Quarks, Partons, and Quantum Chromodynamics

Lecture 8

Quantum Chromodynamics

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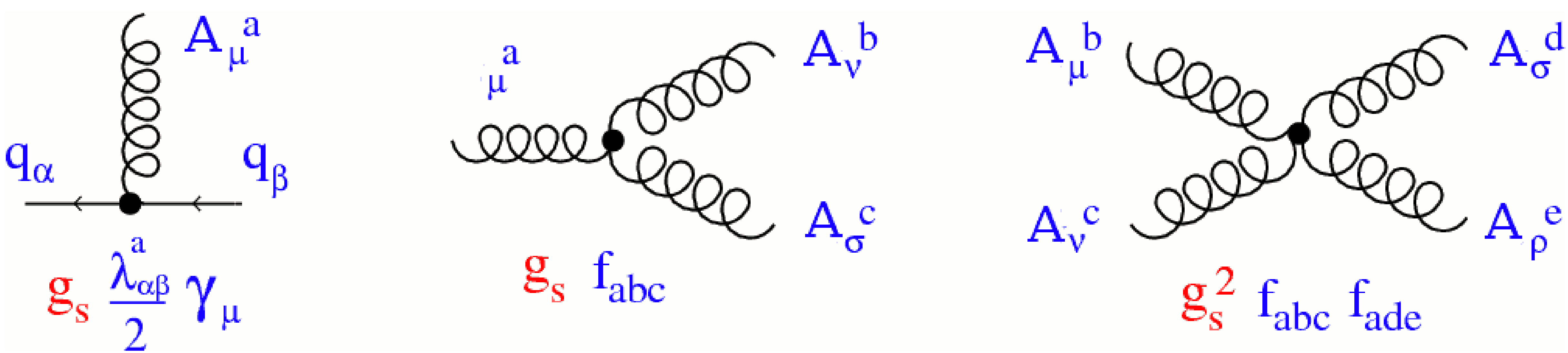
Quantum Chromodynamics - theory of strong interaction

- What kind of quantum field theory (if any) gives:

  confinement at large distances × with **asymptotic freedom** at small scales
- Gross, Wilczek, Politzer (1973) - this behaviour have non-abelian Yang-Mills Quantum Field Theories
- QCD - Yang-Mills QFT with local SU(3) gauge invariance, $\Psi' = \exp[ig\alpha_a(x)T_a]\Psi$

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \bar{\Psi}\left(i\gamma_\mu\mathcal{D}^\mu - m\right)\Psi$$

$$\mathcal{D}^\mu \equiv \partial^\mu - igT_aA^\mu_a\,, \quad A'^\mu_a = A^\mu_a + (1/g)\partial^\mu\alpha_a + gf_{abc}\alpha_bA^\mu_c$$
$$[\mathcal{D}_\mu, \mathcal{D}_\nu] = -igT^aG^a_{\mu\nu}\,,\; G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$



Quantum Electrodynamics

Classical Electrodynamics

(Maxwell 1861-1862)

$$\vec\nabla\cdot\vec E = \frac{\varrho}{\varepsilon_0},\quad \vec\nabla\cdot\vec B = 0,\quad \vec\nabla\times\vec E = -\frac{\partial\vec B}{\partial t},\quad \vec\nabla\times\vec B = \mu_0\,\vec j + \frac{1}{c^2}\frac{\partial\vec E}{\partial t},\qquad \big(\text{in next } c=1\big)$$

- Lorentz force $\vec F = q\left[\vec E + \vec v\times\vec B\right]$

- relativistic formulation in terms of 4-potential $A^\mu = (\varphi,\vec A)$, $\vec E = -\vec\nabla\varphi - \frac{\partial\vec A}{\partial t}$, $\vec B = \vec\nabla\times\vec A$

  equation of motion $\partial^\mu F_{\mu\nu} = \mu_0\, j_\nu,\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,\quad j_\mu = (\varrho,\vec j)$

  generated from $\mathcal{L}_{\text{EM}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + j^\mu A_\mu$ via E.-L. eq $\dfrac{\partial L}{\partial A_\mu} - \dfrac{\partial}{\partial x_\nu}\left(\dfrac{\partial\mathcal{L}}{\partial(\partial A_\mu/\partial x_\nu)}\right) = 0$

- if A^μ solves EoM, then $A'^\mu = A^\mu + \partial^\mu f$ solves it as well (same physics)

Quantum Electrodynamics

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar\Psi(i\,\partial\!\!\!/ - m)\Psi + j^\mu A_\mu$$

- current $j^\mu = e\bar\Psi\gamma^\mu\Psi$ generates right Lorentz force

E.-L. eq. for variables $(\Psi,\,\bar\Psi)$ gives $\quad [i\,\partial\!\!\!/ + e\gamma^\mu A_\mu - m]\,\Psi(x) = 0$

QED

- QED - QFT with U(1) local gauge invariance
- Weyl – interaction free QED has global invariance with respect to U(1): $\Psi' = e^{i\alpha}\Psi, \quad \bar{\Psi}' = \bar{\Psi}\, e^{-i\alpha}$
- What happens if we allow $\alpha(x)$?

$$\bar{\Psi}'(i\not{\partial} - m)\Psi' = \bar{\Psi}\, e^{-i\alpha(x)}(i\not{\partial} - m)e^{i\alpha(x)}\Psi = \bar{\Psi}(i\not{\partial} - m)\Psi - \bar{\Psi}\partial_\mu\alpha(x)\gamma^\mu\Psi$$

- to preserve local invariance, we have to compensate last term with interaction term of type

$\mathcal{L}_{\text{int}} = e\bar{\Psi}\gamma^\mu\Psi A_\mu,$ with gauge field transforming as $A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x)$ (same physics)

- principle of **local gauge invariance** dictates the form of interaction term
- this can be done quickly by replacing ∂_μ with **covariant derivation** $D_\mu = \partial_\mu - ieA_\mu$

transforms as Ψ: $D'_\mu\Psi'(x) = (\partial_\mu - ieA'_\mu)e^{i\alpha(x)}\Psi(x) = e^{i\alpha(x)}D_\mu\Psi(x) = e^{i\alpha(x)}D_\mu e^{-i\alpha(x)}\Psi'(x) \Rightarrow D'_\mu = e^{i\alpha}D_\mu e^{-i\alpha}$

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

- note that $F'_{\mu\nu} = F_{\mu\nu}$ $(\partial_\mu\partial_\nu\alpha = \partial_\nu\partial_\mu\alpha)$ $\Rightarrow$ whole $\mathcal{L}_{\text{QED}}$ is U(1) local gauge invariant
- covariant derivation can be used to generate $F_{\mu\nu}$ (check it!)

$$[D_\mu, D_\nu]\,\Psi = -ieF_{\mu\nu}\Psi$$

Quantization of $\mathcal{L}_{QED}$ - propagator

- propagator for field A_μ

$$0 = \partial_\mu F^{\mu\nu} = \partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = \left[\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu\right] A_\mu$$

- propagator $D_{\varrho\nu}(x-y)$ (Green function) is given by solution of

$$\left[\partial^2 g^{\mu\varrho} - \partial^\mu \partial^\varrho\right] D_{\varrho\nu}(x-y) = -i g^\mu_\nu \delta(x-y)$$

- solution in momentum space (Fourier transformation) - 16 linear equations

$$\left[-k^2 g^{\mu\varrho} + k^\mu k^\varrho\right] D_{\varrho\nu}(k) = -i g^\mu_\nu$$

- matrix $-k^2 g^{\mu\varrho} + k^\mu k^\varrho$ is singular, $[-k^2 g^{\mu\varrho} + k^\mu k^\varrho] k_\varrho = 0$
- gauge fixing conditions $f(A_\mu, \partial_\mu A_\nu) = 0$, technically by adding $\mathcal{L}_{\text{fix}} = -\frac{1}{2\alpha_G} f^2(A_\mu, \partial_\mu A_\nu)$
- **Lorenz (covariant) gauge** $\partial_\mu A^\mu(x) = 0 \Rightarrow$ EoM: $\partial_\mu F^{\mu\nu} + \frac{1}{\alpha_G}\partial_\nu \partial_\mu A^\mu = 0$ $\qquad \frac{\partial L}{\partial A_\nu} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)}\right) = 0$

$$\left[-k^2 g^{\mu\varrho} + \left(1 - \frac{1}{\alpha_G}\right) k^\mu k^\nu\right] D_{\varrho\nu}(k) = -i g^\mu_\nu$$

try to find $D_{\mu\nu}$, hint look for solution in form $D_{\mu\nu} = A g_{\mu\nu} + B k_\mu k_\nu$ and find functions A and B

$$D_{\mu\nu} = -\frac{i}{k^2}\left[g_{\mu\nu} - (1-\alpha_G)\frac{k_\mu k_\nu}{k^2}\right]$$

