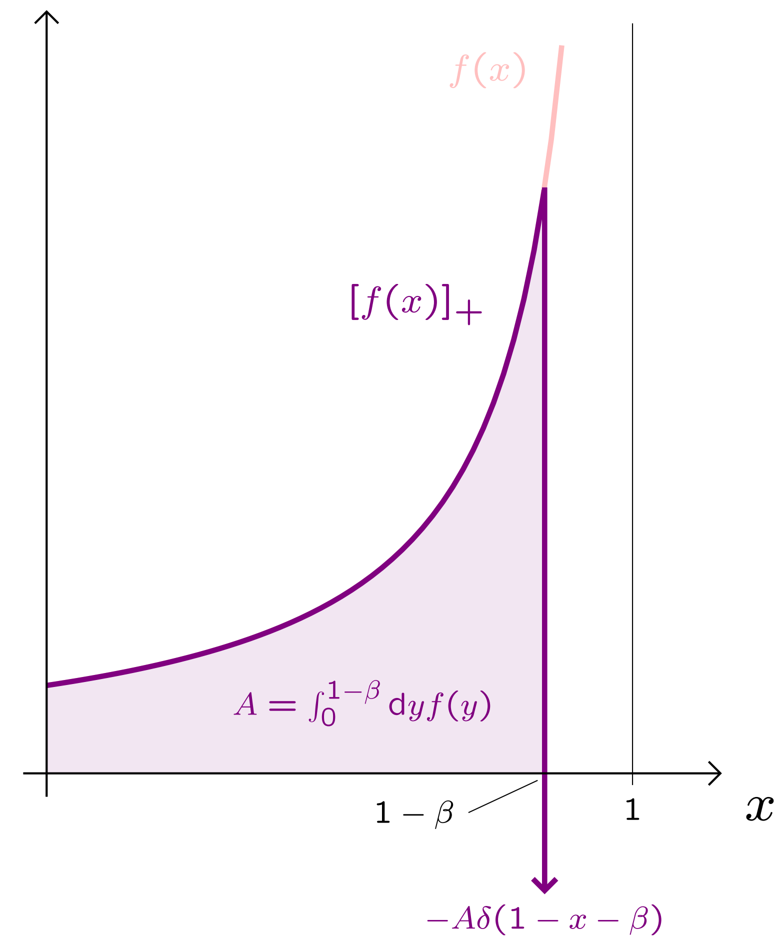
- radiation from quark is then

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = \frac{\mathrm{d}\sigma_{\text{real}}}{\mathrm{d}x} + \frac{\mathrm{d}\sigma_{\text{virt}}}{\mathrm{d}x} \quad \text{more precisely} \quad \frac{\mathrm{d}\sigma}{\mathrm{d}x} = \lim_{\beta\to 0} \left[\frac{\mathrm{d}\sigma_{\text{real}}}{\mathrm{d}x} \theta(1-x-\beta) + \sigma_{\text{virt}} \delta(1-x-\beta) \right]$$

- concept of **"+"-distribution**

$$[f(x)]_+ \equiv \lim_{\beta\to 0} \left[f(x)\theta(1-x-\beta) - \delta(1-x-\beta) \int_0^{1-\beta} \mathrm{d}y f(y) \right]$$

from the definition: $\displaystyle\int_0^1 \mathrm{d}x [f(x)]_+ = 0$



- divergent part ($\ln\beta$ term) in real emission must vanish after adding virtual emission and integration

$$P_{qq}(x) = C_F \frac{1+x^2}{1-x} \to C_F \left[\frac{1+x^2}{1-x} \right]_+ + K\delta(1-x)$$

- $K = 0$ from explicit calculation, or **from a neat trick**

$$P_{qq}(1) + P_{gq}(1) = 0 \qquad \text{and} \qquad 2n_f P_{qg}(1) + P_{gg}(1) = 0$$

Virtual corrections to splitting function (ex. 8.2)

$P_{qq}(z) = C_F \left[\frac{1+z^2}{1-z}\right]_+ + K\delta(1-x)$ is relatated to $P_{gq} = C_F \frac{1+(1-z)^2}{z}$ via $P_{qq}(1) + P_{gq}(1) = 0$

1) the first moment of splitting function $P_{gq}(z)$ is

$$P_{gq}(1) = C_F \int_0^1 \mathrm{d}z\, z \frac{1+(1-z)^2}{z} = C_F \int_0^1 \mathrm{d}z\, [1+(1-z)^2] = C_F \left(1 + \frac{1}{3}\right) = \frac{4}{3} C_F$$

2) In the evaluation of the first moment of splitting function $P_{qq}(z)$, we encounter an integral of type

$$\int_0^1 \mathrm{d}z z \left[\frac{f(z)}{1-z}\right]_+ = \int_0^1 \mathrm{d}z (z-1) \left[\frac{f(z)}{1-z}\right]_+ = -\int_0^1 \mathrm{d}z f(z)$$

prove it!

3) the first moment of splitting function $P_{qq}(z)$ is then

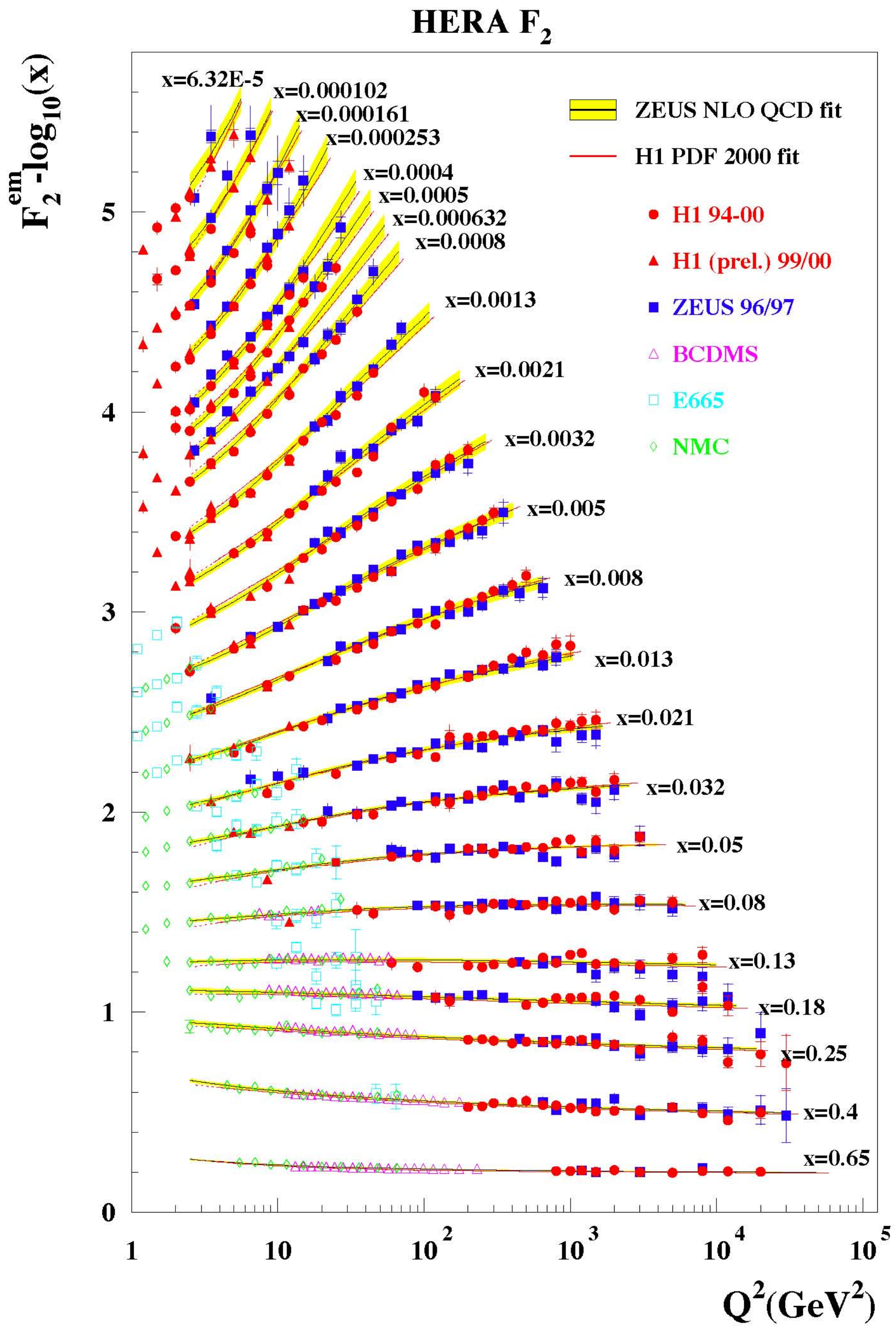
$$P_{qq}(1) = C_F \int_0^1 \mathrm{d}z\, z \left[\frac{1+z^2}{1-z}\right]_+ + K \int_0^1 \mathrm{d}z\, z\delta(1-z) = -C_F \int_0^1 \mathrm{d}z (1+z^2) + K = -\frac{4}{3} C_F + K$$

