Quarks, Partons, and Quantum Chromodynamics

Lecture 6

Proton structure and parton model

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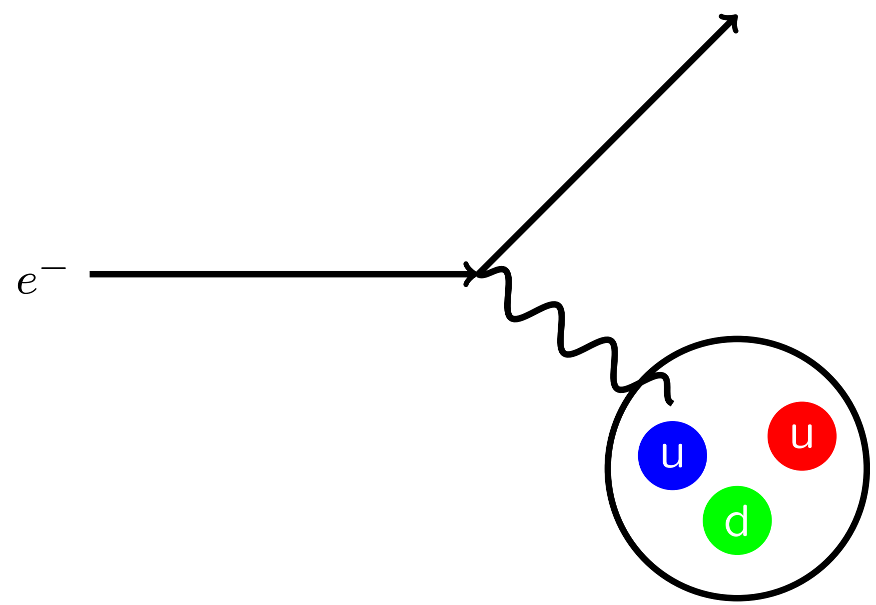
Elastic scattering and proton structure



- if you want to know how object looks like, you just need to look at it - for that you need light
- Heisenberg uncertainty relation

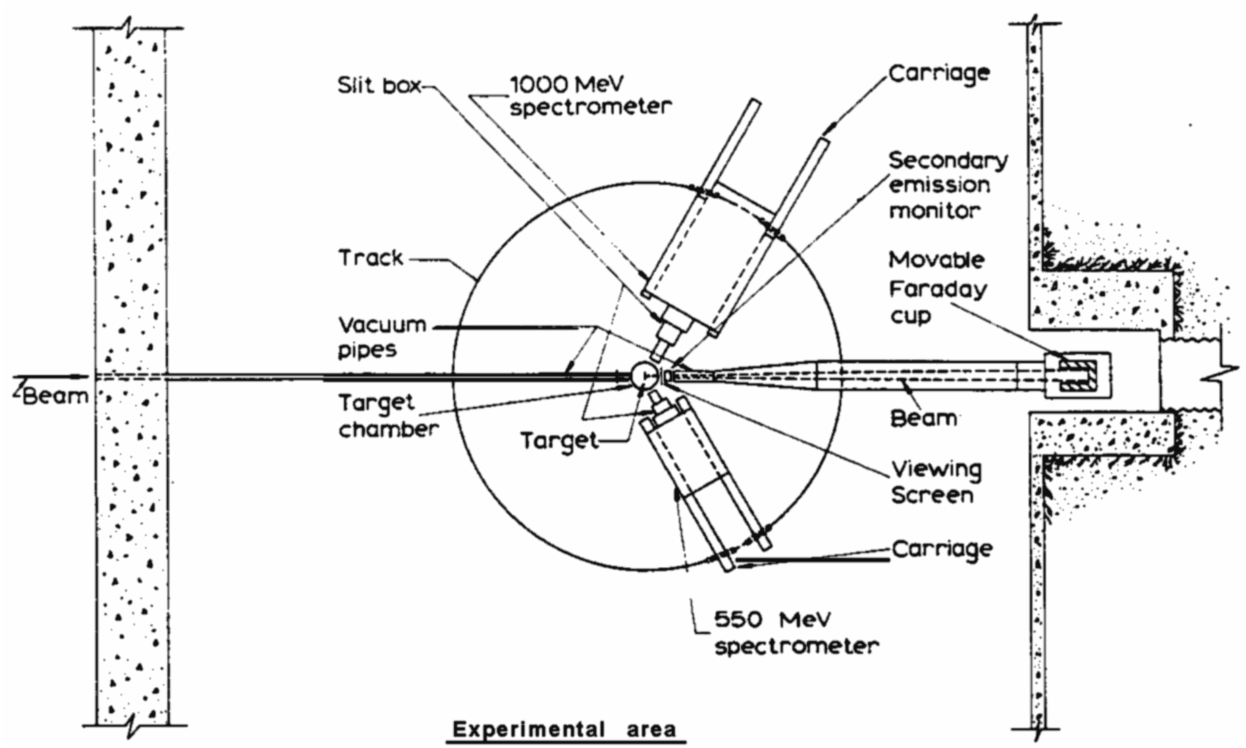
$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta x \sim 1\,\text{fm} \Rightarrow E \sim 200\,\text{MeV}$$

- in relativistic QFT, photon virtuality q^2 plays the role of momentum transfer
- source of light - electron beams

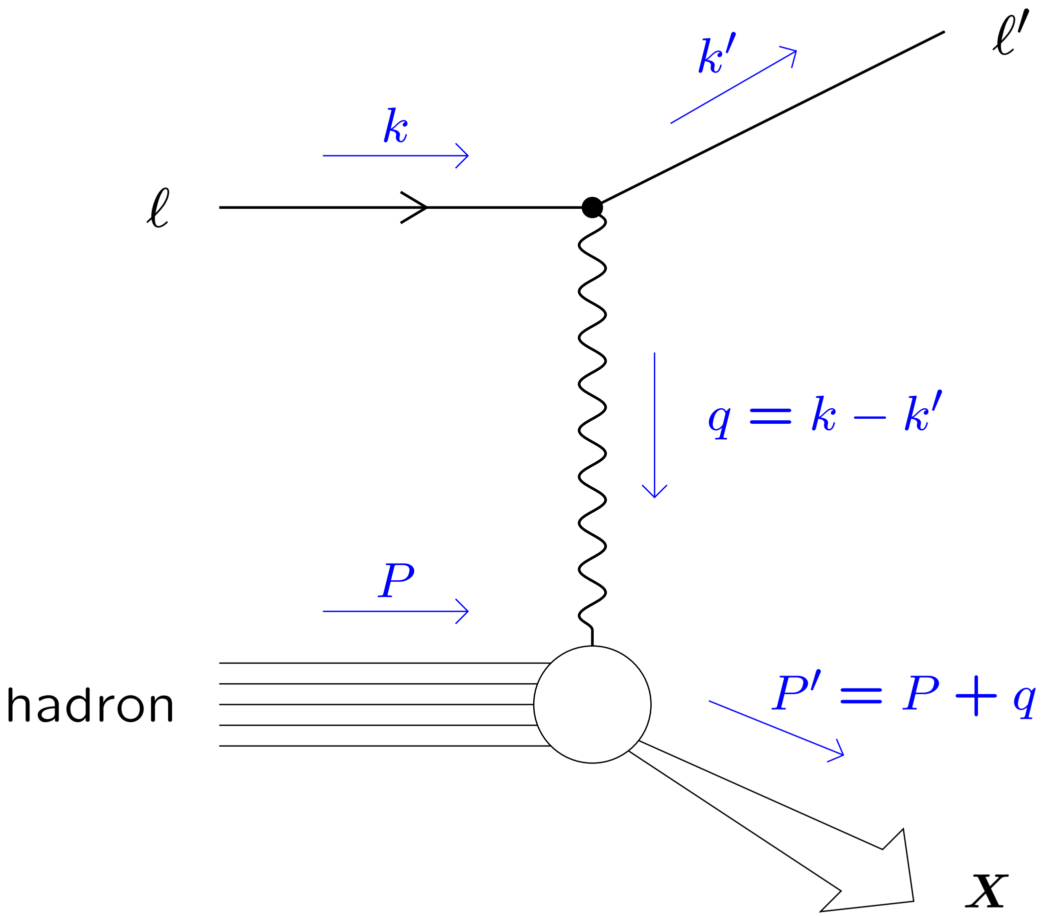
Robert Hofstadter 1956

- conceptually the same experiment as Rutherford's one in addition, measurement of electron momentum
- energy of electron beams (up to 550 MeV)
- enough to get the first measurement of proton size
- not enough to see proton structure



from R. Hofstadter Nobel prize lecture, 1961

Scattering of leptons on hadrons - kinematics



$$\ell(k) + \text{had.}(P) \to \ell'(k') + X, \qquad \ell, \ell' = e, \mu, \nu_e, \nu_\mu$$

