# Quarks, Partons, and Quantum Chromodynamics

# Lecture 5

## Quarks model of hadrons

Alexander Kupčo

http://www-hep2.fzu.cz/~kupco/QCD/

kupco@fzu.cz

# Additive quark model of hadrons

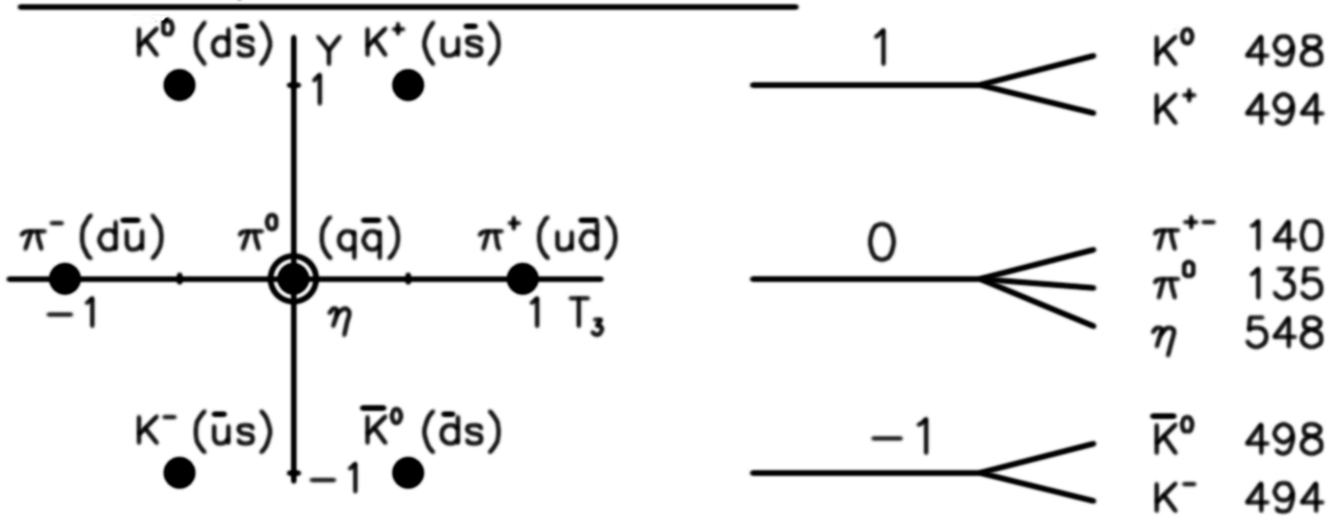
- success with hadron magnetic moments

| p | n | $\Lambda$ | $\Sigma^+$ | $\Sigma^-$ | $\Xi^0$ | $\Xi^-$ | $\Omega$ |
|---|---|---|---|---|---|---|---|
| 2.793 | -1.913 | -0.613±0.004 | 2.458±0.010 | -1.160±0.025 | -1.25±0.014 | -0.651±0.003 | -2.02±0.05 |
| input | input | input | 2.674 | -1.092 | -1.435 | -0.494 | -1.839 |