$\Rightarrow$ $K = 0$ and $P_{qq}(z) = C_F \left[\frac{1+z^2}{1-z}\right]_+$

- similarly, from $2n_f P_{qg}(1) + P_{gg}(1) = 0$ we get

$$P_{gg}(z) = 2C_A \left\{ \left[\frac{z}{1-z}\right]_+ + \frac{1-z}{z} + z(1-z) - \frac{2n_f + 3}{36} \delta(1-z) \right\}$$

$g(x)$ at low x from DIS

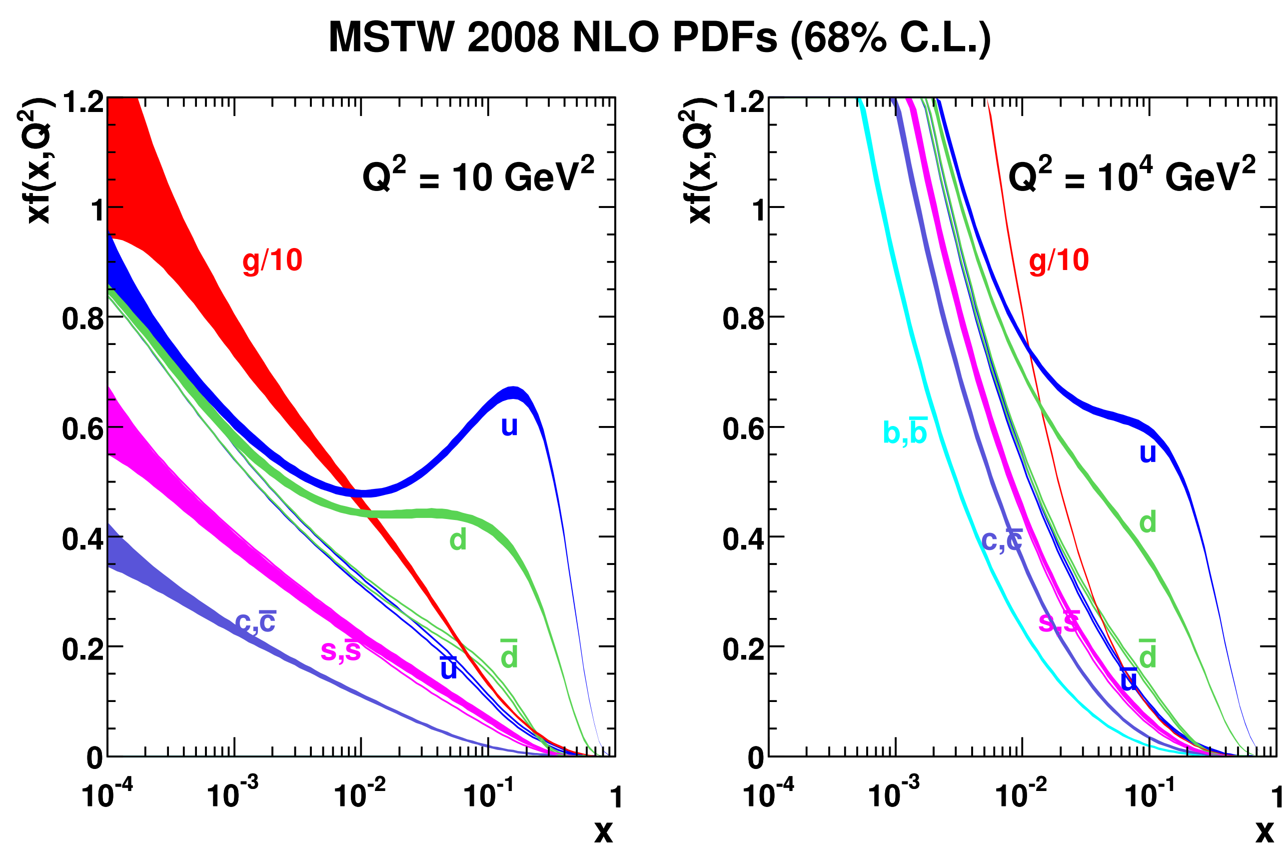


$$F_2(x,Q^2) = x\sum_q^{\text{flav.}} e_q^2[q(x,Q^2)+\bar{q}(x,Q^2)]$$

- at low x, the evolution is driven by gluons

$$\frac{\partial q(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}\left[P_{qq}\left(\frac{x}{y}\right)q(y,Q^2) + P_{qg}\left(\frac{x}{y}\right)g(y,Q^2)\right]$$

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} \overset{x\to 0}{\approx} \frac{\alpha_s(Q^2)}{2\pi}2x\sum e_q^2\int_x^1 \frac{\mathrm{d}y}{y}P_{qg}\left(\frac{x}{y}\right)g(y,Q^2)$$