- **neutral current**: $\ell = \ell'$, exchange of γ or Z
- **charged current**: $\ell \neq \ell'$, exchange of $W^\pm$
- initial state: $S \equiv (k+P)^2 = 2k \cdot P + M_P^2$
- first experiments measured only scattered lepton k'
- final state Lorentz invariants ($m_\ell \to 0$)

$$Q^2 \equiv -q^2 = -(k-k')^2 = 2k \cdot k'$$

$$y \equiv \frac{q \cdot P}{k \cdot P} = \frac{E_{\text{lab}} - E'_{\text{lab}}}{E_{\text{lab}}} = \frac{\nu}{E_{\text{lab}}} = \frac{S - M_p^2}{S} \cdot \frac{1 - \cos\vartheta_\star}{2}$$

$$x \equiv \frac{Q^2}{2P \cdot q} = \frac{Q^2}{Q^2 + W^2 - M_p^2}$$

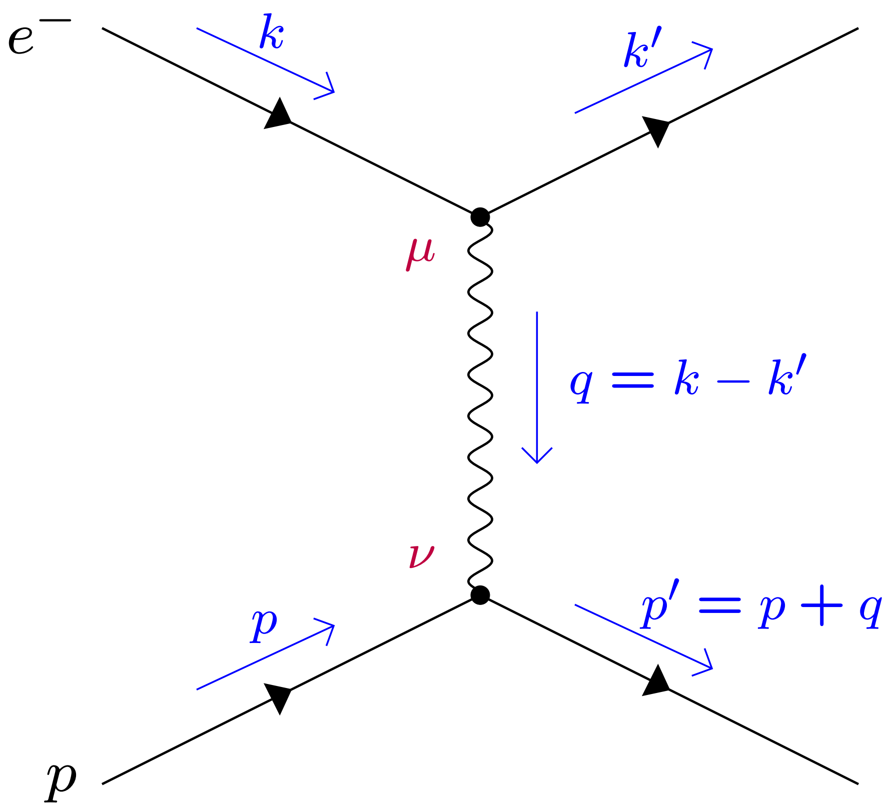
$$W^2 \equiv P'^2 = M_X^2 = (q+P)^2 = -Q^2 + 2P \cdot q + M_p^2$$

- lepton final state characterized for unpolarized beams by 2 scalars (no φ dependence)

$$Q^2 = xy(2k \cdot P) = xy(S - M_p^2)$$

- **elastic scattering** - one more constrain, $W^2 = M_p^2 = -Q^2 + 2P \cdot q + M_p^2 \;\Rightarrow\; 2P \cdot q = Q^2 \;\Rightarrow\; x_{\text{el}} = 1$

Elastic scattering on point-like charge (ex. 2.7)



- QFT: two body process $1 + 2 \to 3 + 4$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{64\pi s}\frac{1}{|\vec{p}_1^{\,\star}|^2}|\mathcal{M}|^2$$

Mandelstam var. $t = (k - k')^2 = q^2$, $\vec{p}_1^{\,\star}$ - incident particle momentum at CMS

$$i\mathcal{M} = iJ_e^{\mu}\frac{ig_{\mu\nu}}{q^2}iJ_p^{\nu}$$

$J_e^{\mu} = \bar{u}(k', s'_e)e\gamma^{\mu}u(k, s_e)$, spin $\frac{1}{2}$ proton: $J_p^{\mu} = \bar{u}(p', s'_p)e\gamma^{\mu}u(p, s_p)$

scalar proton: $J_p^{\mu} = e_q e(p + p')^{\mu}$

$$\overline{|\mathcal{M}|}^2 = (\text{spin factor})\,\frac{1}{q^4}L_e^{\mu\nu}L_{\mu\nu}^{p} \quad \text{with} \quad L_{\mu\nu} = J_{\mu}^{\dagger}J_{\nu}$$

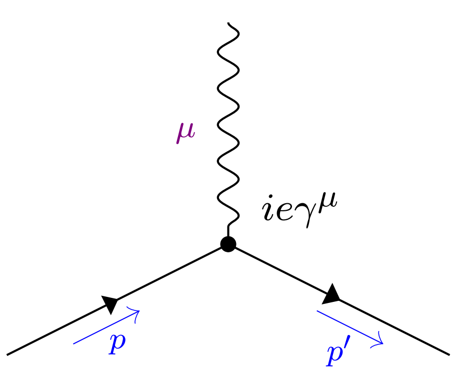
$$L_{\mu\nu}^{e} = e^2\mathrm{Tr}\left[(\not{k}' + m)\gamma^{\mu}(\not{k} + m)\gamma^{\nu}\right] = 4e^2\left[k'^{\mu}k^{\nu} + k'^{\nu}k^{\mu} - g^{\mu\nu}\left(k' \cdot k - m^2\right)\right]$$

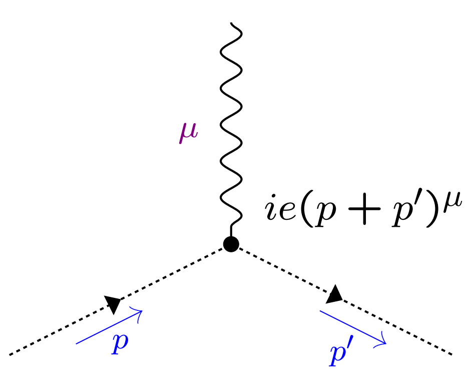
scalar proton: $L_{\mu\nu}^{p} = e_q^2 e^2(p + p')_{\mu}(p + p')_{\nu}$

continuity equation: $\partial_{\mu}J^{\mu} = 0 \;\Rightarrow\; J^{\mu}q_{\mu} = 0 \;\Rightarrow\; L^{\mu\nu}q_{\mu} = 0$

trick: $p + p' = 2p + q$ and $L_{\mu\nu}^{p}$ becomes effectively $L_{\mu\nu}^{p} \to e_q^2 e^2 4p_{\mu}p_{\nu}$.

$$\overline{|\mathcal{M}|}^2_{\text{spin-0}} = \frac{8e_q^2e^4}{q^4}(k \cdot p)^2\left[2(1 - y) - \underbrace{\frac{M_p^2 y}{k \cdot p}}_{\to 0 \text{ for } \sqrt{s} \gg M_p}\right]$$





e mass $m \to 0$, proton mass M_p kept

Elastic scattering on point-like charge (ex. 2.7)

- spin $\frac{1}{2}$ proton: $L^p_{\mu\nu} = 4e_q^2 e^2 \left[p'^{\mu} p^{\nu} + p'^{\nu} p^{\mu} - g^{\mu\nu}\left(p' \cdot p - M_p^2\right)\right]$
- after averaging for proton spin (factor $\frac{1}{2}$) becomes effectively ($p' = p + q$)

$$L^p_{\mu\nu} \to e_q^2 e^2 \left[4p_\mu p_\nu - 2g_{\mu\nu}(p \cdot q)\right]$$

- with this we find

$$\overline{|\mathcal{M}_{1/2}|}^2 = \frac{8e_q^2 e^4}{q^4}(k \cdot p)^2 \left[2(1-y) + y^2 - \frac{M_p^2 y}{k \cdot p}\right]$$