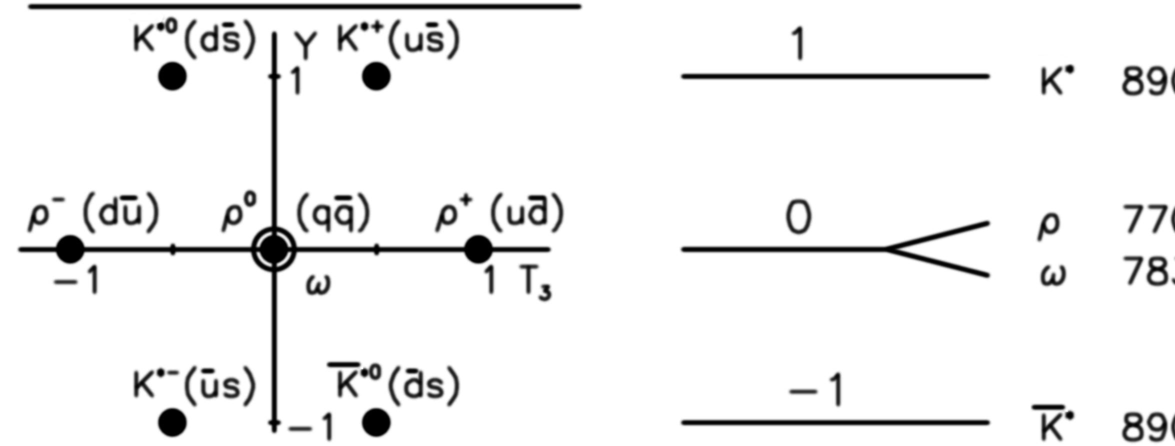
$\Rightarrow$ constituent quark masses: $m_u = 338$ MeV, $m_d = 338$ MeV, $m_s = 510$ MeV

- What about hadron masses?
- simple additive model too cruel in particular for mesons

octet of pseudoscalar mesons




octet of vector mesons




- quark spins must play some role

# Hadron masses

- let's try model with spin-spin interaction (Halsen & Martin)

  analogy with hyperfine structure of hydrogen lines: $\Delta E_{\text{hf}} = \frac{2\pi\alpha}{3}\frac{\vec{\sigma}_p \cdot \vec{\sigma}_e}{m_p m_e}|\psi_e(0)|^2$

- **meson masses**: $m(q_1\bar{q}_2) = m_{q_1} + m_{\bar{q}_2} + a\,\langle\psi_{\text{meson}}|\;\frac{\vec{\sigma}_{q_1}\cdot\vec{\sigma}_{\bar{q}_2}}{m_{q_1}m_{\bar{q}_2}}\;|\psi_{\text{meson}}\rangle$

- **baryon masses**: $m_B(q_1q_2q_3) = m_{q_1} + m_{q_2} + m_{q_3} + a'\,\langle\psi_B|\;\sum_{i<j}^{1,2,3}\frac{\vec{\sigma}_{q_i}\cdot\vec{\sigma}_{q_j}}{m_{q_i}m_{q_j}}\;|\psi_B\rangle$

- we can evaluate the scalar products of Pauli matrices directly (see the last part of exercise 4.3 - Baryon masses)
- or do it in a clever way using quadratic Casimir operators

# Casimir operators (exercise 3.14)

- quadratic, or higher order, forms constructed out of Lie algebra of generators $X_i$, which commute with all its elements
- the simplest form is quadratic Casimir operator, for orthogonal generators $X_i$

$$C = \sum_{1}^{n} X_i X_i\,, \qquad \forall i = 1,\ldots,n:\ [C, X_i] = 0.$$

Schur's Lemma: Casimir operator is for irreducible representation a multiple of unit matrix $\mathbb{1}$

- in SU(2): $C = J_1^2 + J_2^2 + J_3^2$ is square of total orbital momentum $\quad C(\mathcal{D}(j)) = j(j+1)\mathbb{1}$

$$J^2_{\text{tot}} = (\vec{J}_{(1)} + \vec{J}_{(2)})^2 = \vec{J}^2_{(1)} + \vec{J}^2_{(2)} + 2\vec{J}_{(1)}\cdot\vec{J}_{(2)}$$