- **Axial gauge** $c_\mu A^\mu(x) = 0$ has propagator $\qquad D_{\mu\nu} = i\frac{d_{\mu\nu}}{k^2}, \quad d_{\mu\nu} = -g_{\mu\nu} + \frac{\alpha_G - k^2 - c^2}{(c\cdot k)^2} k_\mu k_\nu + \frac{k_\mu c_\nu + k_\nu c_\mu}{k \cdot c}$

Comments on quantization of $\mathcal{L}_{QED}$

- in **classical field theory**
  - adding $\mathcal{L}_{\text{fix}}$ is not completely correct
  - we can have additional solutions that satisfy $\mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{fix}}$ but not the gauge fixing condition
- in **quantum field theory**
  - the whole treatment is consistent
  - observables do not depend on choice of gauge fixing condition and choice α_G
  - proof is not trivial, it is related to the quantization of $\mathcal{L}$ (mostly via Feynman integral technique)
  - in addition, the requirements of renormalizable QFT (up to dimension 4 operators) + local guage invariance fixes (almost) unambiguously the interaction term and the whole Lagrangian

$$[\mathcal{L}] \ = [E]^4,\ [\Psi] = [E]^{3/2},\ [A^\mu] = [E]^1,\ [F^{\mu\nu}] = [E]^2$$

option $\mathcal{L} = \varepsilon_{\mu\nu\sigma\tau} F^{\mu\nu} F^{\sigma\tau}$ leads to renormalizable QFT - it gives zero contribution at all orders of perturbative series

QCD - Yang-Mills QFT with SU(3) local guage invariance

- quarks in 3 color states: $\Psi = \begin{pmatrix} \Psi_R \\ \Psi_G \\ \Psi_B \end{pmatrix}$, SU(3) tranformation $\Psi \to \Psi' = \exp[i\alpha_a T_a]\Psi$,
- T_a $(a = 1, \dots 8)$ generators of SU(3) fundamental representation (color matrices)
- $\mathcal{L}_{\text{free}} = \bar{\Psi}[i\,\partial\!\!\!/ - m]\Psi \quad \to \quad \mathcal{L}'_{\text{free}} = \mathcal{L}_{\text{free}} - \bar{\Psi}\gamma^\mu T_a \Psi \partial_\mu \alpha_a \quad \to \quad \mathcal{L}_{\text{int}} = g\bar{\Psi}\gamma^\mu T_a \Psi A^a_\mu$

$$D_\mu = \partial_\mu - igT_a A^a_\mu$$

- $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$ has global SU(3) invariance if A^a_μ transforms as adjoint repr.

$$A^a_\mu \to A'_\mu = \exp[i\alpha_b F_b]A_\mu, \quad (F^a)_{bc} \equiv -if_{abc}, \qquad \text{for } \alpha_b \ll 1 \quad A'^a_\mu = A^a_\mu + \alpha_b f_{bac} A^c_\mu \quad \text{(ex. 3.17)}$$

- for local guage invariance we find $A'^a_\mu = A^a_\mu + \alpha_b f_{bac} A^c_\mu + \frac{1}{g}\partial_\mu \alpha^a$
- non-abelian character of SU(3) (some f_{abc} are non-zero) $\Rightarrow F'^{\mu\nu}_a F'^a_{\mu\nu} \neq F^{\mu\nu}_a F^a_{\mu\nu}$
- use $G^{\mu\nu}_a$ defined by $[D_\mu, D_\nu]\,\Psi = -igT^a G^a_{\mu\nu}\Psi$

$$\text{prove } D'_\mu = SD_\mu S^{-1}, \text{ with } S = e^{i\alpha_a T_a} = 1 + i\alpha_a T_a \quad \Rightarrow \quad [D'_\mu, D'_\nu] = S[D_\mu, D_\nu]S^{-1}$$

$$G^{\mu\nu}_a G^a_{\mu\nu} = 2\text{Tr}[T^a G^a_{\mu\nu} T_b G^{\mu\nu}_b] \quad \text{which follows from } \text{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}$$

$$-g^2 G'^{\mu\nu}_a G'^a_{\mu\nu} = -2g^2\text{Tr}(T_a G'^{\mu\nu}_a T^b G'^b_{\mu\nu}) = 2\text{Tr}([D'^\nu, D'^\nu][D'_\mu, D'_\nu]) = 2\text{Tr}(S[D^\mu, D^\nu][D_\mu, D_\nu]S^{-1}) = -g^2 G^{\mu\nu}_a G^a_{\mu\nu}$$

Gluon field Lagrangian

$$[D_\mu, D_\nu]\,\Psi = -igT^a G^a_{\mu\nu}\Psi$$

$$D_\mu D_\nu \Psi = (\partial_\mu - igT_a A^a_\mu)(\partial_\nu - igT_b A^b_\nu)\Psi = [\partial_\mu\partial_\nu - igT_b(\partial_\mu A^b_\nu) - igT_b A^b_\nu \partial_\mu - igT_a A^a_\mu \partial_\nu - g^2 T_a T_b A^a_\mu A^b_\nu]\Psi$$

$$D_\nu D_\mu = \partial_\nu\partial_\mu - igT_b(\partial_\nu A^b_\mu) - igT_a A^a_\mu \partial_\nu - igT_b A^b_\nu \partial_\mu - g^2 T_a T_b A^a_\nu A^b_\mu \quad (\mu \leftrightarrow \nu,\ \text{sometimes } a \leftrightarrow b)$$

$$[D_\mu, D_\nu] = -igT^a F^a_{\mu\nu} - g^2[T_a, T_b]A^a_\mu A^b_\nu = -igT^a F^a_{\mu\nu} - ig^2 f_{abc} T_c A^a_\mu A^b_\nu$$

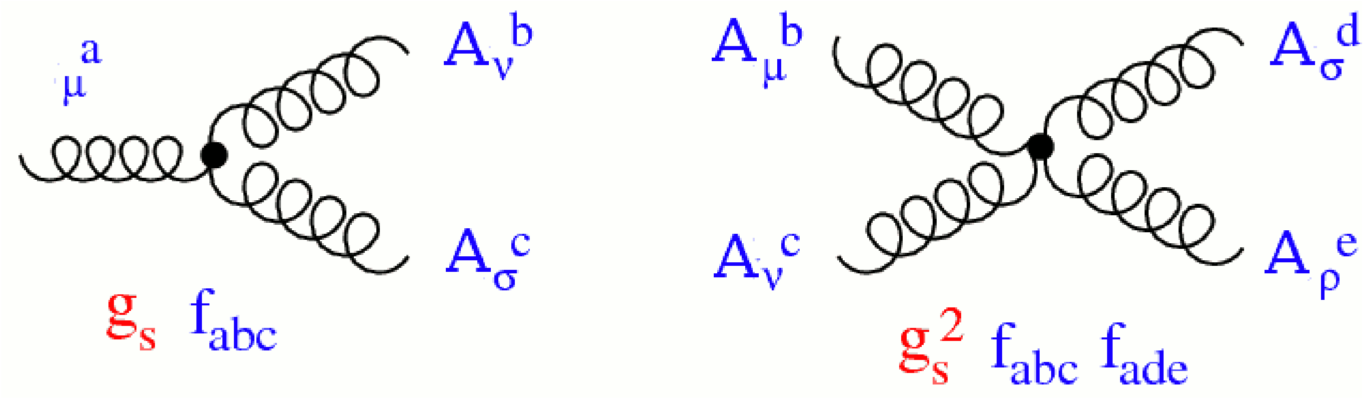
$$= -igT^a F^a_{\mu\nu} - ig^2 f_{bca} T_a A^b_\mu A^c_\nu = -igT_a\left(F^a_{\mu\nu} + gf_{abc}A^b_\mu A^c_\nu\right)$$

$$G^a_{\mu\nu} = F^a_{\mu\nu} + gf_{abc}A^b_\mu A^c_\nu$$

- gluon field Lagrangian preserving local guage SU(3) invariance $\mathcal{L}_{\text{gluon}} = -\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu}$

$$\mathcal{L}_{\text{gluon}} = -\frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu} - \frac{1}{2}gf_{abc}F^{\mu\nu}_a A^b_\mu A^c_\nu - \frac{1}{4}g^2 f_{abc}f_{ade}A^b_\mu A^c_\nu A^\mu_d A^\nu_e$$

- unlike in QED, the gluons interact with themselves



Feynman rules for pQCD - propagators

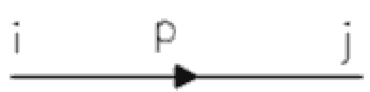
Rules for external particles:

- $u(p,s)$ for each incoming fermion with momentum p and spin s,
- $\overline{v}(p,s)$ for each incoming antifermion with momentum p and spin s,
- $\overline{u}(p,s)$ for each outgoing fermion with momentum p and spin s,
- $v(p,s)$ for each outgoing antifermion with momentum p and spin s.