- and for cross section we get

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left(1 - y - \frac{M_p^2 y}{2\,k \cdot p}\right) \qquad \frac{\mathrm{d}\sigma_{1/2}}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left(1 - y + \frac{1}{2}y^2 - \frac{M_p^2 y}{2\,k \cdot p}\right)$$

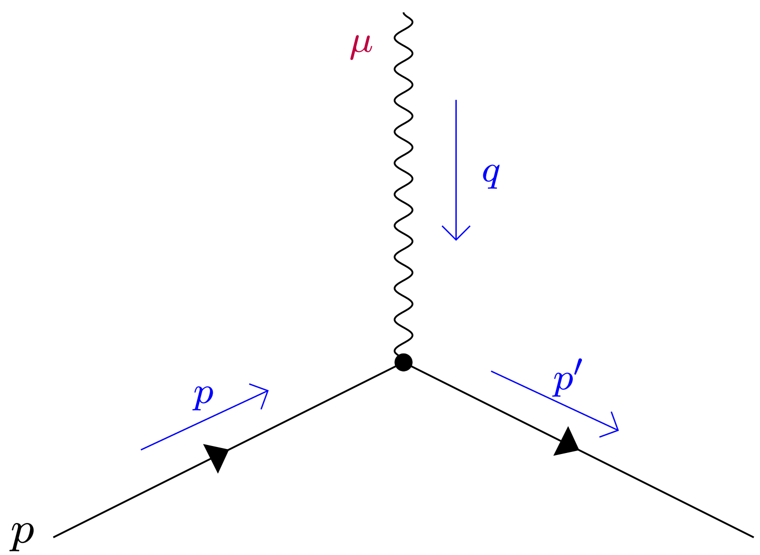
- using relation $Q^2 = 2y\,k \cdot p$ $(x_{\text{el}} = 1)$

$$\frac{\mathrm{d}\sigma_{1/2}}{\mathrm{d}y} = 2(k \cdot p)\frac{4\pi\alpha^2 e_q^2}{Q^4}\left(1 - y + \frac{1}{2}y^2 - \frac{M_p^2 y}{2\,k \cdot p}\right)$$

- in lab:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\text{lab}}} = \underbrace{\frac{\alpha^2 \overbrace{\cos^2(\vartheta_{\text{lab}}/2)}^{\text{relativistic correction}}}{4E^2 \sin^4(\vartheta_{\text{lab}}/2)}}_{\text{Rutherford formula}} \underbrace{\frac{E'}{E}}_{\text{kinematic factor}} \underbrace{\left[1 + \frac{Q^2}{2M_p^2}\tan^2\frac{\vartheta_{\text{lab}}}{2}\right]}_{\text{spin } 1/2}, \qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\text{Mott}}} = \frac{\alpha^2 \cos^2(\vartheta_{\text{lab}}/2)}{4E^2 \sin^4(\vartheta_{\text{lab}}/2)}\frac{E'}{E}$$

Elastic scattering on real proton (ex. 2.8)



- charged current of spin-$1/2$ Dirac point-like particle: $J_\mu = e\bar{u}(p')\gamma_\mu u(p)$
- for proton with structure we need to generalize J_μ
- for unpolarized proton we have only p_μ, p'_μ, and γ^μ

  not $\varepsilon_{\mu\nu\sigma\tau}$ as this would violate parity conservation

$$J_\mu = e\bar{u}(p')\left[A(q^2)\gamma_\mu + B_1(q^2)p_\mu + B_2(q^2)p'_\mu\right]u(p)$$

- other possible cases can be always converted to this using Dirac equation

- $A(q^2)$, $B_1(q^2)$, $B_2(q^2)$ are **form factors**, depend on only one kinematic variable (elastic + unpolarized proton)
- charge conservation $J_\mu q^\mu = 0$ leads to $B_1(q^2) = B_2(q^2) = B(q^2)$
- most general form of current: $J_\mu = e\bar{u}(p')\left[A(q^2)\gamma_\mu + B(q^2)(p_\mu + p'_\mu)\right]u(p)$

  using Gordon decomposition $\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{(p+p')^\mu}{2m} + \frac{i}{2m}\sigma^{\mu\nu}(p'-p)_\nu\right]u(p)\,, \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

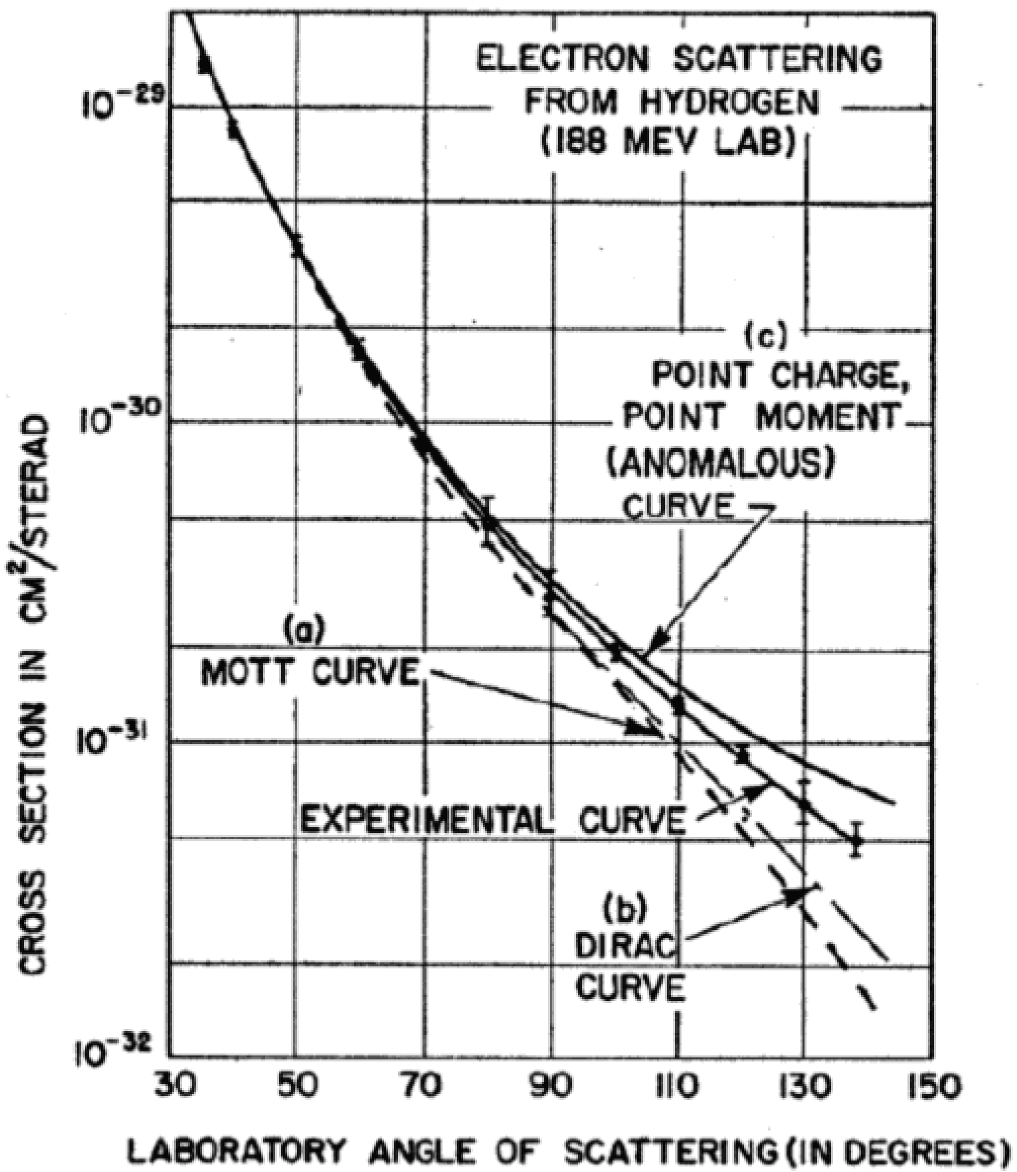
$$J^\mu = e\bar{u}(p')\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m}\kappa\sigma^{\mu\nu}q_\nu\right]u(p),$$

- **elastic form factors**: $F_1(q^2\!\to 0) = 1$ (e - total charge), $F_2(q^2\!\to 0) = 1 \;\Rightarrow$ (κ - anomalous magnetic moment)

  other choices: **electric** $G_E = F_1 - \frac{Q^2}{4M_p^2}\kappa F_2$ and **magnetic** $G_M = F_1 + \kappa F_2$ form factors