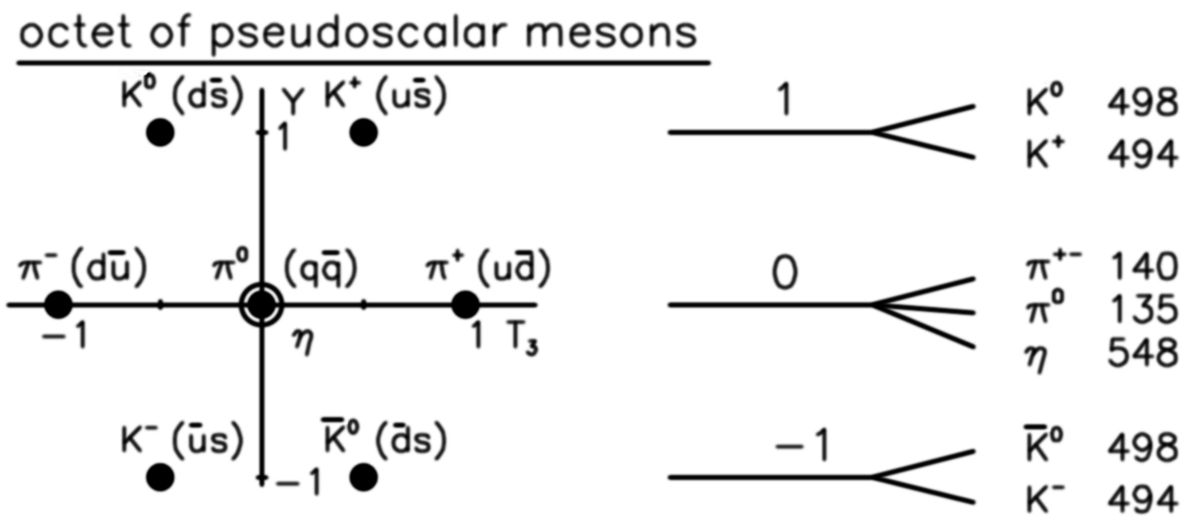
- as $J_i = \sigma_i/2$ :

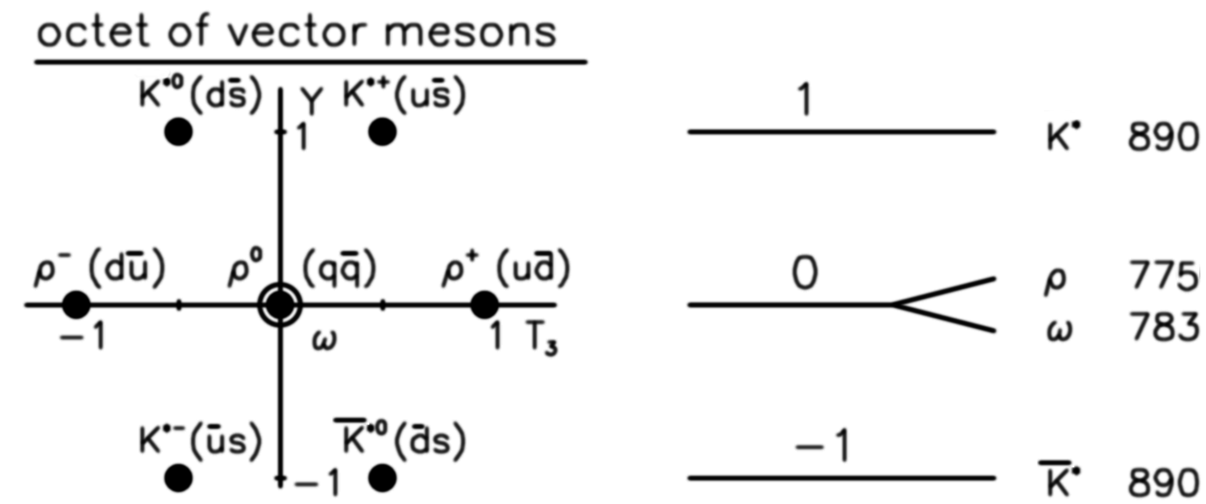
$$\vec{\sigma}_{(1)}\cdot\vec{\sigma}_{(2)} = 4\,\vec{J}_{(1)}\cdot\vec{J}_{(2)} = 2\,(C_{\text{tot}} - C_{(1)} - C_{(2)})$$

$$\sum_{i<j}^{1,2,3} \vec{\sigma}_{(i)}\cdot\vec{\sigma}_{(j)} = 4\sum_{i<j}^{1,2,3} \vec{J}_{(i)}\cdot\vec{J}_{(j)} = 2\,(C_{\text{tot}} - C_{(1)} - C_{(2)} - C_{(3)})$$