Statistical factors in loops:

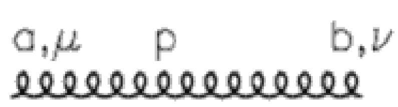
- -1 for each closed fermion loop.
- $1/2!$ for each gluon loop with two internal gluons.
- $1/3!$ for each gluon loop with three internal gluons.

Rules for propagators



$$\delta^{ij}\frac{i}{(\not p-m+i\varepsilon)}$$

$$\delta^{ab}\frac{i}{(p^2+i\varepsilon)}$$ ghost propagator in loops only



$$\delta^{ab}\frac{i}{(p^2+i\varepsilon)}\left[-g_{\mu\nu}+(1-\alpha_G)\frac{p_\mu p_\nu}{(p^2+i\varepsilon)}\right]$$

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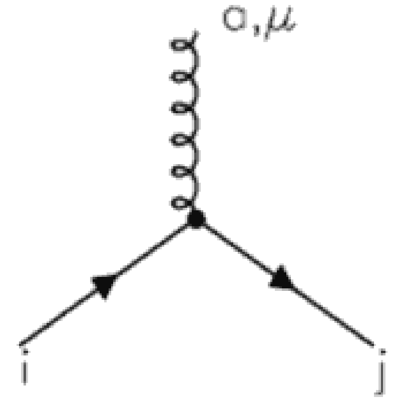
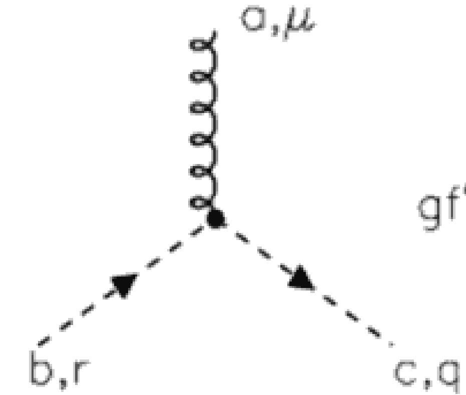
- factor -1 for
  a) exchange of two identical final state fermions
  b) or exchange $f \leftrightarrow \bar{f}$ in initial $\leftrightarrow$ final state
- Fadeev-Poppov ghosts
  - result of quantization
  - appear in certain gauges (not in axial gauge)
  - appear only in loops

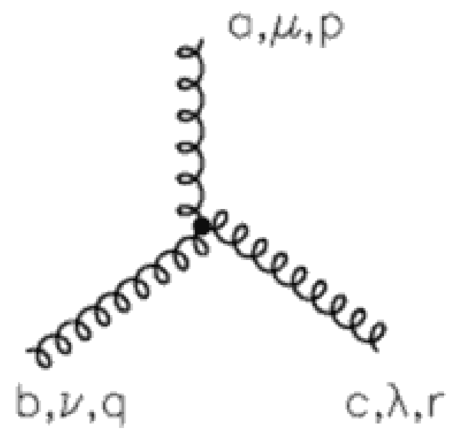
Feynman rules for pQCD - vertices



$-ig(T^a)_{ji}\gamma_\mu$



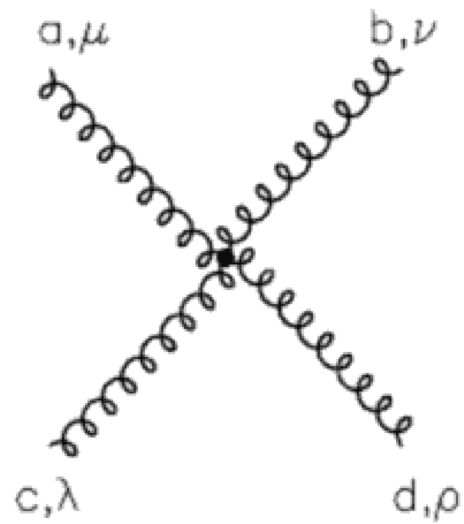
$gf^{abc}r^\mu$



all momenta ~~outgoing~~ incoming

$-gf^{abc}C^{\mu\nu\lambda}(p,q,r)$

$C^{\mu\nu\lambda}(p,q,r)=g^{\mu\nu}(p-q)^\lambda+g^{\nu\lambda}(q-r)^\mu+g^{\lambda\mu}(r-p)^\nu$



$-ig^2f^{eac}f^{ebd}(g^{\mu\nu}g^{\lambda\rho}-g^{\mu\rho}g^{\nu\lambda})$

$-ig^2f^{ead}f^{ebc}(g^{\mu\nu}g^{\lambda\rho}-g^{\mu\lambda}g^{\nu\rho})$

$-ig^2f^{eab}f^{ecd}(g^{\mu\lambda}g^{\nu\rho}-g^{\mu\rho}g^{\nu\lambda})$

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Color matrices

(example 6.1 and Appendix C)

- starting point: $T_a T_b = \frac{1}{6}\delta_{ab}\mathbb{1} + \frac{1}{2}(d_{abc} + i f_{abc}) T_c$

  with anti-symmetric f_{abc} and symmetric d_{abc} defined via $[T_a, T_b] = i f_{abc} T_c$ and $\{T_a, T_b\} = \frac{1}{3}\delta_{ab}\mathbf{1} + d_{abc} T_c$
- compare with $\sigma_i \sigma_j = \delta_{ij}\mathbb{1} + i\varepsilon_{ijk}\sigma_k$ for the SU2 case

- most common traces

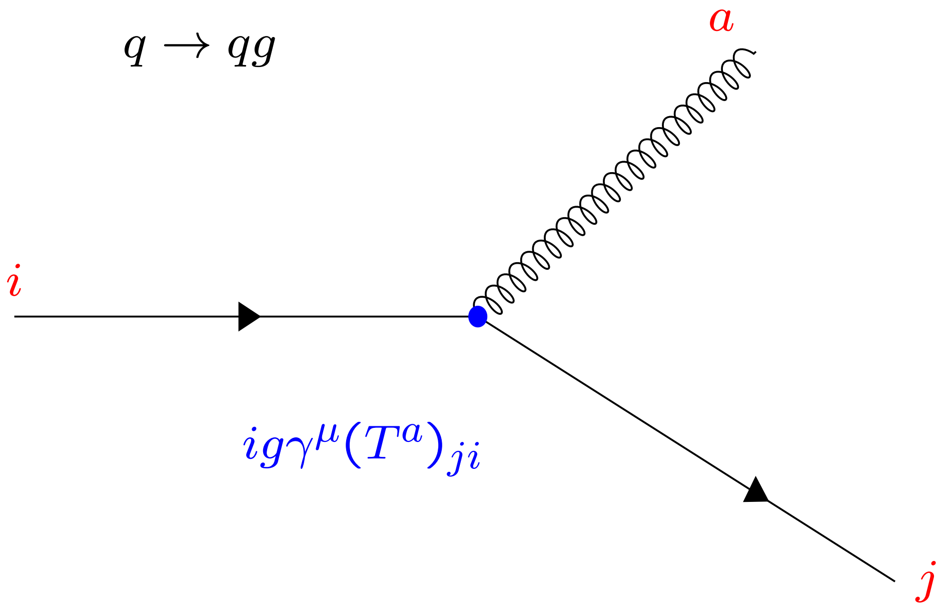
$$\mathrm{Tr}(T^a T^{b\dagger}) = \mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta_{ab}$$
$$\mathrm{Tr}(T^a T^b T^c) = \frac{1}{4}(d_{abc} + i f_{abc})$$
$$\mathrm{Tr}(T^a T^b T^a T^c) = -\frac{1}{12}\delta_{bc}$$