Target with a structure behaves at $q^2 \to 0$ like a point-like particle with total charge (e) and with **anomalous** magnetic moment

Elastic ep - proton size (Hofstadter 1956)



from R. Hofstadter Nobel prize lecture, 1961

- Mott curve: relativistic scalar point-like proton
- Dirac curve: point-like proton with $\kappa = 0$

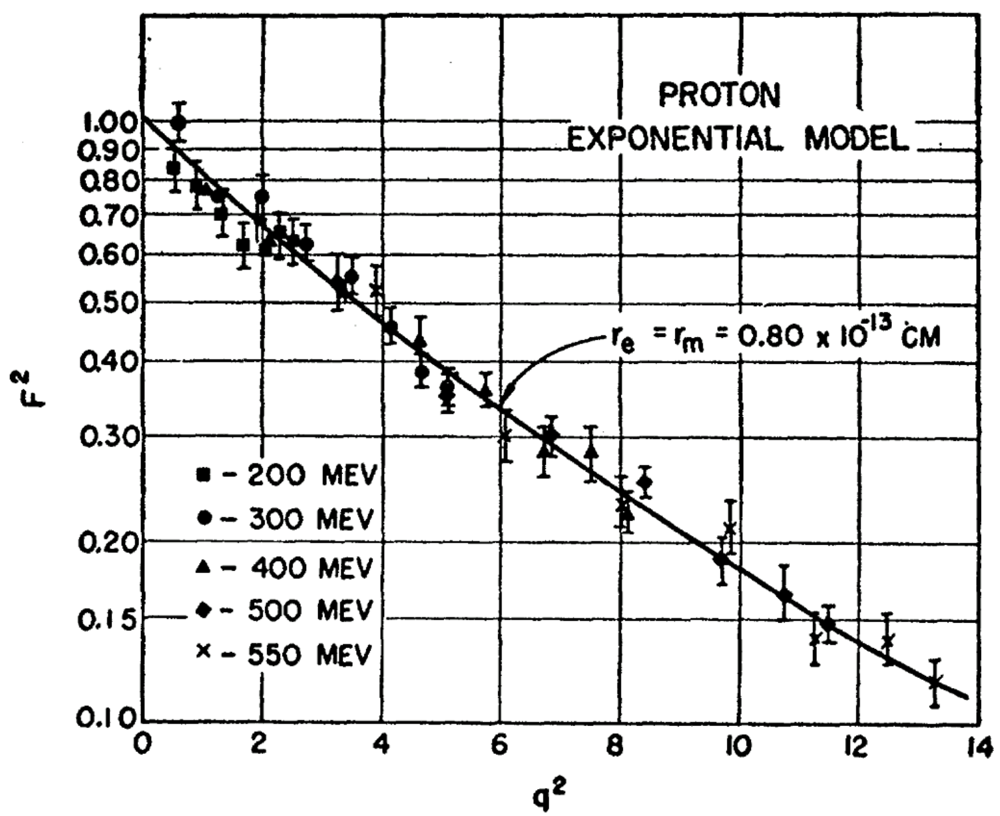
  Data still above!

- Corect curve to define form factors

  Point charge with proton anomalous magnetic moment

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\text{lab}}} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\text{Mott}}} \left(F_1^2 + \frac{Q^2}{4M_p^2} \left[\kappa^2 F_2^2 + 2(F_1 + \kappa F_2)^2 \tan^2 \frac{\vartheta_{\text{lab}}}{2} \right] \right)$$

Elastic ep - proton size



Chambers and Hofstadter, Phys. Rev. 103 (1956), 1454-1464

> the parameter corresponding to an rms size, although, of course, the exponential has to be expressed in dimensionless units. For example, ρ_{expon} can be expressed as $\rho_0 \exp(-r/a)$, where $3.48a$ is the rms radius. The assumption that the magnetic-moment distribution has a shape and size equal to that of the charge distribution
>
> Chambers and Hofstadter, Phys. Rev. 103 (1956), 1454-1464

$$\sqrt{< r^2 >} = \sqrt{12}\, a \approx 3.46\, a$$

Hofstadter (1956)

- series of exepriments at Standford University
- first analyses of data $F_1(Q^2) \approx F_2(Q^2)$

  later on with more data $G_E \approx G_M/\mu_p$
- proton charge density model $\varrho(\vec{x}) \sim \exp(-\mu|\vec{x}|)$

  $\Rightarrow$ dipole form factor

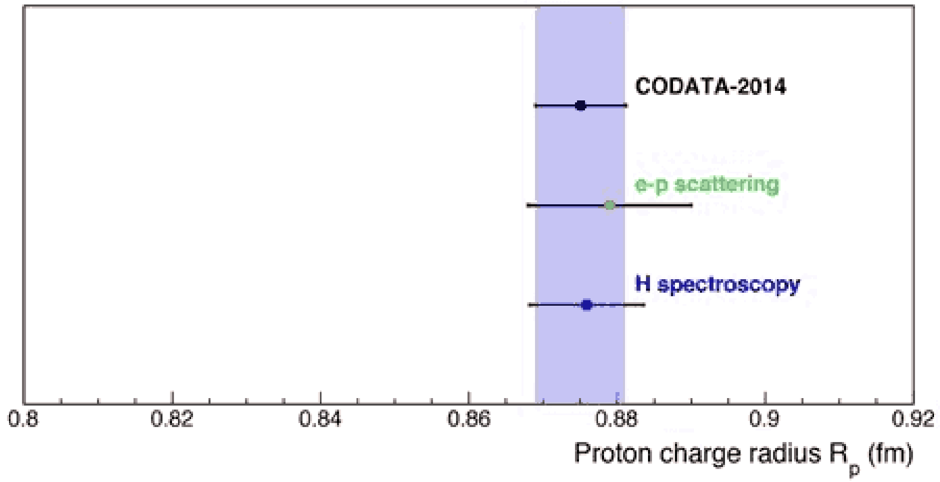
$$F(q^2) = \frac{1}{(1 + q^2/\mu^2)^2}$$

- 1953 - electon beam up to 125 MeV (gold atom size)
- 1955 - beam energy 200 MeV, first estimates of proton size (0.7 fm)
- 1956 - beams up to 550 MeV $\Rightarrow$ $r_p \approx 0.8$ fm
- Nober Prize 1961
- to see proton structure we need higher energies
- SLAC - plans for linear accelerator with $E_e = 20$ GeV

Situation before 2010

- agreement between electron scattering and spectroscopy

  (electron energy levels depend slightly on nucleus size)



CODATA average: 0.8751 ± 0.0061 fm
ep-scattering average (CODATA): 0.879 ± 0.011 fm
Regular H-spectroscopy average (CODATA): 0.859 ± 0.0077 fm

Pohl et al. experiment in 2010

- spectroscopy on muonic hydrogen : $r_{spec}^{(\mu)} = 0.84087 \pm 0.00039$ fm

  muon orbits closer to the proton, spending in proton more time $(r_H \sim \frac{1}{\alpha m_e})$
- 5σ discrepancy with respect to electron spectroscopy

Situation in 2019

- new measurements tends to agree mostly with new muonic results
  - PRad (e-p scat.) $r_p = 0.831 \pm 0.014$ fm