- SU(3) for $\mathcal{D}(q_3, q_{\bar{3}})$: $C(q_3, q_{\bar{3}}) = \frac{1}{3}\left(q_3^2 + q_3 q_{\bar{3}} + q_{\bar{3}}^2\right) + (q_3 + q_{\bar{3}})$
- SU(N): $\quad C_F(N) = \frac{N^2-1}{2N}$, $C_A(N) = N$ (exercise 3.16)

# Meson masses




- $\pi$ mass: $$m_\pi = 2m_u + \frac{a}{m_u^2} 2\left(0\cdot 1 - \frac{1}{2}\cdot\frac{3}{2} - \frac{1}{2}\cdot\frac{3}{2}\right) = 2m_u - 3A$$
  $m_u = m_d$ $\quad A \quad J_\pi = 0 \quad J_u = 1/2$

- $\varrho$ mass: $$m_\varrho = 2m_u + \frac{a}{m_u^2} 2\left(1\cdot 2 - \frac{1}{2}\cdot\frac{3}{2} - \frac{1}{2}\cdot\frac{3}{2}\right) = 2m_u + A$$
  $J_\varrho = 1$

$$m_u = \frac{3m_\varrho + m_\pi}{8} \approx 306 \text{ MeV}$$
$$A = \frac{m_\varrho - m_\pi}{4} \approx 159 \text{ MeV}$$
$$a \approx 0.015 \text{ GeV}^3$$

- Kaon masses: $m_K = m_u + m_s - 3A\frac{m_u}{m_s}, \quad m_{K^\star} = m_u + m_s + A\frac{m_u}{m_s} \quad \Rightarrow \quad m_{K^\star} - m_K = \frac{m_u}{m_s}(m_\varrho - m_\pi)$
- difference supressed by $m_u/m_s$: $m_s \approx 496$ MeV $\Rightarrow m_K = 508$ MeV, $m_{K^\star} = 900$ MeV
- works for baryons as well (exercise 4.3): $m_u = 362$ MeV, $m_s = 539$ MeV, $A' = 49$ MeV $\Rightarrow a' = 0.0064$ GeV$^3$

# Color

- Why are hadrons colorless and why we don't observe free quarks (confinment)?

**Nambu 1965** - quarks are confined in hadrons due to new fundamental force

*"mediated by an octet of gauge fields $A^i_\mu$ coupled to infinitesimal SU(3) generators (currents) $\lambda_i$ of the triplets with strength $g$"* **QCD!**

- we don't observe free quarks because they are very (infinitely) heavy, due to new force dynamics (concept of running mass and coupling in QFT) - $M_q \to \infty$

$\mathcal{L}_{int} = \bar{\psi}\lambda_i\gamma_\mu\psi A^\mu_i \;\Rightarrow$ (in analogy with spin-spin interaction) potential $V_{ij} = v(\vec{r}_{ij})\,\vec{T}_{(i)} \cdot \vec{T}_{(j)}$

- binding energy of hadron made out of $n$ quarks

$$V(n) \sim \langle n| \sum_{\text{pairs}} V_{ij} |n\rangle \sim \langle n| \sum_{i<j} \vec{T}_{(i)} \cdot \vec{T}_{(j)} |n\rangle \sim \langle n| \vec{T}^2_{\text{tot}} - \vec{T}^2_{(1)} - \ldots \vec{T}^2_{(n)} |n\rangle \sim C_{\text{tot}} - nC_F$$

- hadron mass: $M(n) = nM_q + k(C - nC_F) = kC + n(M_q - kC_F)$
- the only chance to produce small hadron mass is to compansate large $M_q$ with large $k$ ($k \sim M_q/C_F$)
- $M(n) \sim \frac{C}{C_F} M_q \;\Rightarrow$ small hardon masses only for $C = 0$, which is the case of **color singlet** only

$$C(q_3, q_{\bar{3}}) = \frac{1}{3}\left(q_3^2 + q_3 q_{\bar{3}} + q_{\bar{3}}^2\right) + (q_3 + q_{\bar{3}}) \qquad C(0,0) = 0 \qquad C_F = C(1,0) = C(0,1) = \frac{4}{3}$$