- other most common formulae

$$d_{abc} f_{abd} = 0$$
$$f_{abc} f_{abd} = \mathrm{Tr}(F_c F_d) = 3\delta_{cd}$$
$$d_{abc} d_{abd} = \frac{5}{3}\delta_{cd}$$

example 6.2

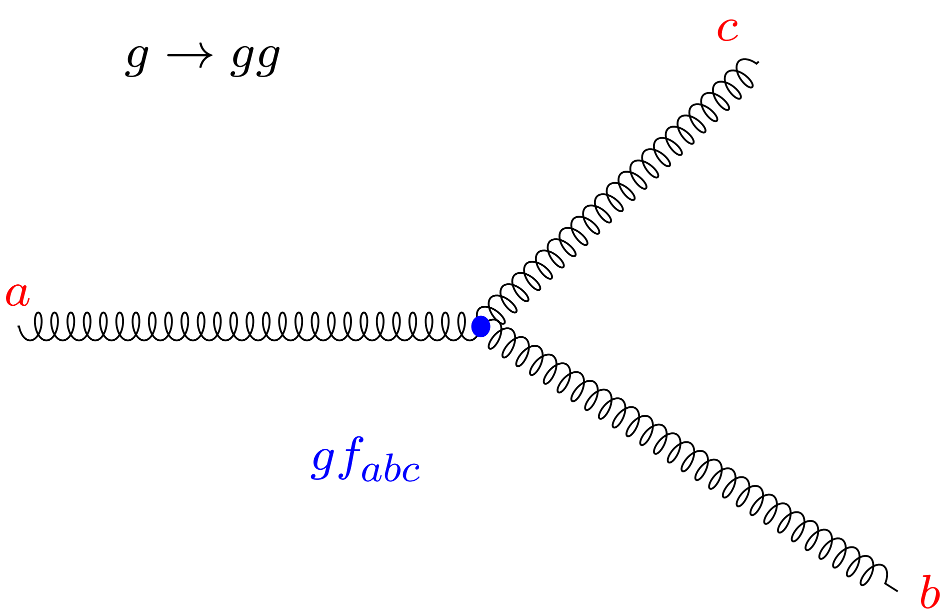
$q \to qg$



- color factor in $\overline{|\mathcal{M}|}^2$

$$C_{q\to qg} = \frac{1}{3}T^a_{ji}(T^a_{ji})^\star = \frac{1}{3}T^a_{ji}(T^a_{ji})^T = \frac{1}{3}T^a_{ji}T^a_{ij}$$
$$= \frac{1}{3}\mathsf{Tr}(T^aT^a) = \frac{1}{3}\cdot\frac{1}{2}\delta_{aa} = \frac{8}{6} = \frac{4}{3}$$
$$= \frac{1}{3}\mathsf{Tr}(T^aT^a) = \frac{1}{3}\mathsf{Tr}(C_F\mathbf{1}_3) = C_F$$

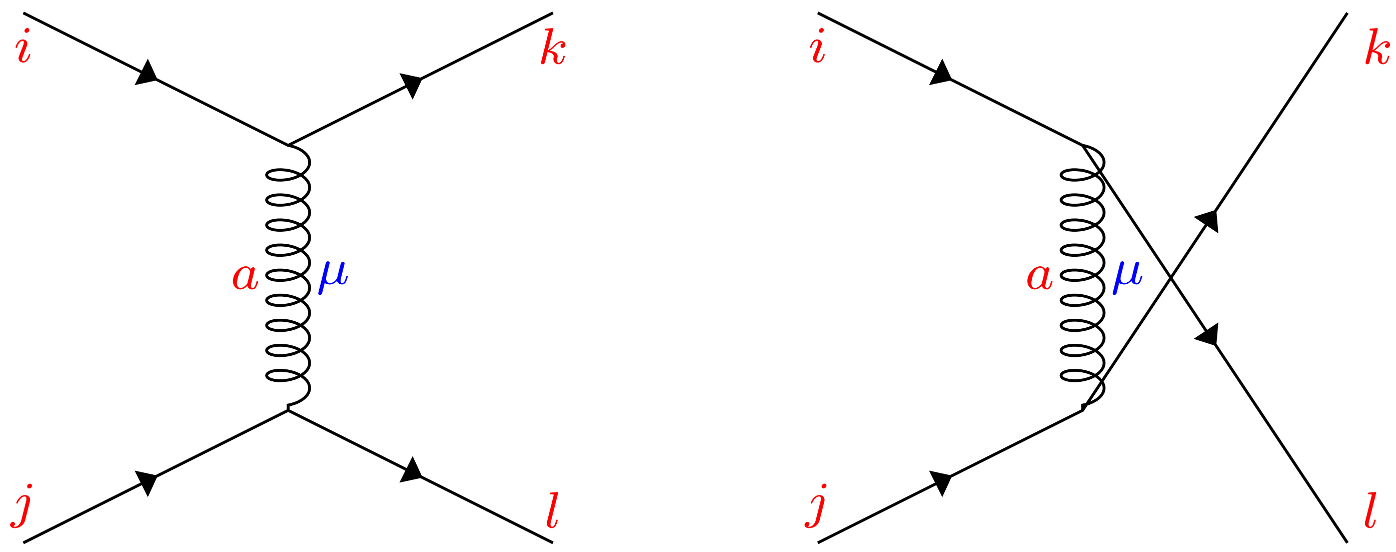
$g \to gg$



generators of adjoint repr.: $(F^a)_{bc} = -if_{abc}$

$$C_{g\to gg} = \frac{1}{8}f_{abc}f_{abc} = -\frac{1}{8}f_{abc}f_{acb} = \frac{1}{8}(-if_{abc})(-if_{acb})$$
$$= \frac{1}{8}\mathsf{Tr}(F^aF^a) = \frac{1}{8}\mathsf{Tr}(C_A\mathbf{1}_8) = C_A = 3$$

Elementary QCD processes - $uu \to uu$ (ex. 6.3)



- process similar to QED $ee \to ee$ scattering

$$\overline{|\mathcal{M}_{\mathsf{QED}}|}^2 = 2e^4 \left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + 2\frac{s^2}{tu} \right)$$

- try to derive the QED result by yourself

- in QCD, there are in addition just color factors from $q \to qg$ vertices

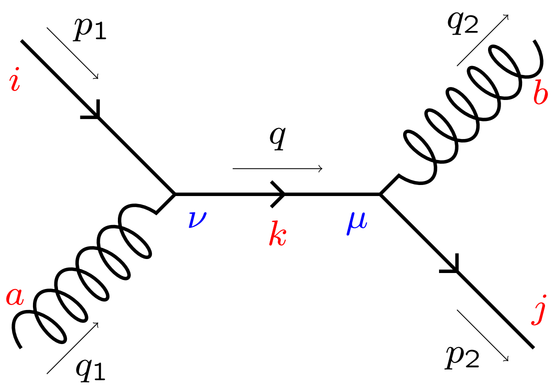
$$C_t = \frac{1}{9} T^a_{ki} T^a_{lj} (T^b_{ki} T^b_{lj})^\star = \frac{1}{9} T^a_{ki} T^b_{ki} T^a_{lj} T^b_{jl} = \frac{1}{9} \mathsf{Tr}(T^a T^b) \mathsf{Tr}(T^a T^b) = \frac{1}{9} \cdot \frac{1}{2}\delta_{ab} \frac{1}{2}\delta_{ab} = \frac{1}{4\cdot 9}\delta_{aa} = \frac{8}{4\cdot 9} = \frac{2}{9}$$

$$C_u = \frac{1}{9} T^a_{li} T^a_{kj} (T^b_{kj} T^b_{li})^\star = C_t$$

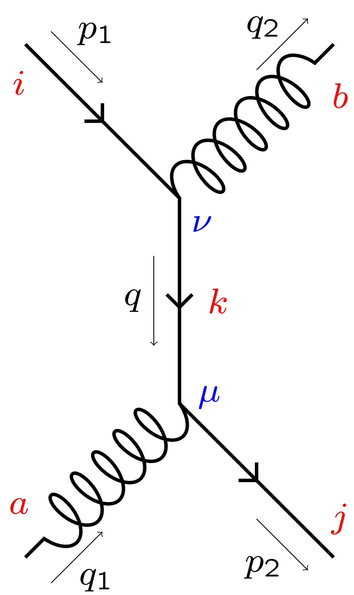
$$C_{tu} = \frac{1}{9} T^a_{ki} T^a_{lj} (T^b_{li} T^b_{kj})^\star = \frac{1}{9} T^a_{ki} T^b_{il} T^a_{lj} T^b_{jk} = \frac{1}{9} \mathsf{Tr}(T^a T^b T^a T^b) = \frac{1}{9}\left(-\frac{1}{12}\right)\delta_{bb} = -\frac{8}{9\cdot 12} = -\frac{2}{9}\cdot\frac{1}{3}$$