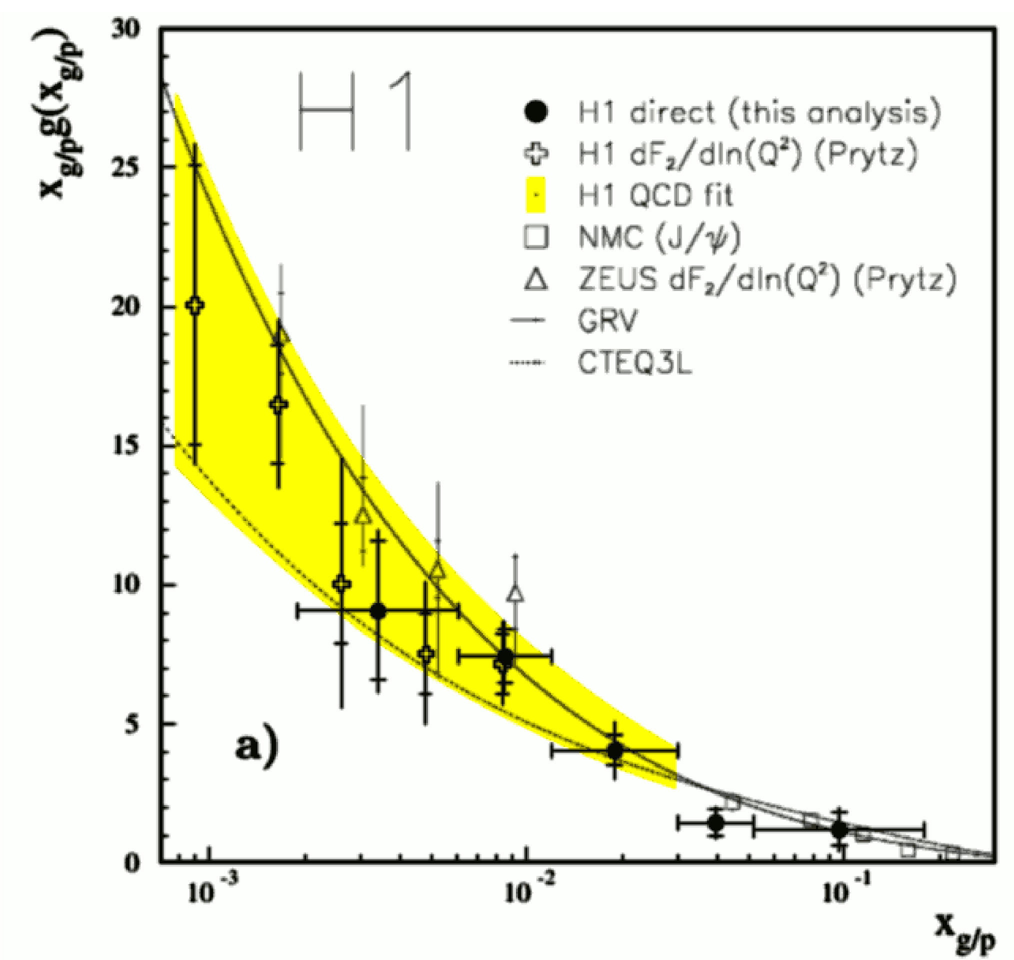


$g(x)$ at low x from DIS

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} \overset{x\to 0}{\approx} \frac{\alpha_s(Q^2)}{2\pi} 2x \sum e_q^2 \int_x^1 \frac{\mathrm{d}y}{y} P_{qg}\left(\frac{x}{y}\right) g(y,Q^2) \overset{y=x/z}{=} \frac{\alpha_s(Q^2)}{2\pi}\left(2\sum e_q^2\right) \int_{x\approx 0}^1 \frac{\mathrm{d}z}{z} x g(x/z) P_{qG}(z)$$

- $P_{qg}(z) = (z^2 + (1-z)^2)/2$ is symmetric around $z = 1/2$
- expand $(x/z)g(x/z,Q^2)$ around this point, keeping first two terms of Taylor expansion

$$H(x/z,Q^2) \equiv (x/z)g(x/z,Q^2) = H(2x,Q^2) + H'(2x,M)(z-1/2) + H''(2x,M)(z-1/2)^2/2! + \dots$$



- due to symmetry of $P_{qg}(z)$ around $z = 1/2$ the integral of the second term vanishes

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} \approx \frac{\alpha_s(Q^2)}{2\pi} \underbrace{\left(2\sum_q^{u,d,s,c} e_q^2\right)}_{\frac{20}{9}} 2xg(2x,Q^2) \underbrace{\int_{\approx 0}^1 \mathrm{d}z P_{qg}(z)}_{\frac{1}{3}}$$

$$= \frac{20}{27}\frac{\alpha_s(Q^2)}{2\pi} 2xg(2x,Q^2)$$

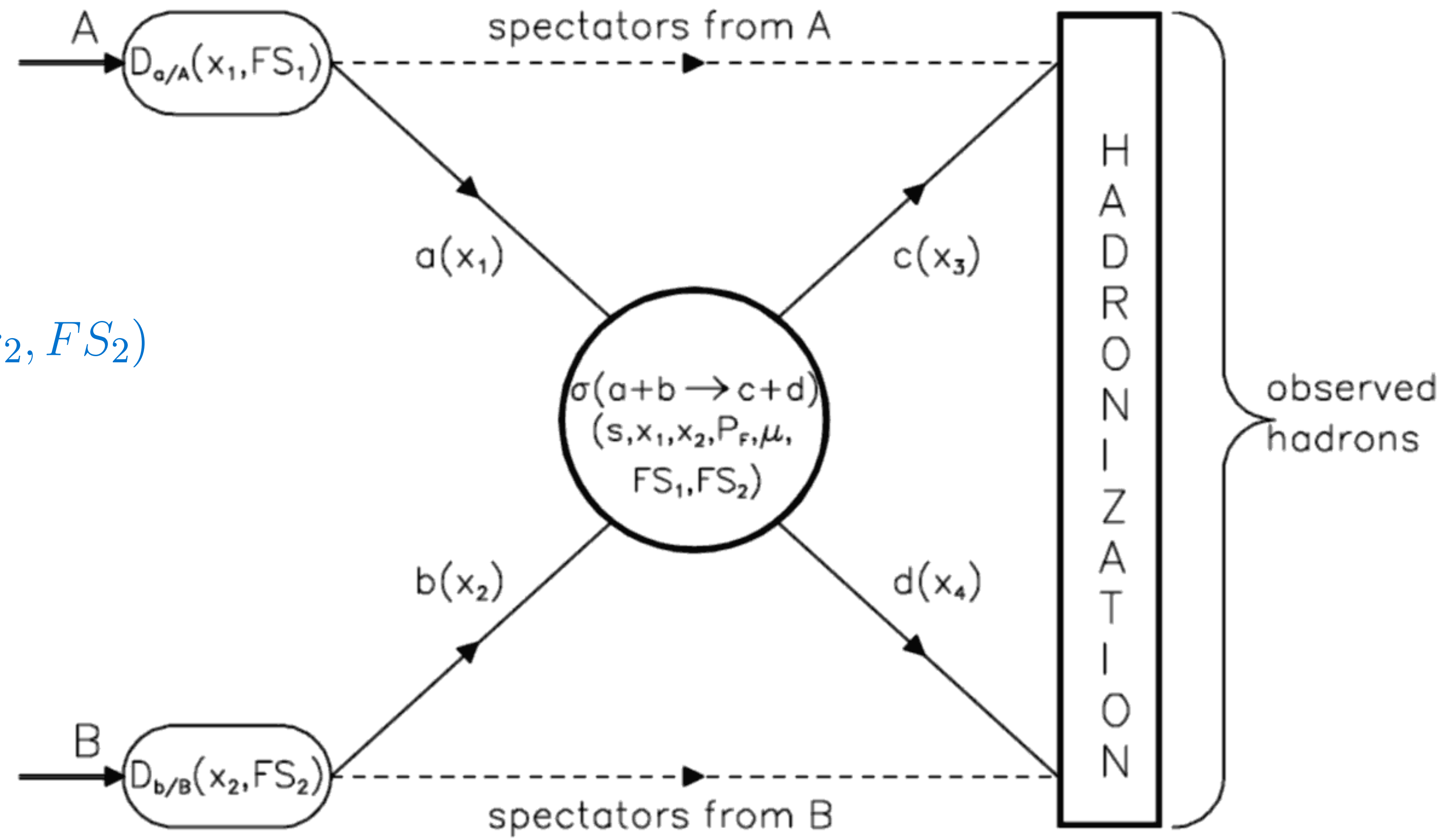
General framework for x-section calculations in QCD