# Color

- we find singlets in: $3 \otimes \bar{3} = 8 \oplus 1$ ($q\bar{q}$ - mesons), $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ ($qqq$ - baryons)
- or in: $3 \otimes \bar{3} \otimes 3 \otimes \bar{3}$ ($q\bar{q}q\bar{q}$ - tetraquarks), $3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3}$ ($qqqq\bar{q}$ - pentaquarks)
  - $Z(4430)^+$ $c\bar{c}d\bar{u}$ state (Belle 2007, LHCb 2014), $P_c(4380)^+$ $c\bar{c}uud$ state (LHCb 2015)
- or in: $8 \otimes 8$ ($gg$ - Pomeron), $8 \otimes 8 \otimes 8$ ($ggg$ - Oderon), $8 \otimes 8 \otimes \ldots \otimes 8$ (glueballs) (exercise 6.5)
  - BESIII announced in 2025 a discovery of glueball candidate X(2370)
- but **not** in: $3 \otimes 3 = 6 \oplus \bar{3}$ (no $qq$ states), $3 \otimes 3 \otimes \bar{3}$, $3 \otimes 3 \otimes 3 \otimes 3$, (exercise 3.10)
- binding energy between color and anti-color in meson

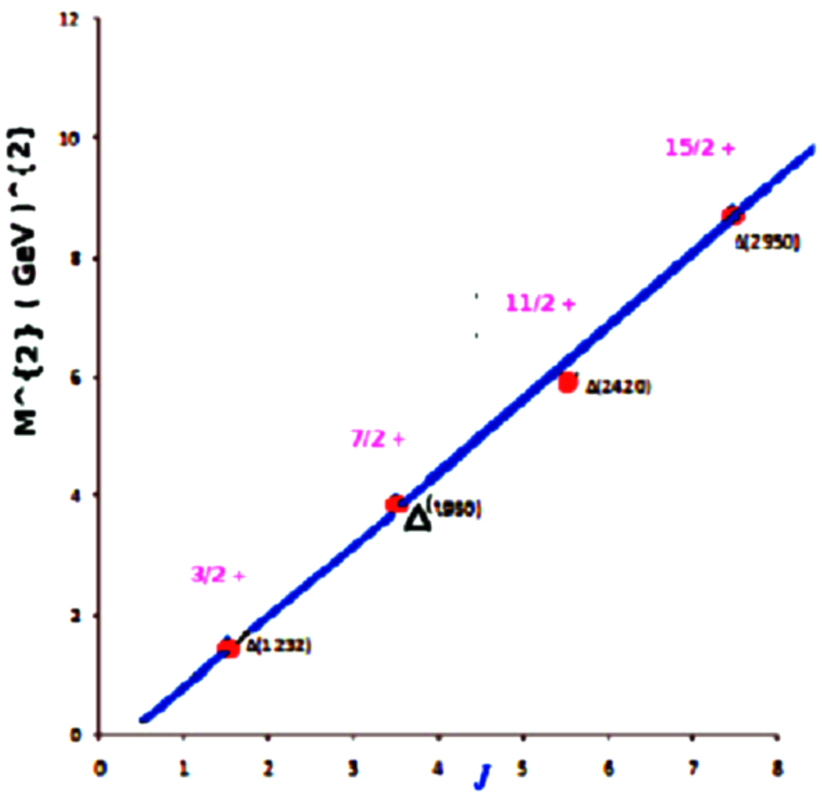
$$V_{q\bar{q}}^{meson} \sim 0 - 2 \cdot \frac{4}{3} = -\frac{8}{3}$$