- this leads to

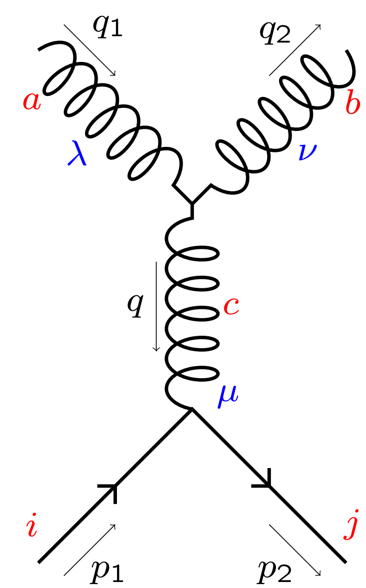
$$\overline{|\mathcal{M}_{\mathsf{QCD}}|}^2 = 2g^4 \frac{2}{9} \left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + \left(-\frac{1}{3}\right) 2\frac{s^2}{tu} \right)$$



$\mathcal{M}_s$



$\mathcal{M}_u$



$\mathcal{M}_t$

QED

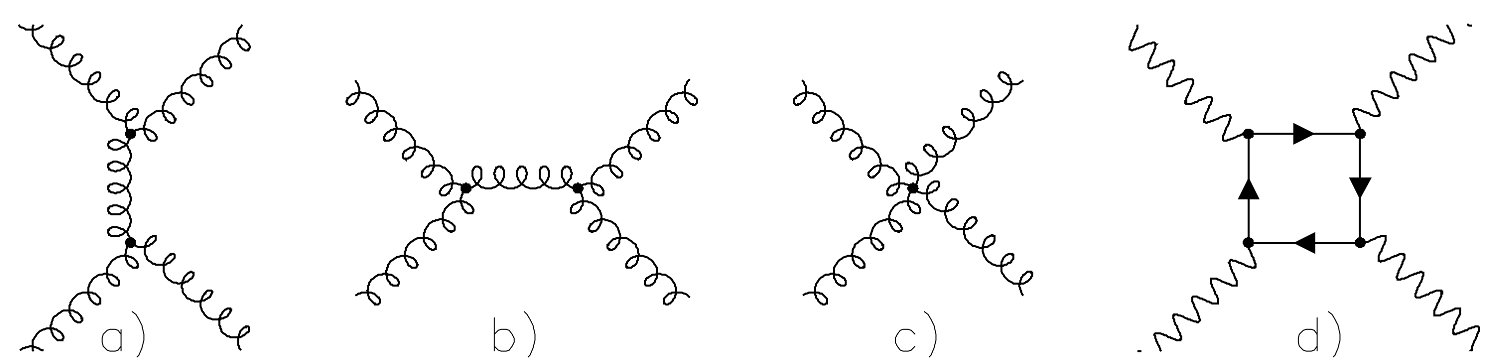
- only $\mathcal{M}_s$ and $\mathcal{M}_u$
- $\mathcal{M} = \mathcal{M}_s + \mathcal{M}_u$ vanishes for longitudinaly polarized outgoing photon $\varepsilon_\mu(q_2) = K q_{2\mu}$

QCD - three tree level processes

- show that $\mathcal{M} = \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t$ vanishes for longitudinaly polarized outgoing gluon (ex. 6.4)