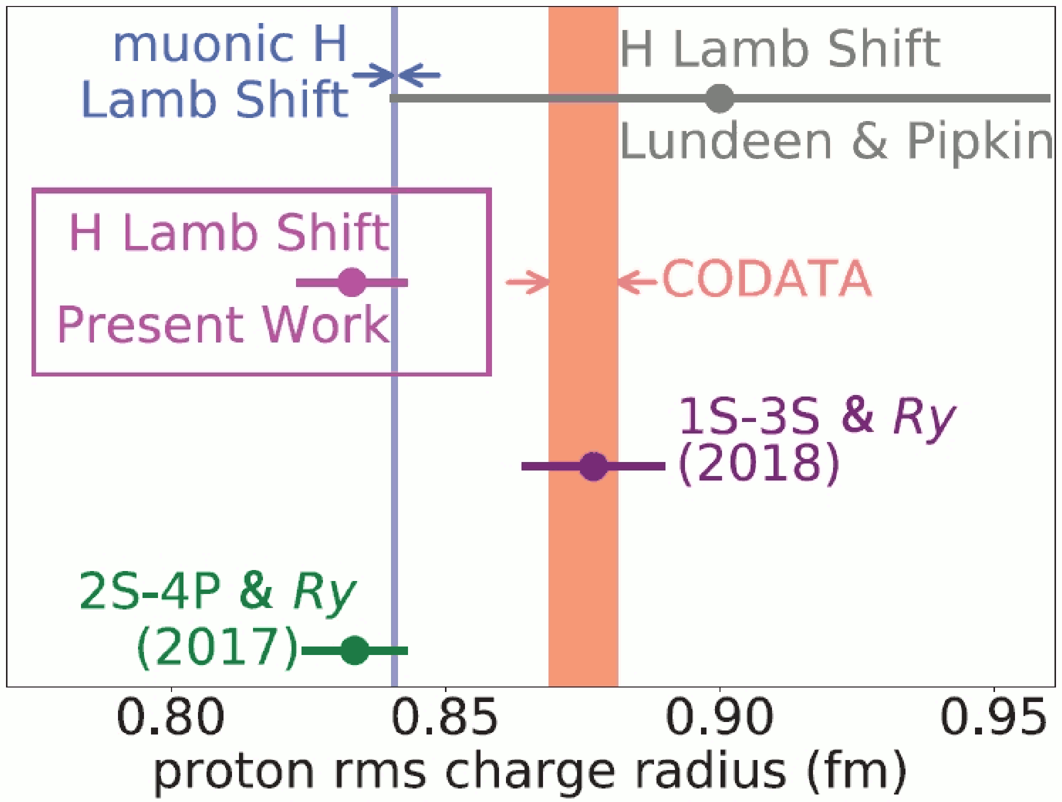
    Xiong, W. et al (2019) Nature 575, 147
  - H-spectroscopy:

    0.833 ± 0.010 fm (Bezingov 2019)

    0.8335 ± 0.0095 fm (Beyer 17)

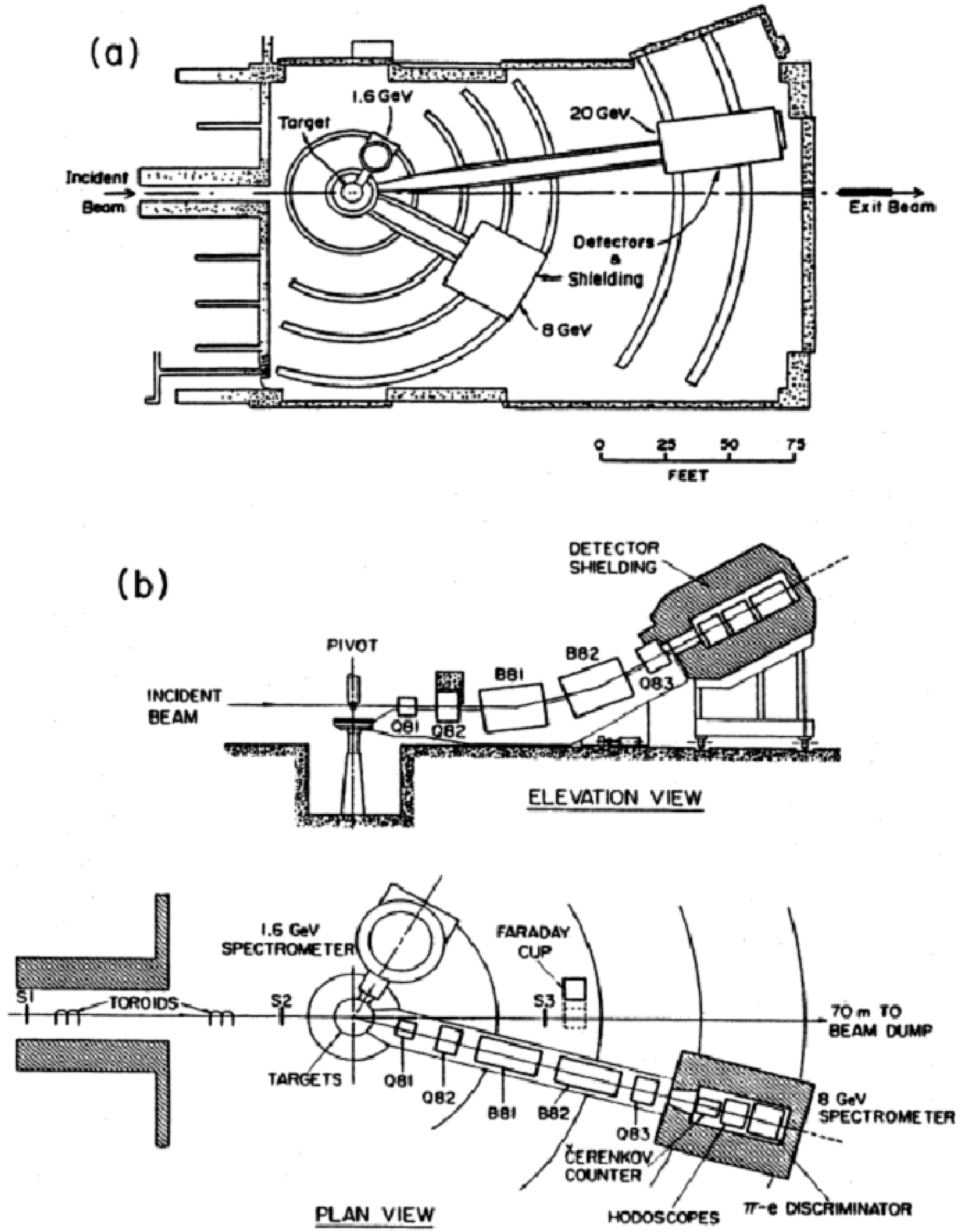
CODATA 2018: $r_p = 0.8414 \pm 0.0019$ fm

PDG 2024: $r_p = 0.8409 \pm 0.0004$ fm



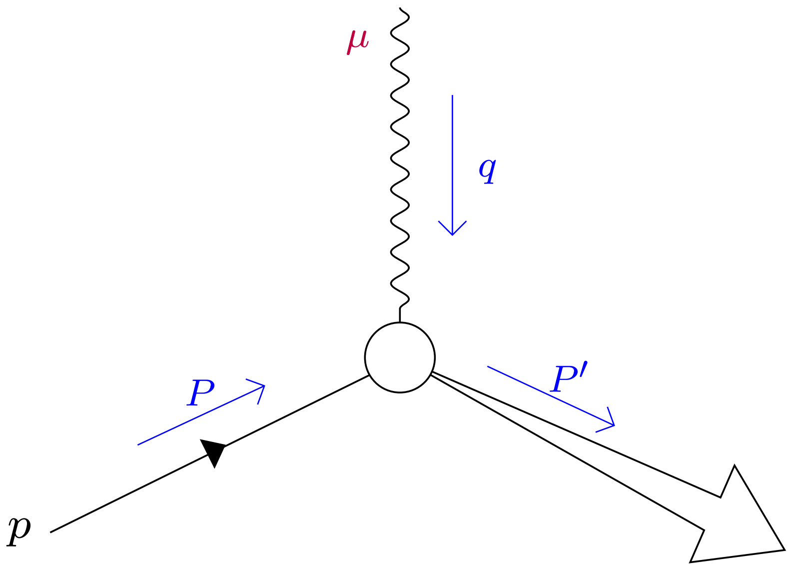
Science 365 (2019) 6457, 1007-1012

SLAC and the first DIS experiments (1968)



from H. Kendall Nobel prize lecture, 1990

Deep inelastic scattering



- proton is destroyed $\Rightarrow$ no current J^μ associated with vertex
  $\Rightarrow$ need to construct general hadron tensor $L_p^{\mu\nu} \equiv W^{\mu\nu}$
- we have $g_{\mu\nu}$, P_μ, q_μ, and parity violating $\varepsilon_{\mu\nu\sigma\tau}$ (for CC)

$$W_{\mu\nu} = -W_1 g_{\mu\nu} + W_2 \frac{P_\mu P_\nu}{M_p^2} + iW_3 \varepsilon_{\mu\nu\sigma\tau} P^\sigma q^\tau$$
$$+ W_4 q_\mu q_\nu + W_5(p_\mu q_\nu + p_\nu q_\mu) + iW_6(p_\mu q_\nu - p_\nu q_\mu)$$

- charge conservation $W^{\mu\nu} q_\nu = 0$ and hermiticity of $W^{\mu\nu}$ fix $W_{4,5,6}$