$$A + B \to F$$

$$\sigma = \sum_{a,b,c,d} \int \mathrm{d}x_1 D_{a|A}(x_1, FS_1) \sum_{a,b,c,d} \int \mathrm{d}x_2 D_{b|B}(x_2, FS_2)$$

$$\sigma_{a+b\to c+d}(s, x_1, x_2, \mu, FS_1, FS_2, P_F)$$

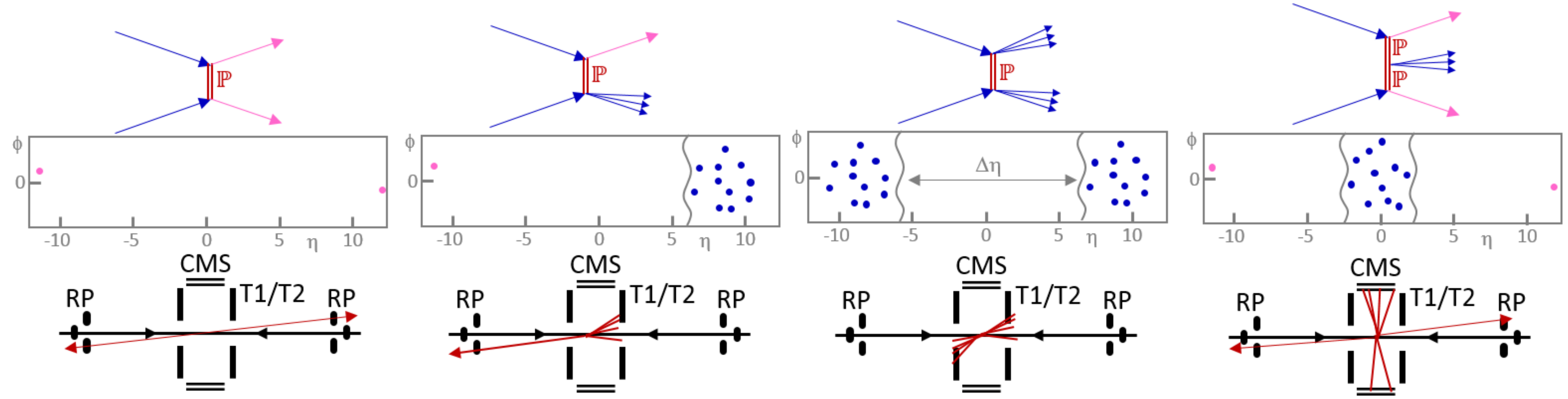
$$\otimes D_{\mathsf{hadr}}(p_c, p_d, P_F)$$



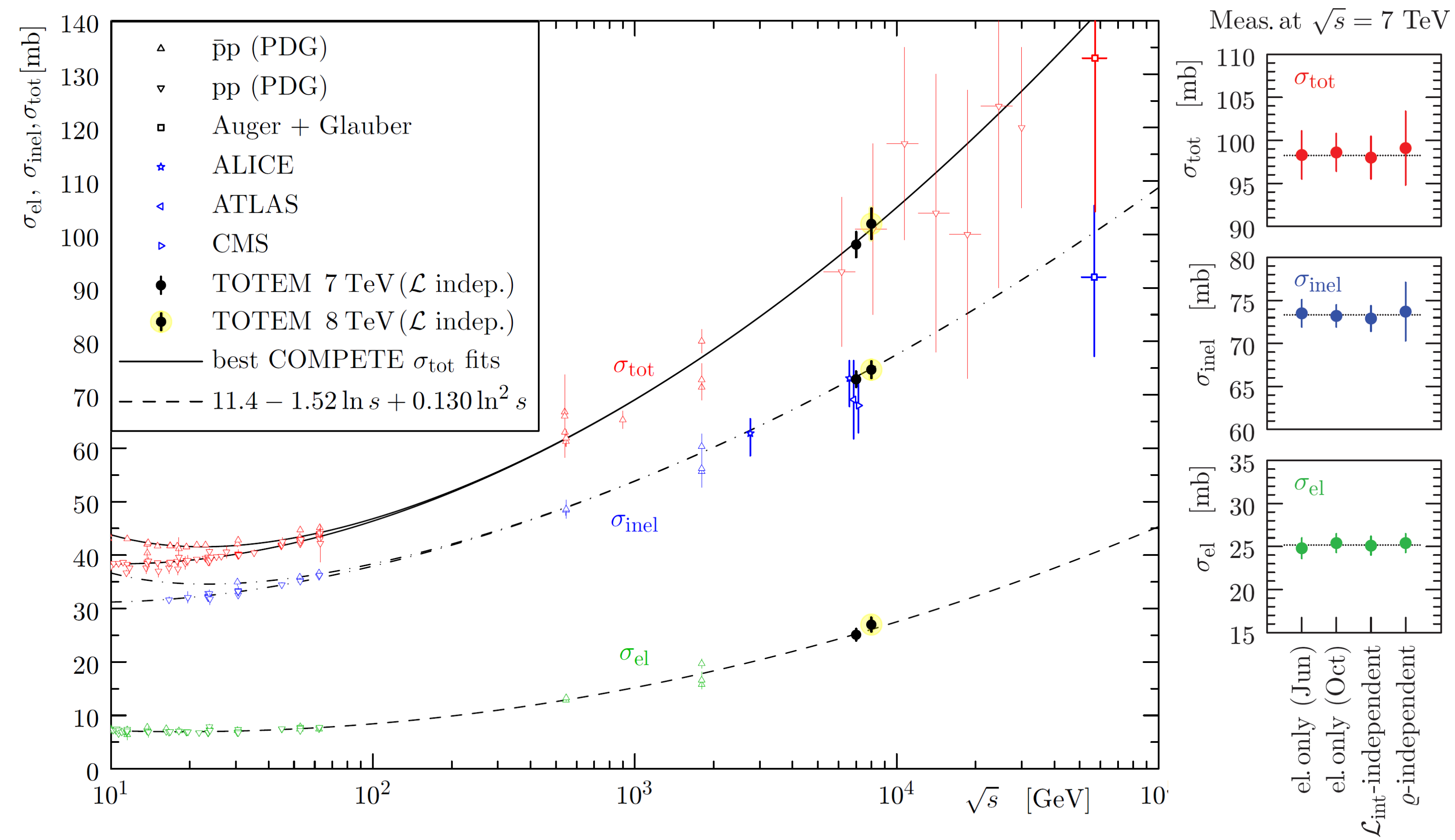
- **initial state** evolution
  factorization scheme and scale (FS)
- **hard scattering** of partons - depends on renormalization scheme, FS, fragmentation scheme P_F
- **factorization and hadronization** - final state parton showers (pQCD) + models for hadronization
- this factorization was proven only for certain processes (very inclusive, like DIS)
- it does no hold in general - diffractive final states, elastic scattering