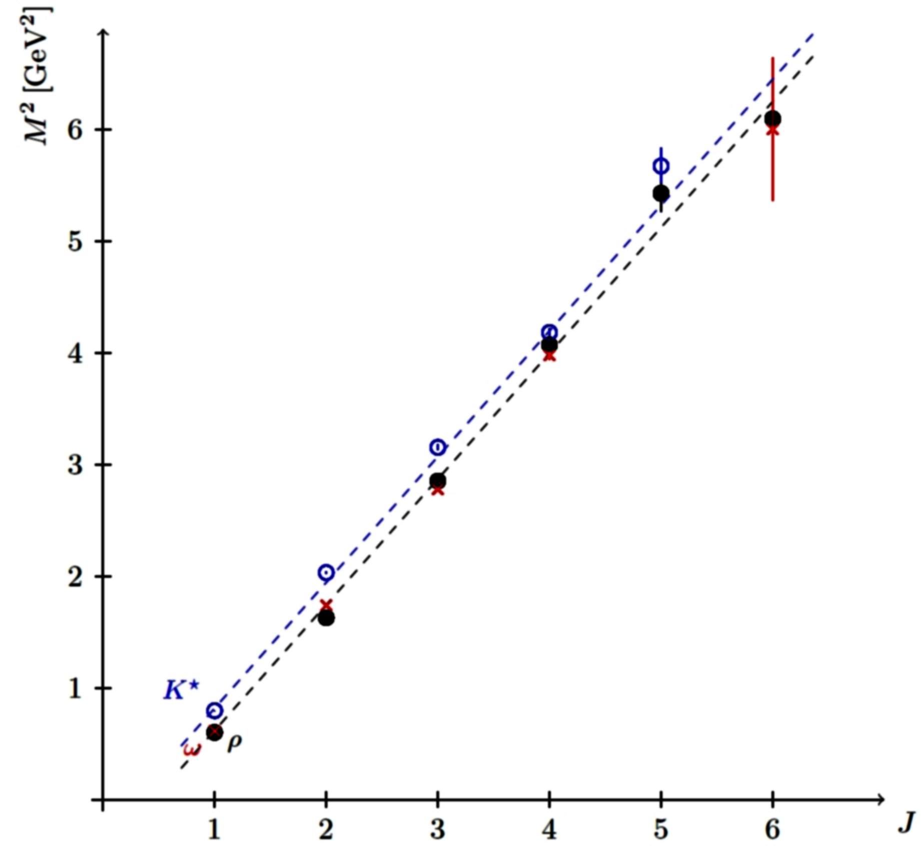
- two colors in baryon can be either in 6 or $\bar{3}$
- cannot be in 6, as $6 \otimes 3 = 10 \oplus 8$ has no singlet

$$V_{qq}^{baryon} \sim \frac{4}{3} - 2 \cdot \frac{4}{3} = -\frac{4}{3}$$

- Nambu models explains why $\alpha \sim 2\alpha'$

# Hadron spin and masses and string theory




$M^2 \sim J$ (Regge trajectory)

Answer to *"Why?"* led to **string theory**

**Inelastic (non-relat.) solid body**

$$E_k = \frac{1}{2}L\omega\,, \quad L = I\omega \;\Rightarrow\; E \sim L^2$$

**Elastic (non-relat.) solid body**

- maximal orbital speed $\omega_{\text{max}}$ at which the body streches to infinity $(l \to \infty)$

  for $\omega \to \omega_{\text{max}}$:

$$L \to \infty,\; E \sim L\omega_{\text{max}} \;\Rightarrow\; E \sim L$$

**Rotating relat. elastic string** - maximal orbital speed at which the edge moves with $c$: $\omega_{\text{max}} = 2/l$

- string rest density $\varrho(2x/l)$, near $\omega_{\text{max}} = 2/l$: $\varrho(\xi = \omega_{\text{max}}x)$