Gluon-gluon scattering

- no tree level contribution in QED
- in QCD, the most strong process



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Quark and gluon scattering

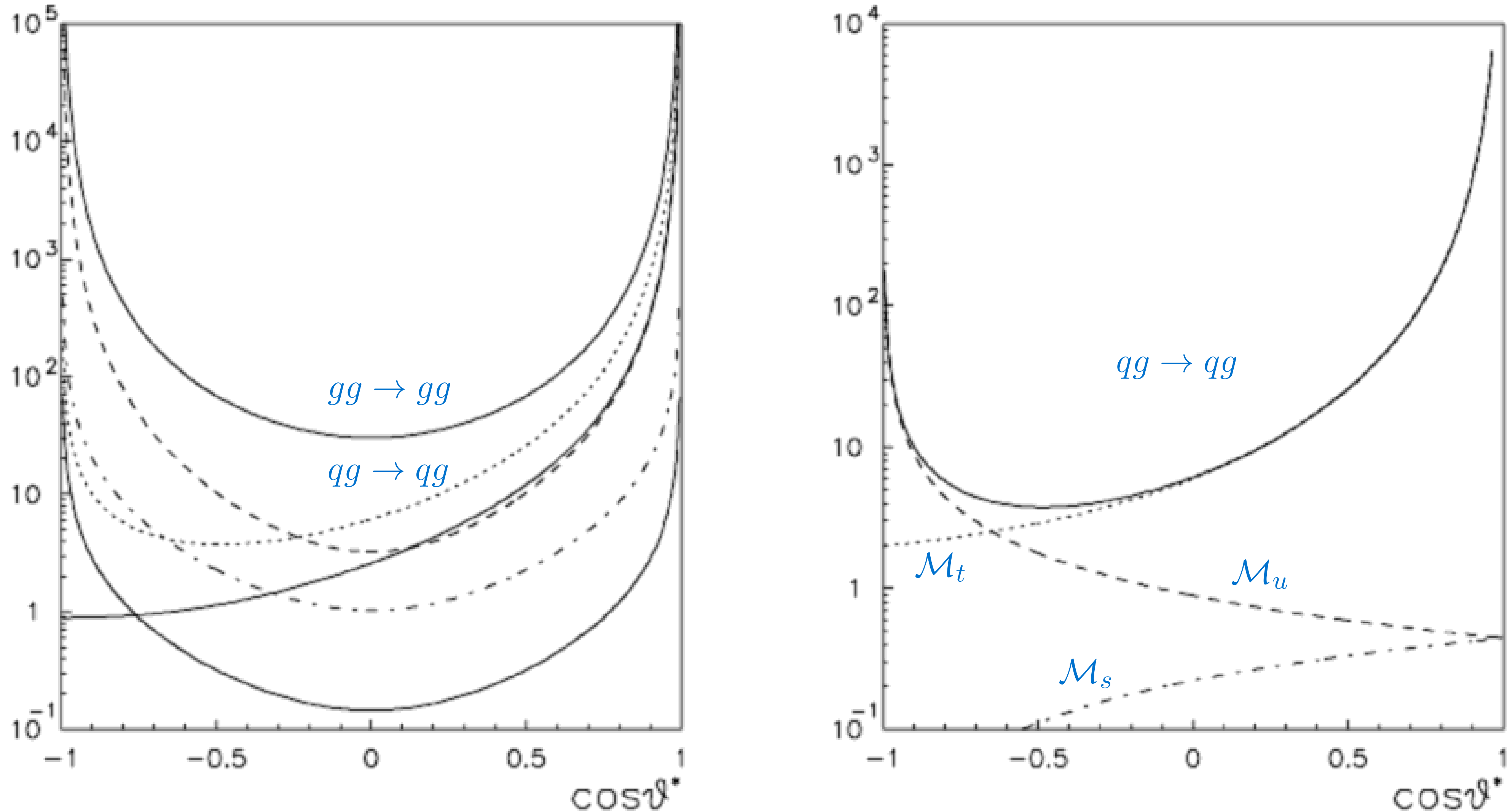


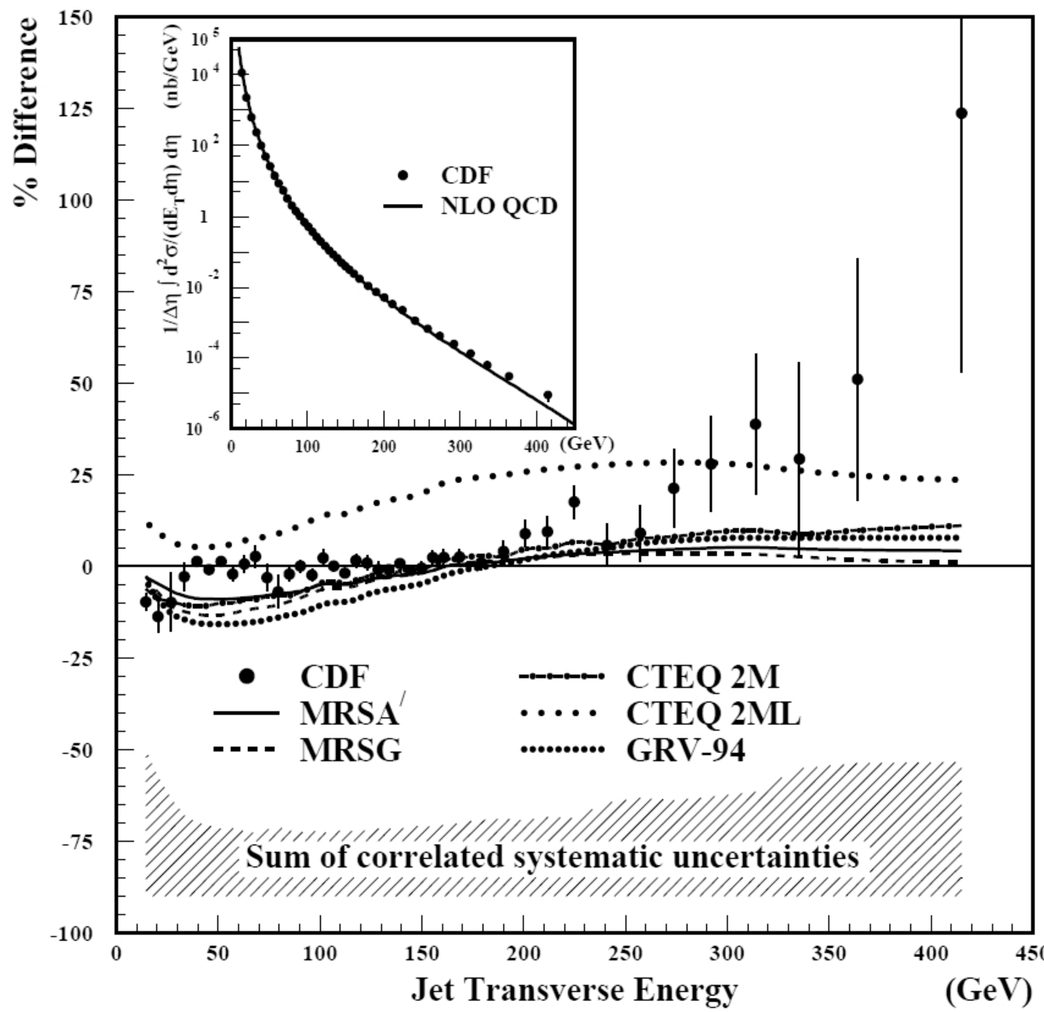
Figure 7.4: a): $\langle | M |^2 \rangle / g^4$ plotted for different 2 body processes as given in the Table below. The curves correspond, in decreasing order of their values for $\cos\vartheta = 0.5$, to $gg \to gg$, $qg \to qg$, $q\overline{q} \to q\overline{q}$, $qq \to qq$, $q\overline{q} \to gg$, $gg \to q\overline{q}$;b): Individual contributions of M_s (dash-dotted line), M_u (dashed line) and M_t (dotted line) to the full $qg \to qg$ cross-section (solid line)

Test QCD - jet production - 1996 (CDF experiment)

- *Rutherford experiment* in $p\bar{p}$ collisions

$$\frac{\mathrm{d}\sigma_{\mathsf{QPM}}}{\mathrm{d}p_{\mathsf{T}}} = \sum_{a,b,c,d}^{q,\bar{q},g} \int \mathrm{d}x_1 \mathrm{d}x_2\, d_a(x_1) d_b(x_2) \sigma_{ab\to cd} \left[\delta(p_{\mathsf{T}} - p_{\mathsf{T}}^c) + \delta(p_{\mathsf{T}} - p_{\mathsf{T}}^d)\right]$$

- CDF experiment on Tevatron reported much more high p_T quarks scattered at large angles $\eta \sim 0$ ($\vartheta \sim 90°$)



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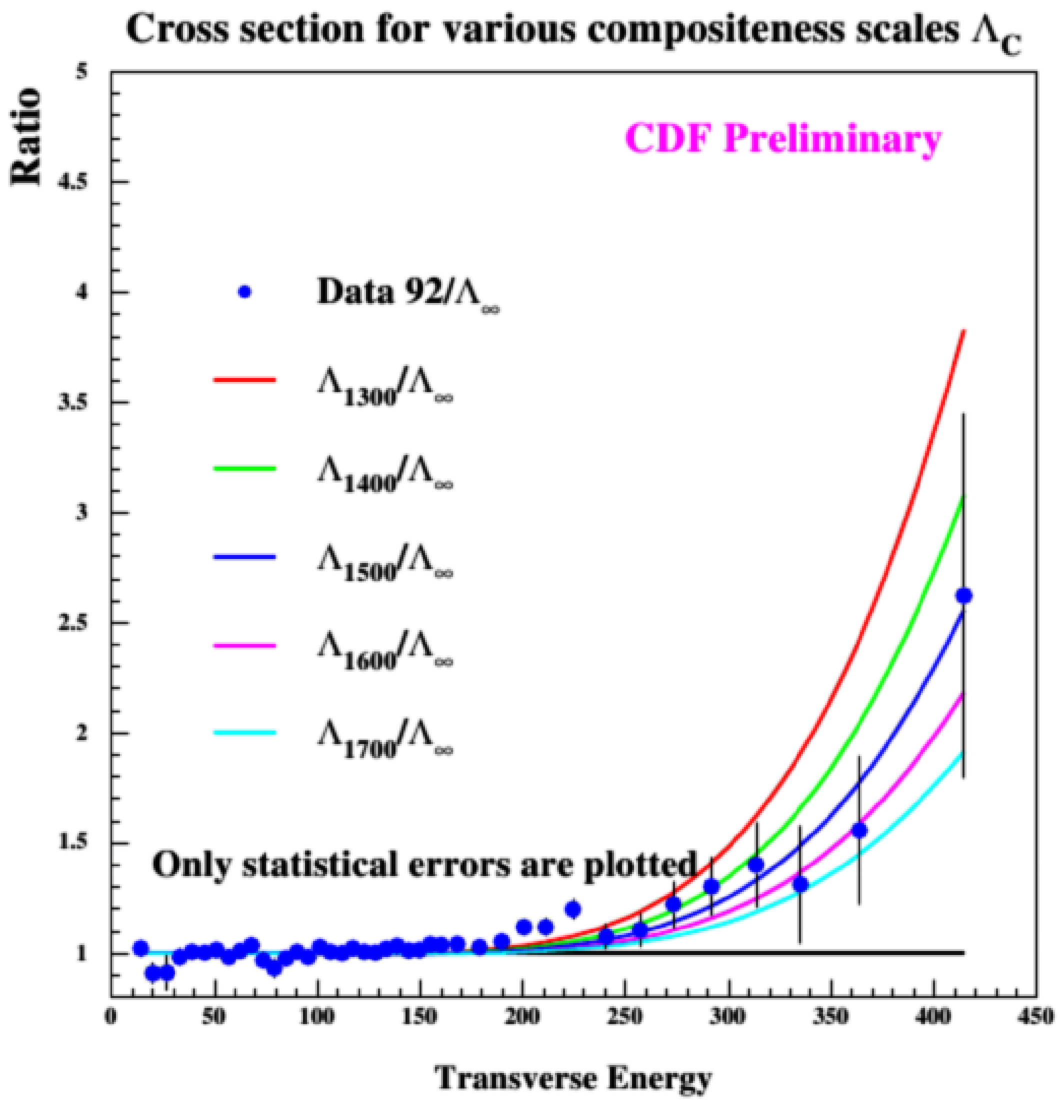
Test QCD - jet production - 1996 (CDF experiment)

- *Rutherford experiment* in $p\bar{p}$ collisions

$$\frac{\mathrm{d}\sigma_{\mathrm{QPM}}}{\mathrm{d}p_{\mathrm{T}}} = \sum_{a,b,c,d}^{q,\bar{q},g} \int \mathrm{d}x_1 \mathrm{d}x_2\, d_a(x_1) d_b(x_2) \sigma_{ab\to cd} \left[\delta(p_{\mathrm{T}} - p_{\mathrm{T}}^c) + \delta(p_{\mathrm{T}} - p_{\mathrm{T}}^d)\right]$$