Diffraction

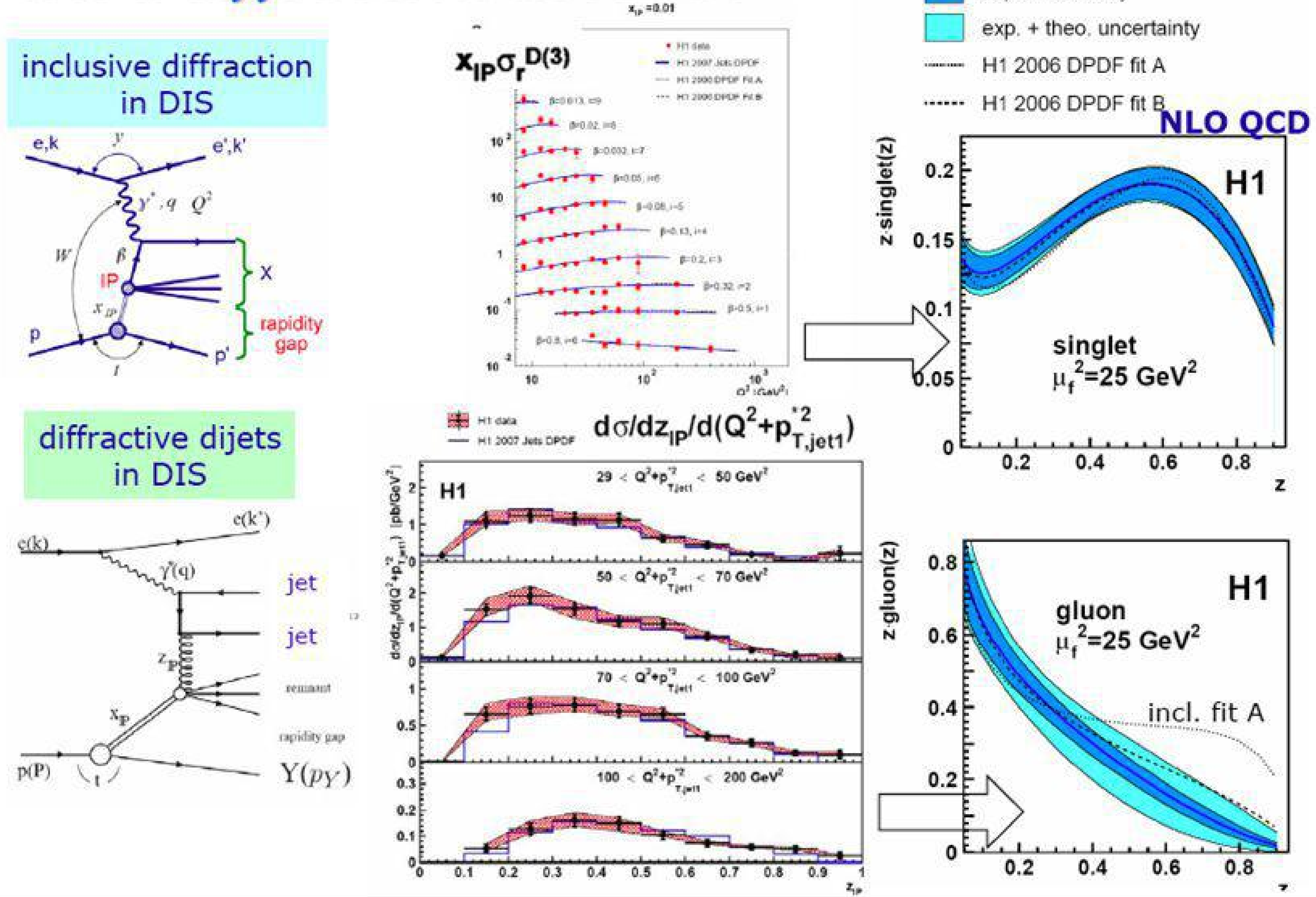
Elastic (~25 mb) | Single diffractive (~10 mb) | Double diffractive (~5 mb) | Central- diffractive (~1 mb)



- exchange of object with quantum numbers of vacuum
- description of elastic x-sec
  - **Pomeron** 2 gluon exchange
  - **Odderon** 3 gluons (ex. 5.5 - glueballs)
- diffractive events
  - large rapidity gaps
  - both - soft and **hard**