- 3 form factors (**structure function**) describing proton in inelastic scat.: $W_1(x, Q^2)$, $W_2(x, Q^2)$,$W_3(x, Q^2)$
- functions of two scalars
- for parity conserving QED only two ($W_3 = 0$)
- we don't need to know W_4, W_5, W_6 as $L_e^{\mu\nu} q_\nu = 0$ $\quad \Rightarrow \quad$ $W_{\mu\nu}^{\text{eff}} = -W_1 g_{\mu\nu} + W_2 \frac{P_\mu P_\nu}{M_p^2}$

spin-0

spin-1/2

for $S \gg M_p^2$, i.e. neglecting term $\frac{M_p^2 y}{2k \cdot P}$

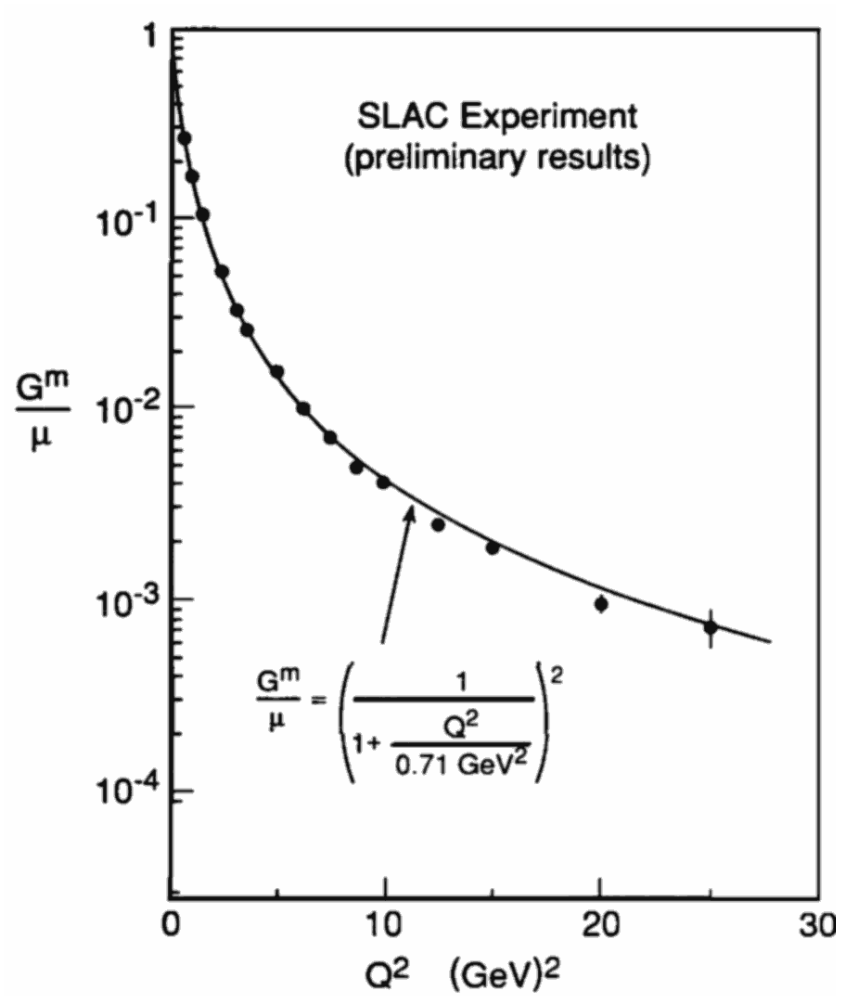
$$\frac{\mathrm{d}\sigma_{\text{el}}^{\text{point}}}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[1 - y + \frac{1}{2} y^2\right] \quad \Rightarrow \quad \frac{\mathrm{d}\sigma_{\text{inel}}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + 2F_1(x,Q^2)\frac{1}{2}y^2\right]$$

try it by evaluating $L_e^{\mu\nu} W_{\mu\nu}$ and using $F_1 \equiv W_1$, $F_2 = W_2 \frac{\nu}{M_p}$ (textbook 5.2)

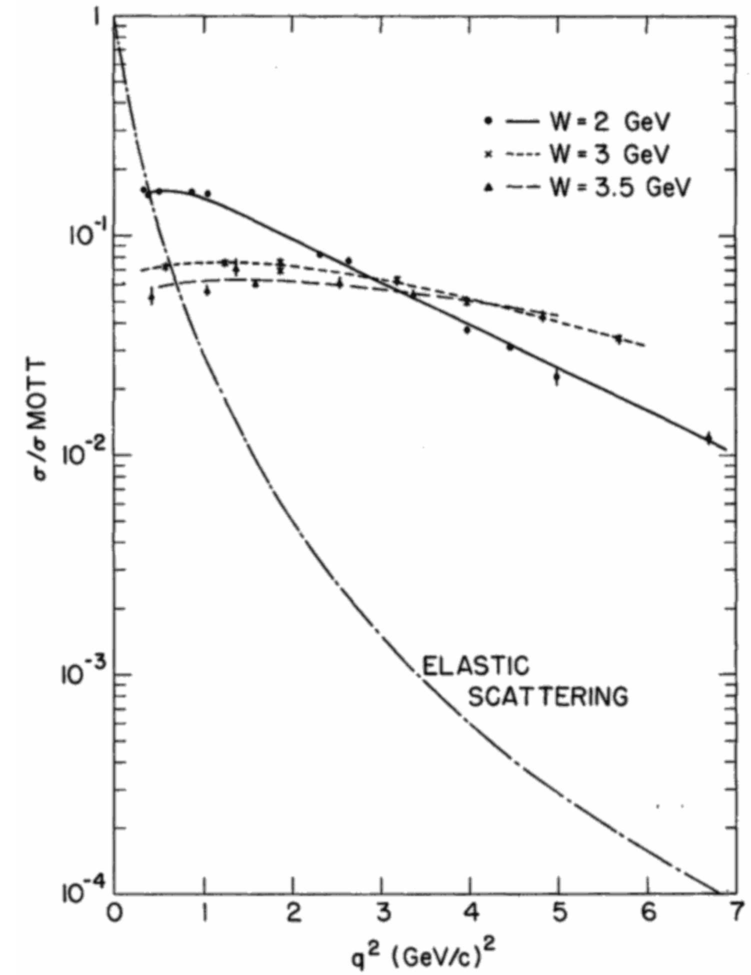
$\downarrow$ $\quad$ $\downarrow$

conventional choice

First results from SLAC on DIS (1968)



from W. Kendall Nobel prize lecture (1990)



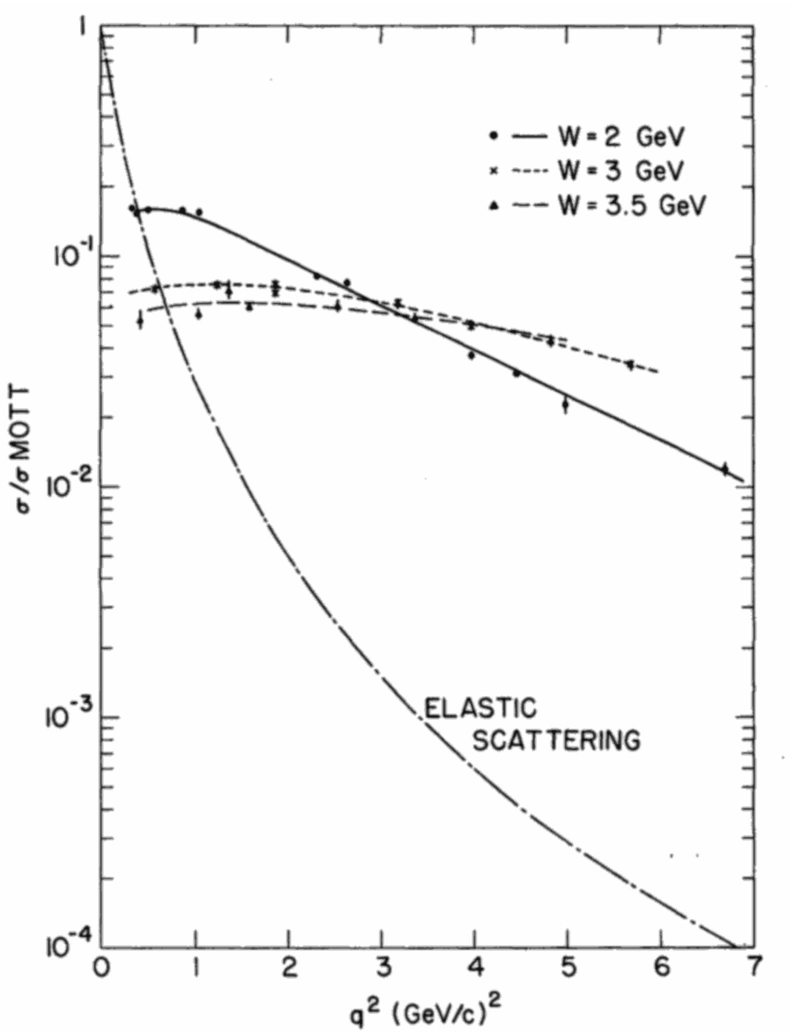
PRL 23 (1969) 935

- main experimental program: elastic, and quasi-elastic form factors (like for $ep \to e\Delta^+$)
- inelastic measurements just part of supplementary program
- steeply falling elastic form factor followed the power law dipole formula even at very high Q^2
- in strinking contrast, inelastic cross section at high Q^2 behaved like a scattering on point-like target