- CDF experiment on Tevatron reported much more high p_T quarks scattered at large angles $\eta \sim 0$ ($\vartheta \sim 90°$)
- CDF interpreted the data as a possible sign of new interaction between quarks
- other, less intriguing possibility, was that there are in proton more gluons at high x

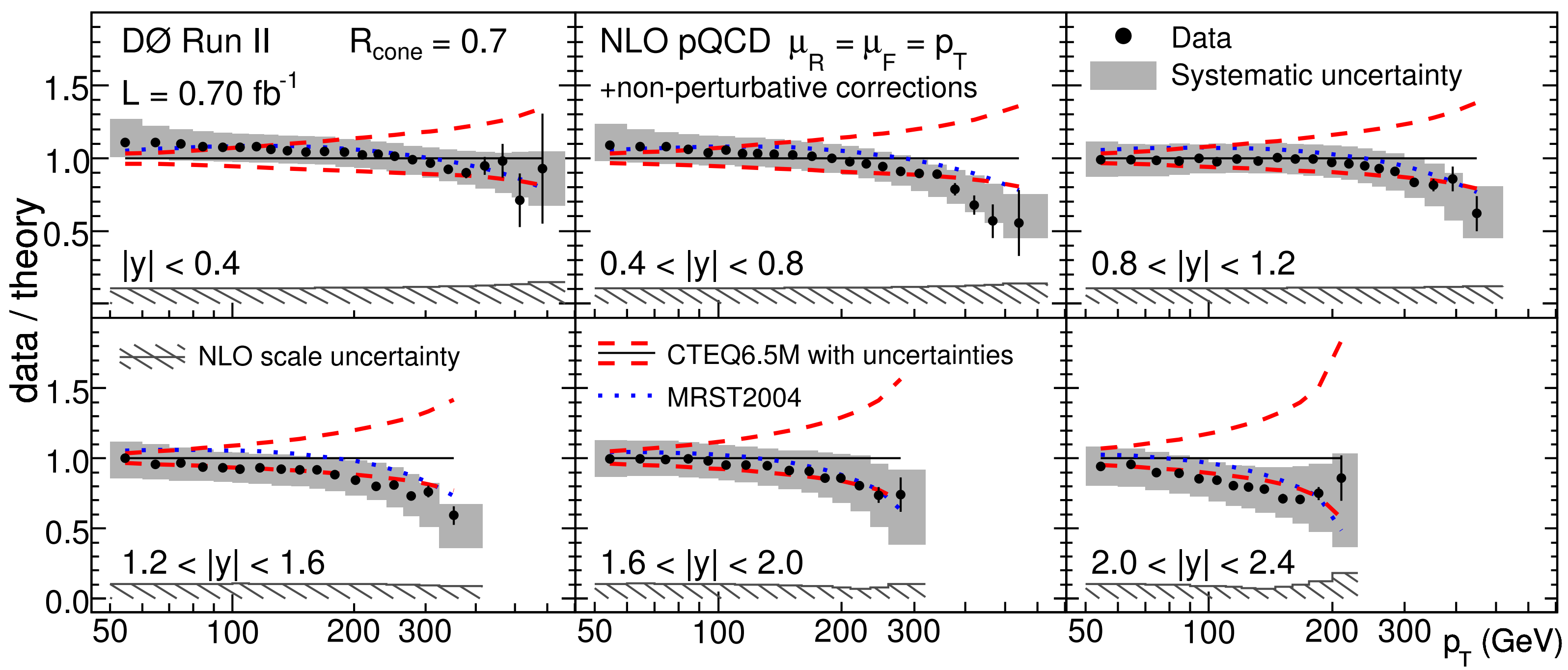


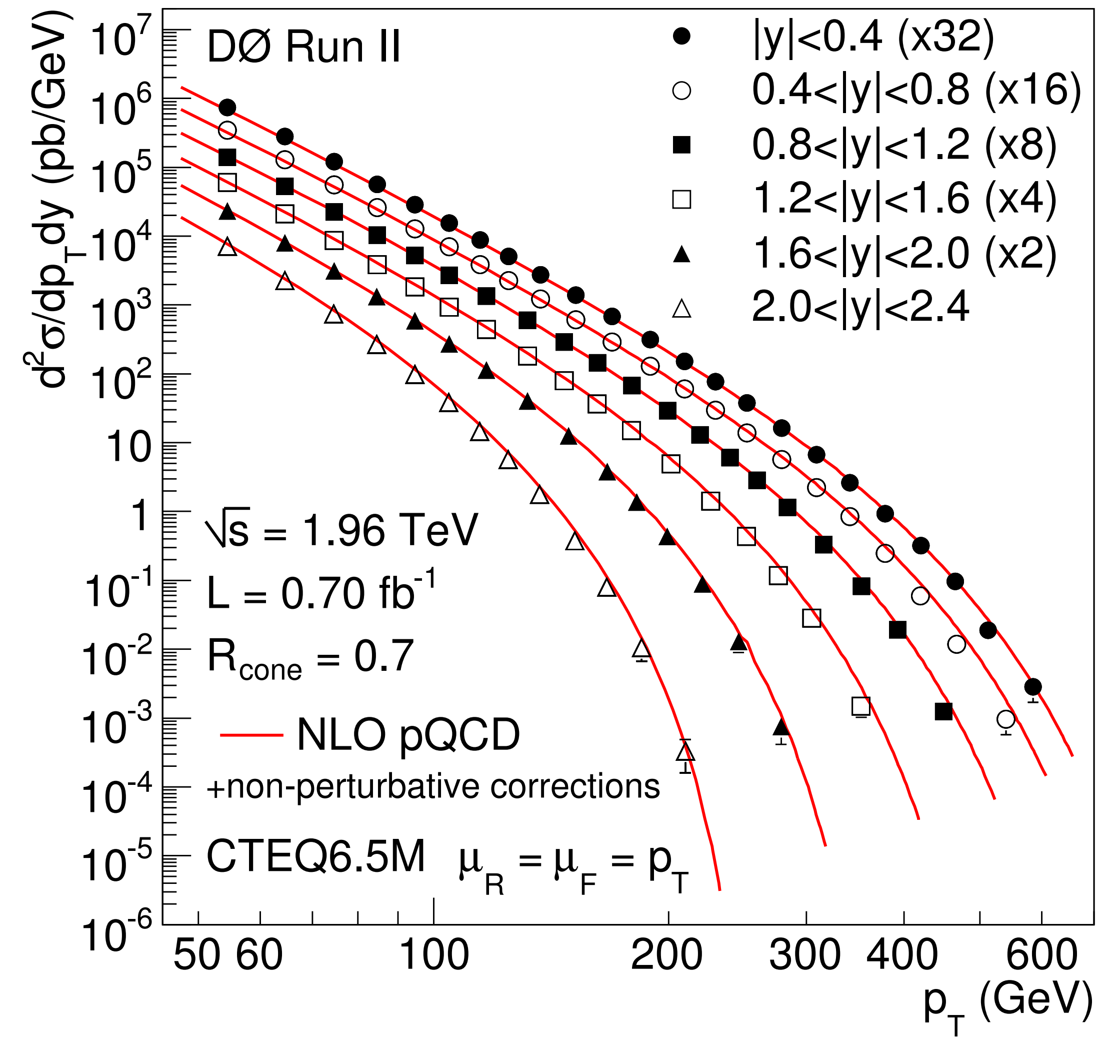
Test QCD - jet production - 2008 (DØ experiment)

- "Rutherford experiment" in $p\bar{p}$ at $\sqrt{s} = 1.96$ TeV

$$\frac{d\sigma_{\mathrm{QPM}}}{dp_{\mathrm{T}}} = \sum_{a,b,c,d}^{q,\bar{q},g} \int dx_1 dx_2\, d_a(x_1) d_b(x_2) \sigma_{ab\to cd} \left[\delta(p_{\mathrm{T}} - p_{\mathrm{T}}^c) + \delta(p_{\mathrm{T}} - p_{\mathrm{T}}^d)\right]$$

- the highest momentum transfer with $p_{\mathrm{T}} \approx 600$ GeV probing QCD at disctance scales of $\Delta x \sim 10^{-3}$ fm