Hard diffraction at HERA

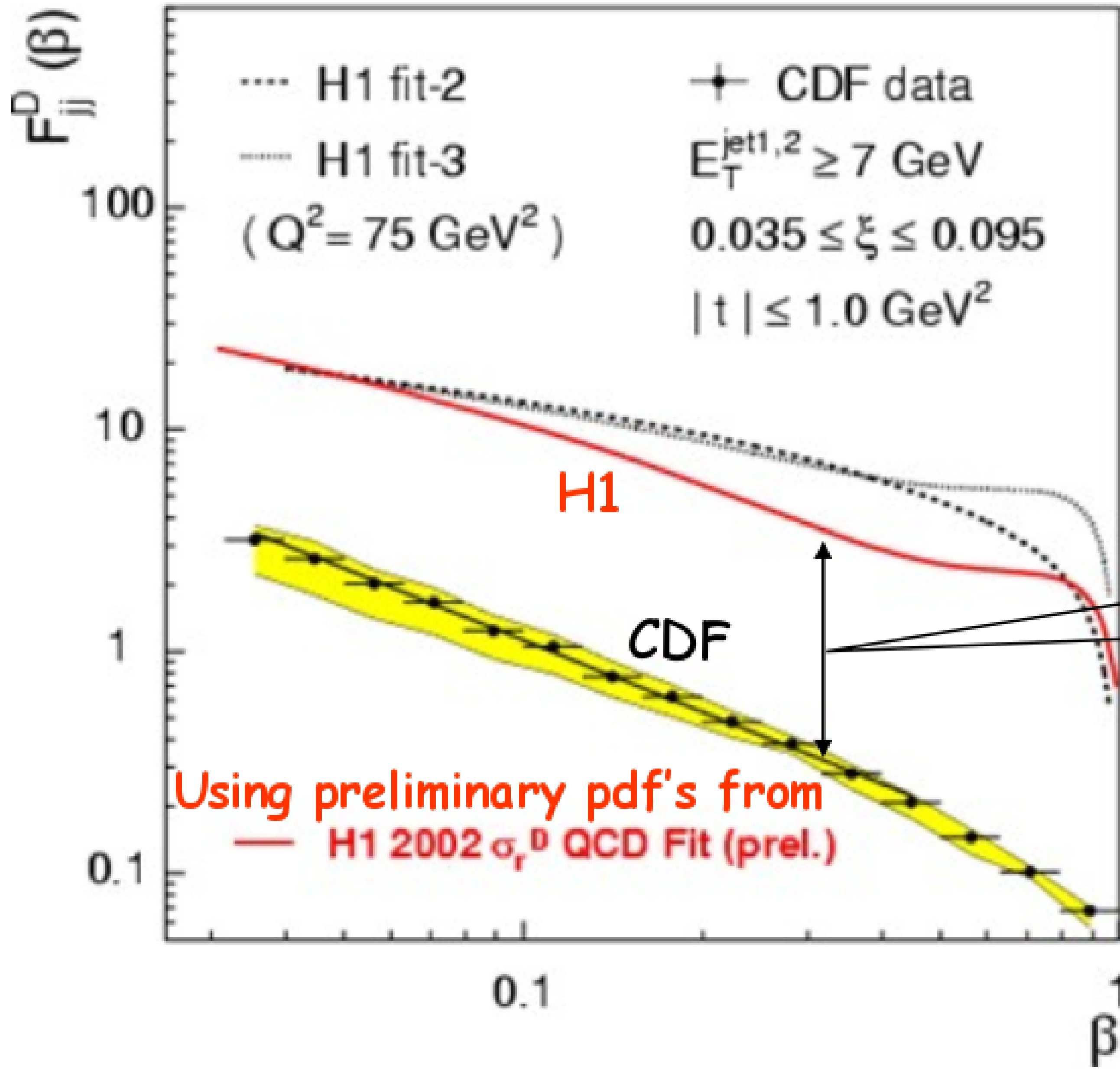


U.Klein / 9.2007

Max Klein HERA-LHeC Manchester 31/1/08

Diffractive Structure Function

Breakdown of QCD factorization

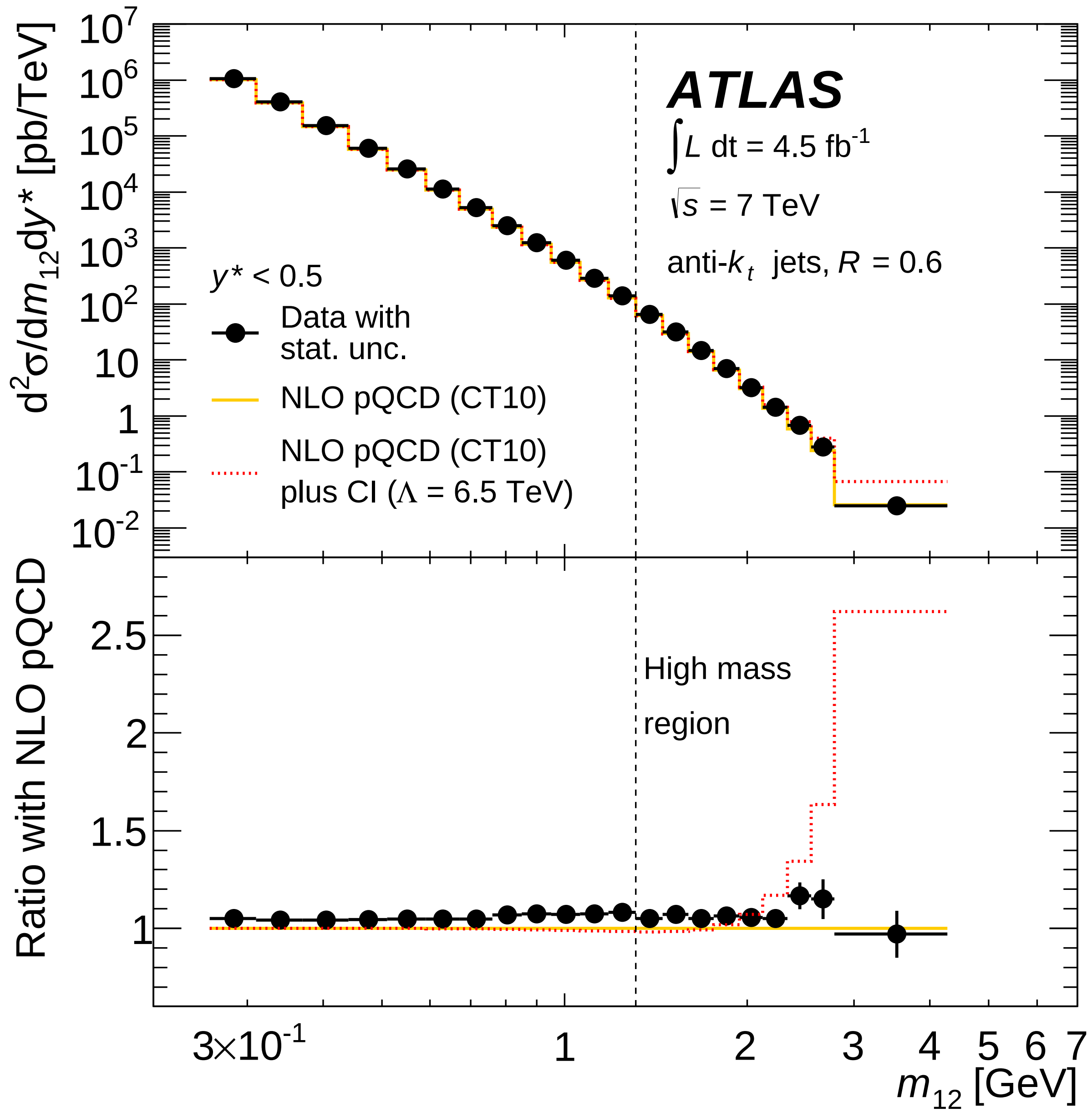


$$\bar{p}p \to \bar{p} + \text{dijet} + X$$

same suppression
as in soft diffraction - why?

momentum fraction of parton
in "Pomeron" - note quotes

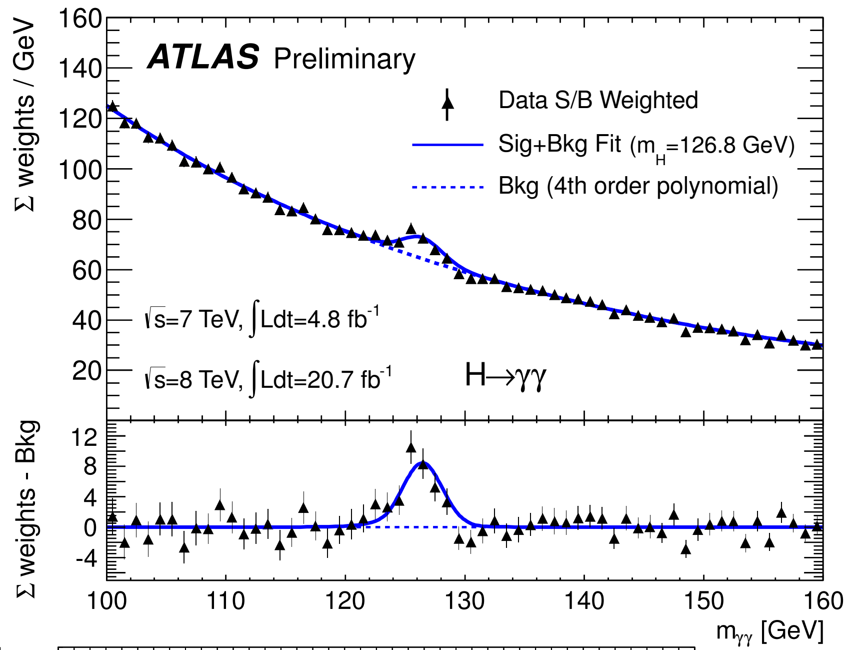
Collisions induced by strong interaction