$\Rightarrow$ only mild dependence of $F_1(x, Q^2)$ and $F_2(x, Q^2)$ on Q^2 - **Björken scaling** $(F_2(x, Q^2 \to \infty) = \text{const} > 0)$

Taylor, Kendall, Friedman - NP in 1990

DIS and proton structure



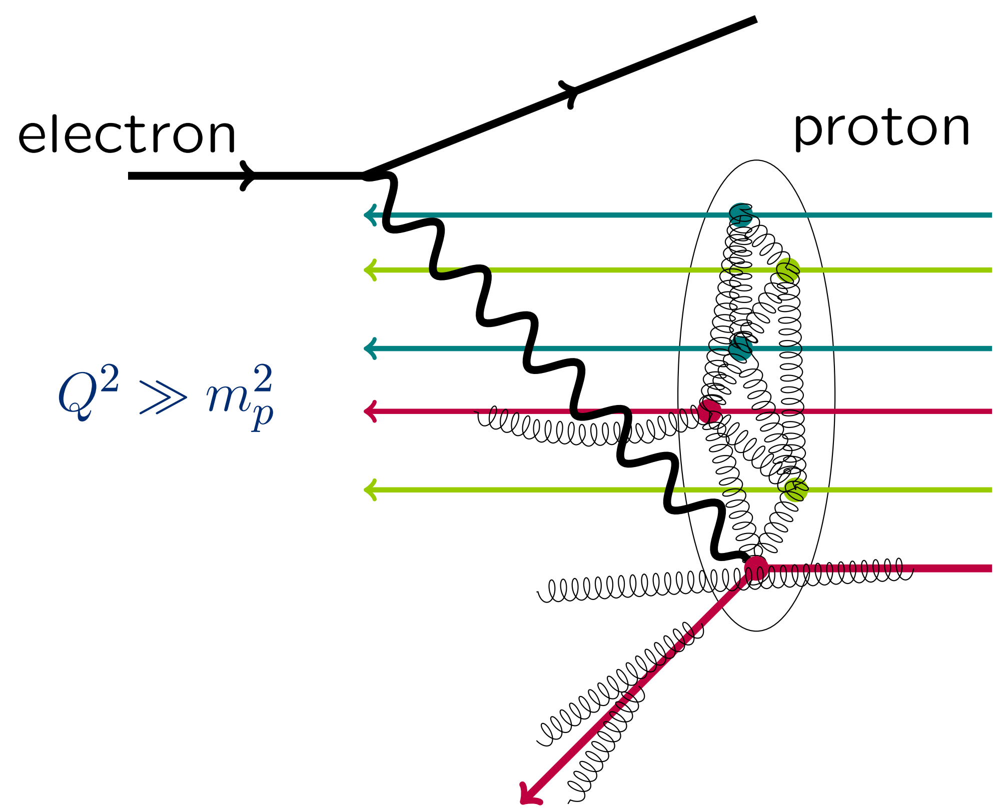
PRL 23 (1969) 935

- J. Björken in 1967

  But the indication is that there does not yet seem to be any large cross sections which this model of point-like constituents suggests. Additional data is necessary and very welcome in order to completely destroy the picture of elementary constituents.

- early 1968 - with new data from SLAC, first evidence that

  data might give evidence on the behaviour of point-like charged structures in nucleon



$Q^2 \gg m_p^2$

- proton made of point-like sources with charge α_i
- Reminder from Lect 1:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_{\text{point}}}{\mathrm{d}\Omega}\left|\sum_i \alpha_i \mathrm{exp}(i\vec{q}\cdot\vec{r}_i)\right|^2 = \frac{\mathrm{d}\sigma_{\text{point}}}{\mathrm{d}\Omega}\left[\sum_i \alpha_i^2 + 2\mathrm{Re}\sum_{i<j}\alpha_i\alpha_j \mathrm{exp}(i\vec{q}\cdot\vec{\Delta}_{ij})\right]$$

$$\xrightarrow{|q|\gg\Delta_{ij}} \frac{\mathrm{d}\sigma_{\text{point}}}{\mathrm{d}\Omega}\sum_i \alpha_i^2 = \sum_i \frac{\mathrm{d}\sigma_{\text{point}}(\alpha_i)}{\mathrm{d}\Omega}$$

this is Björken scalling

Feynman parton model of proton

- summer 1968 - SLAC results presented on conference in Viena
- autumn 1968 - Feynman formulated basic ideas of parton model (for hadron-hadron collisions)
- 1969 - first formulation for DIS (Björken, Paschos: Phys. Rev. 185 (1969) 1975; full credit given to Feynman)

Parton model

- proton behaves in deep inelastic scattering ($Q^2 \gg M_p^2$) as a bag of free point-like constituents, **partons**
- final cross section is incoherent sum of cross sections on individual proton constituents
- formulated for very fast protons, in so called **infinite momentum frame**, in this frame

- we can neglect transversal motion of partons ($k_T \sim 200$ MeV, due to their binding in 1 fm radius)
- we can neglext proton and parton masses, $p = \xi P$, i.e. ξ fractional momentum carried by parton

  $x = \frac{Q^2}{2P\cdot q}$, scattering on parton is elastic $x_{\text{el}} = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{2\xi P\cdot q} = 1$, but $x = \frac{Q^2}{2P\cdot q} \;\Rightarrow\; x = \xi$

> In parton model, kinematic variable $x = \frac{Q^2}{2P\cdot q}$ describes fractional longitudinal momentum of the parton that was kicked by the incident electron

- proton is described by **parton distribution functions** (PDF) $d_A(x)$

  $d_A(x)\mathrm{d}x$ is number of partons of type A carrying fractional momentum $(x, x + \mathrm{d}x)$

  - partons are free, i.e. on mass shell, therefore PDFs are functions of x only

    partons are not free, at the end we get $d_A(x, Q^2)$ as described by QCD

- charged partons were identified with quarks, neutral with gluons $\Rightarrow$ **quark parton model** (QPM)

Structure functions F_2 and F_1 in QPM

- QPM prescription: final x-sec is sum of x-secs on individual proton constituents

$$\frac{\mathrm{d}\sigma^{\mathrm{QPM}}}{\mathrm{d}Q^2} = \sum_i^{\text{partons}} \underbrace{d_i(x)\mathrm{d}x}_{N_{\text{partons}}} \cdot \frac{\mathrm{d}\sigma^{\text{point}}}{\mathrm{d}Q^2}(e_i) \quad \Rightarrow \quad \frac{\mathrm{d}\sigma^{\mathrm{QPM}}}{\mathrm{d}x\mathrm{d}\Omega} = \frac{4\pi\alpha^2}{Q^4}\sum_i^{\text{part.}} e_i^2 d_i(x)\left[1-y+\frac{y^2}{2}\right]$$