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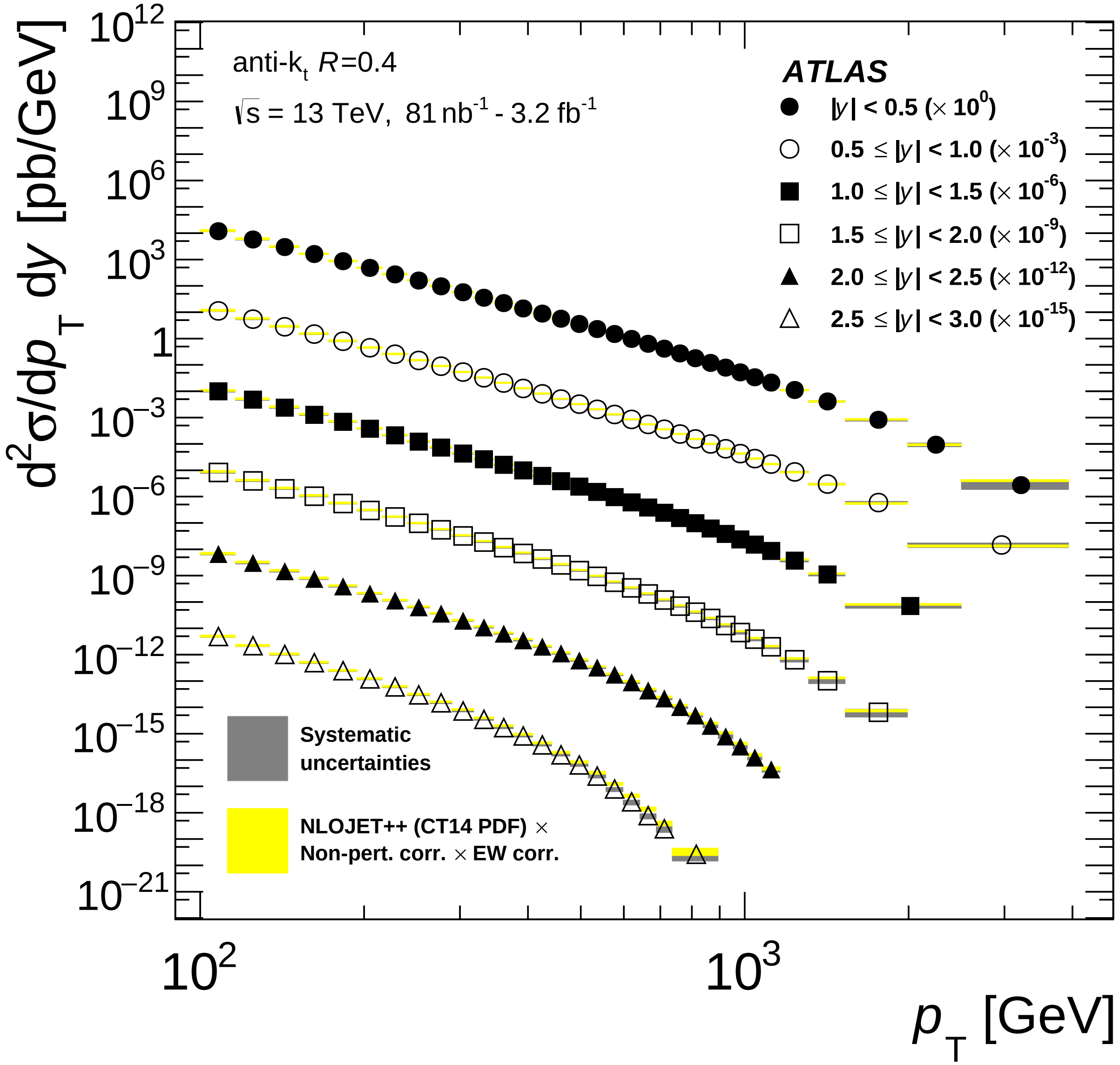
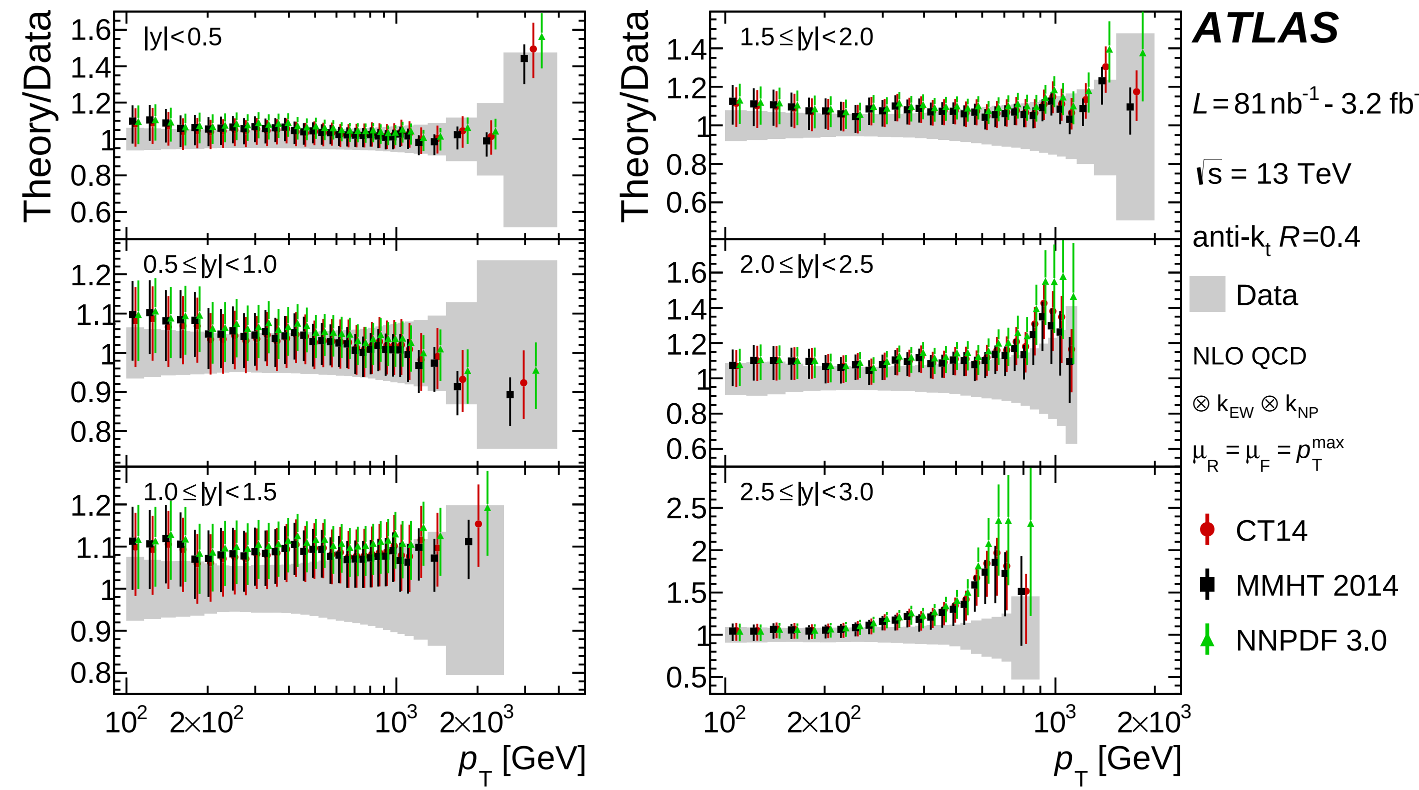
- improved knowledge of proton structure (more data from HERA)
- very good agreement with QCD

Jet production from LHC





- QCD works at least up to 2 TeV scales
- quarks are point-like at least up to 10^{-4} fm

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For reading and for exercises

- Chapter 6 (Basics of QCD)
- Exercises
  - ex. 6.1 (Color matrices formulae)
  - ex. 6.2 (Color factors for basic QCD processes)
  - ex. 6.3 ($uu \to uu$ scattering)
  - ex. 6.4 ($qg \to qg$ scattering)
  - reminder of exercise 3.17 (invariance of interaction lagrangian)
  - derive photon propagator in Lorenz gauge $\partial_\mu A^\mu = 0$ and try the same for axial gauge $c_\mu A^\mu = 0$ (J. Chyla scripts)
  - prove covariance of D'_μ in SU(3), i.e. that $D'_\mu = S D_\mu S^{-1}$, with $S = e^{i\alpha_a T_a} = 1 + i\alpha_a T_a$