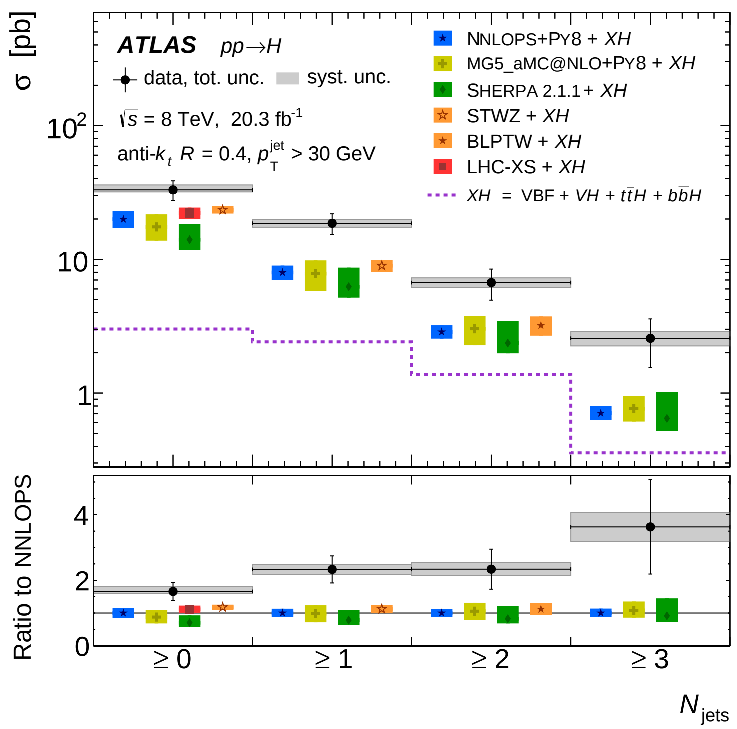


$$y^\star = \frac{|y_1 - y_2|}{2}$$

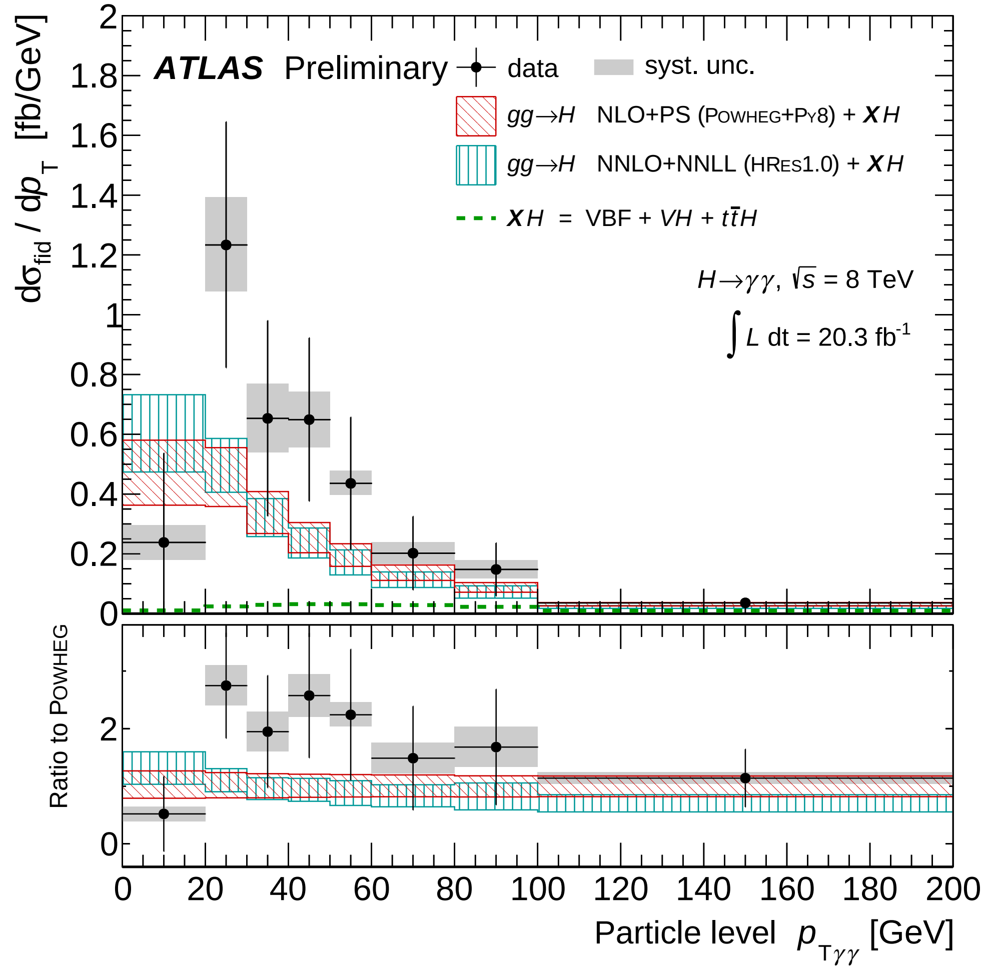
- collisions induced by strong interaction are the most violent ones at LHC (largest momentum exchange)
- we can the structure of matter and space-time at scales of several TeV ($\sim 1/10000$ of proton size)