- comparing with general formula

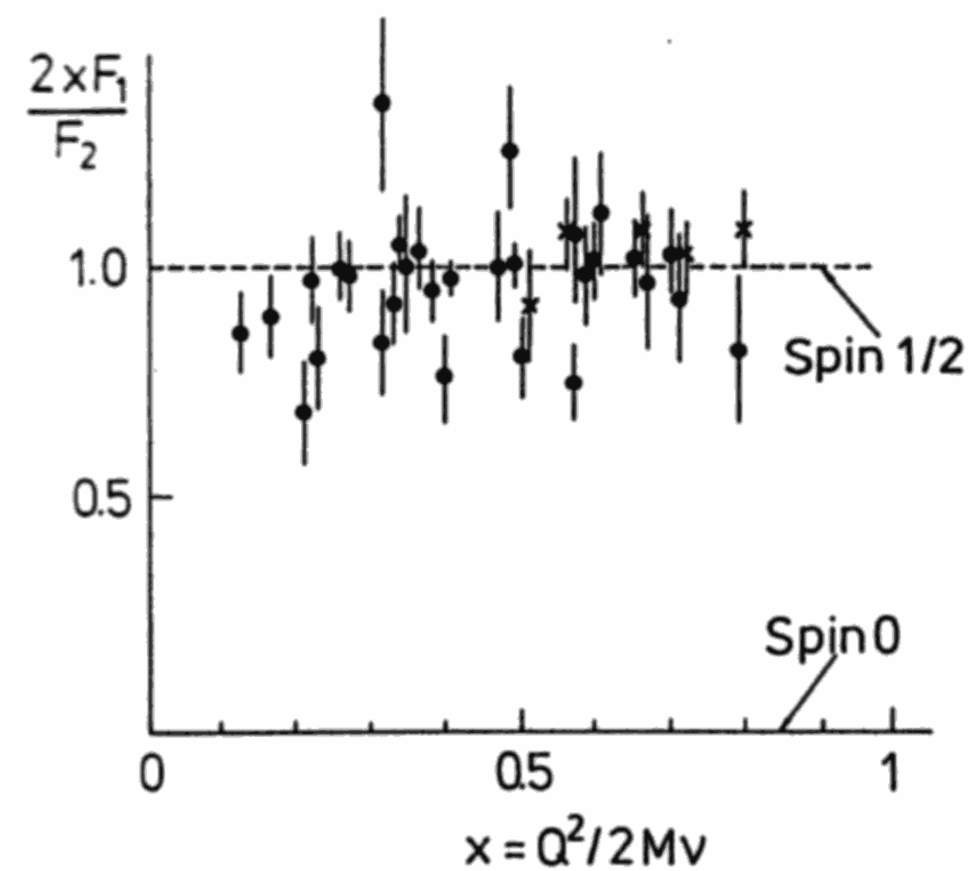
$$\frac{\mathrm{d}\sigma_{\text{inel}}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + 2F_1(x,Q^2)\frac{1}{2}y^2\right] \quad \Rightarrow \quad F_2(x,Q^2) = \sum_i^{\text{part.}} e_i^2 x d_i(x)$$

- prediction for $F_1(x,Q^2)$

$$\text{scalar parton: } F_1(x,Q^2) = 0, \quad \Rightarrow$$

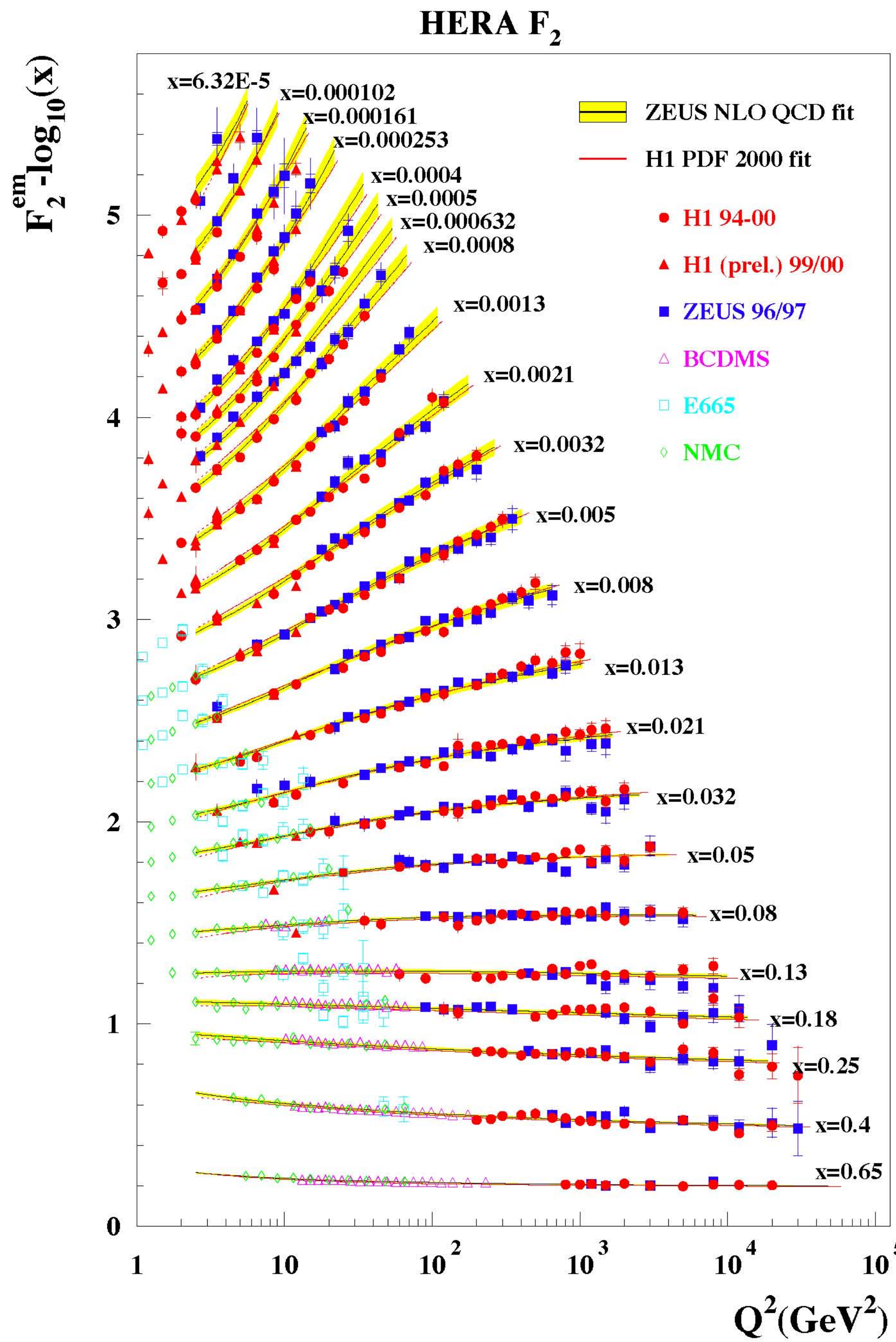
$$\text{spin } \frac{1}{2} \text{ parton: } 2xF_1(x,Q^2) = F_2(x,Q^2) \quad \text{(Callan-Gross relation)}$$

- to measure F_1 and F_2, DIS has to be performed at two different $\sqrt{S}$ as ($Q^2 = xy(S-M_p^2)$)
- measurements confirmed spin $1/2$ for charged partons



P. Schmuser, Feynman-Graphen und Euchtherein fur Experimentalphysiker (1988)

Proton structure after HERA

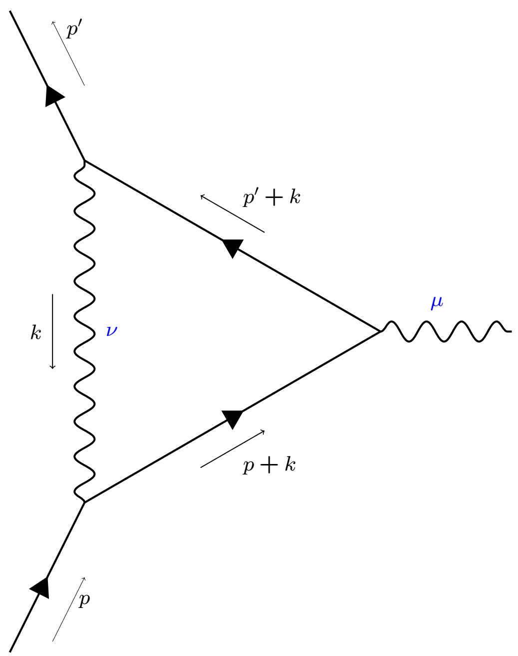


HERA

- $e^-/e^+ + p$ collider
- $E_{cms} = 318$ GeV
  - 27.5 GeV electrons
  - 920 GeV protons

For reading and for exercises

- Chapters 5.1-5.7 from J. Chyla's scripts (5.3 optional)
- Exercises
  - ex. 2.7 (Elastic scattering of electron on point-like particle in QFT)
  - ex. 2.8 (Proton anomalous magnetic moment)
  - recapitulate ex. 2.6 (Dipole form factor and proton size)
- Extra exercises for further study
  - Appendix B (Electron magnetic moment at NLO)



$$J^\mu = e\bar{u}(p')\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m}\kappa\sigma^{\mu\nu}q_\nu\right]u(p)$$