Higgs boson at LHC

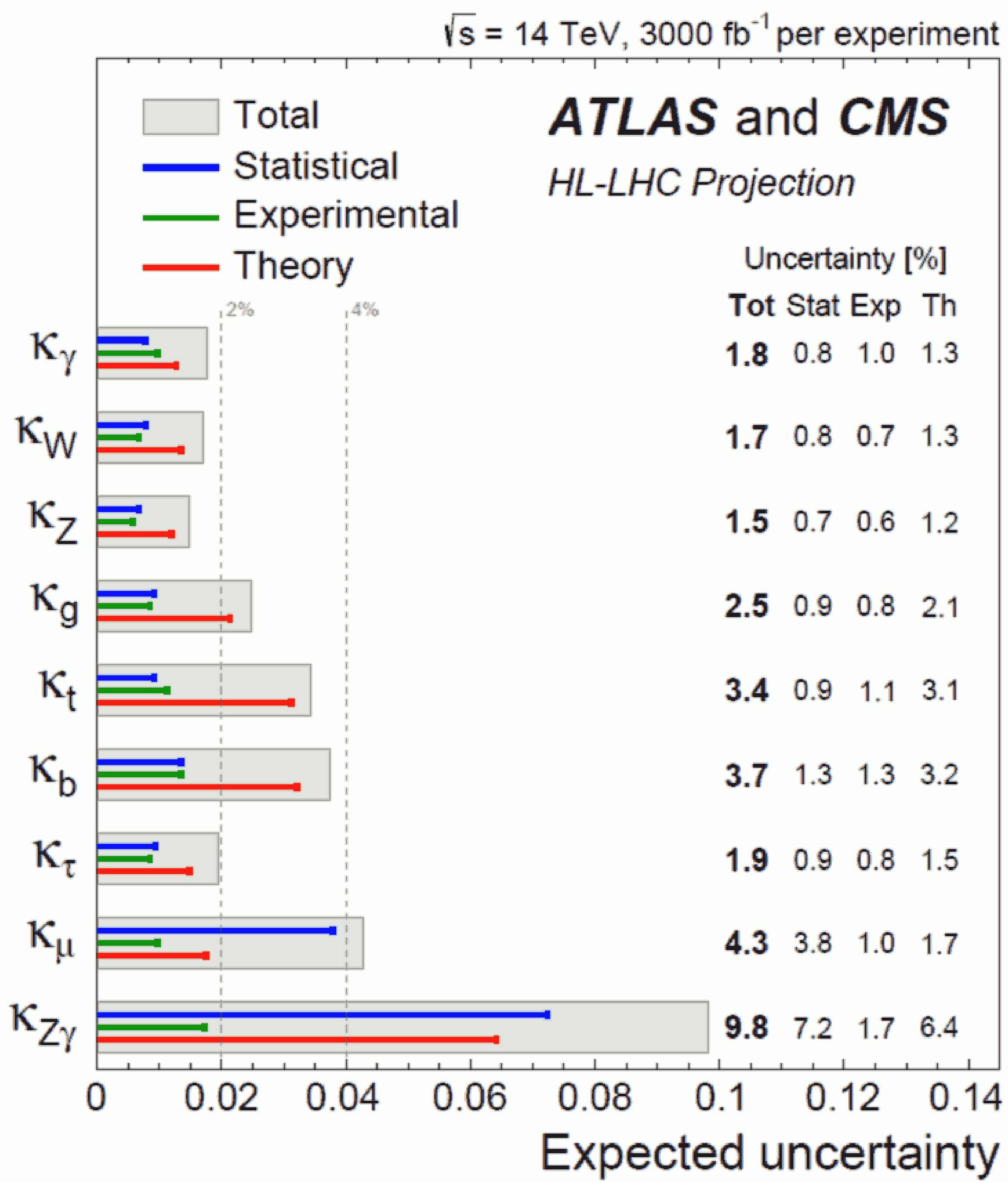


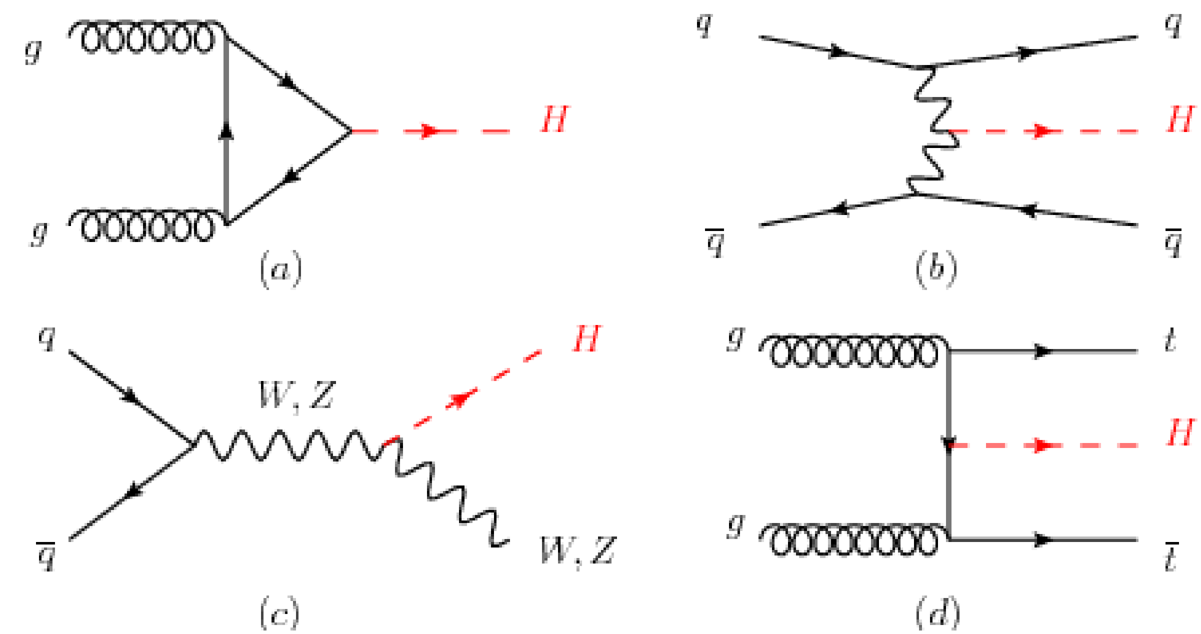


- even in channel $H \rightarrow \gamma\gamma$ - the Higgs production is associated with jets in more than half of the events
  - example of QCD NNLO+NNLL



Higgs boson at LHC





- theoretical uncertainty (mostly from QCD) - at HL-LHC, dominant error for many Higgs channels
  - uncertainty on PDF and α_S,
  - limited precision of calculations
    only(!) NNLO + NNLL

For reading and for exercises

- For reading (J. Chyla textbook)
  - chapter 10.2.2 (Splitting functions in QCD)
  - chapter 10.3 (Partons within hadrons) - subsections 10.3.1, 10.3.2, 10.3.3
  - chapter 10.1 (General framework)
- Exercise
  - exercise 8.2 (Virtual corrections to branching functions)

Exam requirements

- **Group Theory** - practical aspects of Lie groups
  - SU(2) a SU(3), generators (Pauli and Gell-Man matrices), structure coefficients, adjoint representation
  - basic irreducible multiplets of SU(2) and SU(3)
  - decomposition of direct product of repr. into irreducible multiplets, like $3 \otimes 3 = 6 \oplus \bar{3}$, etc.
  - Casimir operator $C = T^a T^a$ in SU(2) and SU(3)

- **Quark model of hadrons**
  - knowledge of at least two basic multiplets of baryons and mesons (baryon octet a decuplet + pseudoscalar a pseudovector meson octets)
  - wave functions of hadrons
  - applications - magnetic moments and hadron masses, Zweig's rule, other hadron properties (J^{CP}), decay channels, decay widths, etc.
  - Nambu model of color confinement

Exam requirements

- **Elastic scattering of electron on proton**
  - theory, including elastic formfactors
  - Hofstadter's experiment and proton size
- **Deep inelastic scattering**
  - kinematics, theory, including structure functions F_1 and F_2
  - first experiments at SLAC, Björken scaling
  - modifications to νDIS (structure function F_3)
- **Quark Parton Model**
  - formulation of model and its assumptions
  - basic properties of parton distribution functions (PDF) - low and high x behavior for valence quarks, see quarks and gluons
  - application to DIS (relation between PDFs and structure functions)
  - other applications: $e^+e^- \to$ hadrons, Drell-Yan
  - model of independent fragmentation

Exam requirements

- **QCD**
  - QCD Lagrangian as a Yang-Mills QFT, differences with respect to QED
  - ability to calculate basic QCD processes at tree level (Feynman rules, traces of color matrices)
  - running of α_S, beta function, solution
- **Mass singularities in QCD**
  - mass singularities in $e^+e^- \to q\bar{q}g$, splitting functions
  - cancellation of mass singularities and virtual singularities ($e^+e^- \to$ hadrons in NLO, KLN theorem)
  - Weizsacker-Williams approximation in DIS and splitting functions
  - clustering jet algorithms (JADE, k_T, anti-k_T)
  - DGLAP evolution equations - at least the derivation with one parton emission at the level of α_S
  - calculations in QCD - factorization theorem
  - determination of PDFs from experiments, global fits, gluons in DIS