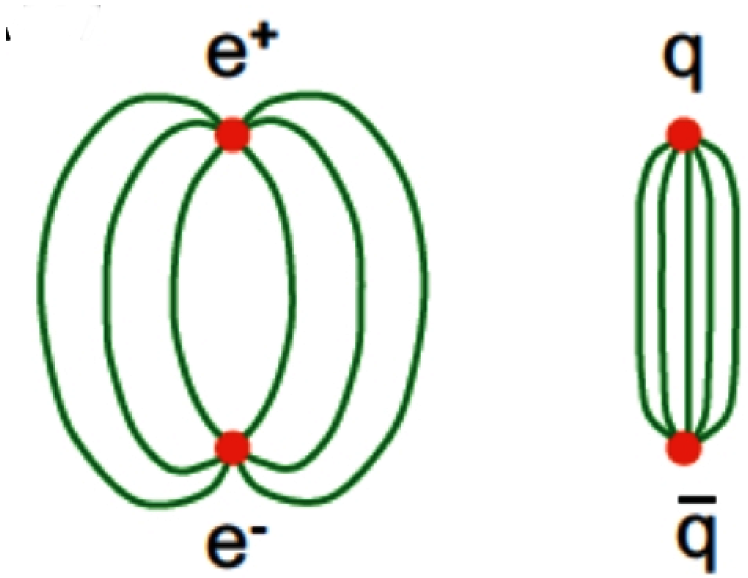
$$M = E = \int_{-l/2}^{l/2} \mathrm{d}x \frac{\varrho(\omega_{\text{max}}x)}{\sqrt{1-(\omega_{\text{max}}x)^2}} = \frac{2}{\omega_{\text{max}}}\int_0^1 \mathrm{d}\xi \frac{\varrho(\xi)}{\sqrt{1-\xi^2}} \sim l, \qquad L = \int_{-l/2}^{l/2} \mathrm{d}x \frac{\varrho(\omega_{\text{max}}x)x^2\omega_{\text{max}}}{\sqrt{1-(\omega_{\text{max}}x)^2}} = \frac{2}{\omega_{\text{max}}^2}\int_0^1 \mathrm{d}\xi \frac{\varrho(\xi)\xi^2}{\sqrt{1-\xi^2}} \sim l^2$$

- linear increase of energy with string length and $M^2 \sim L$
- relativistic rotating disc: $M^{3/2} \sim L$, sphere $M^{4/3} \sim L$ (exercise 4.7)

**L. Susskind** - Lecture 1 on String Theory and M-Theory

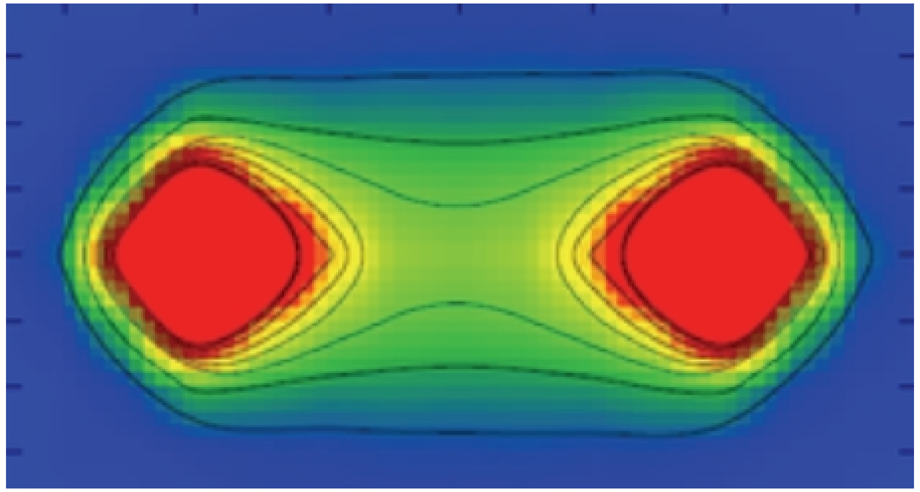
www.youtube.com/watch?v=25haxRuZQUk

# Color string

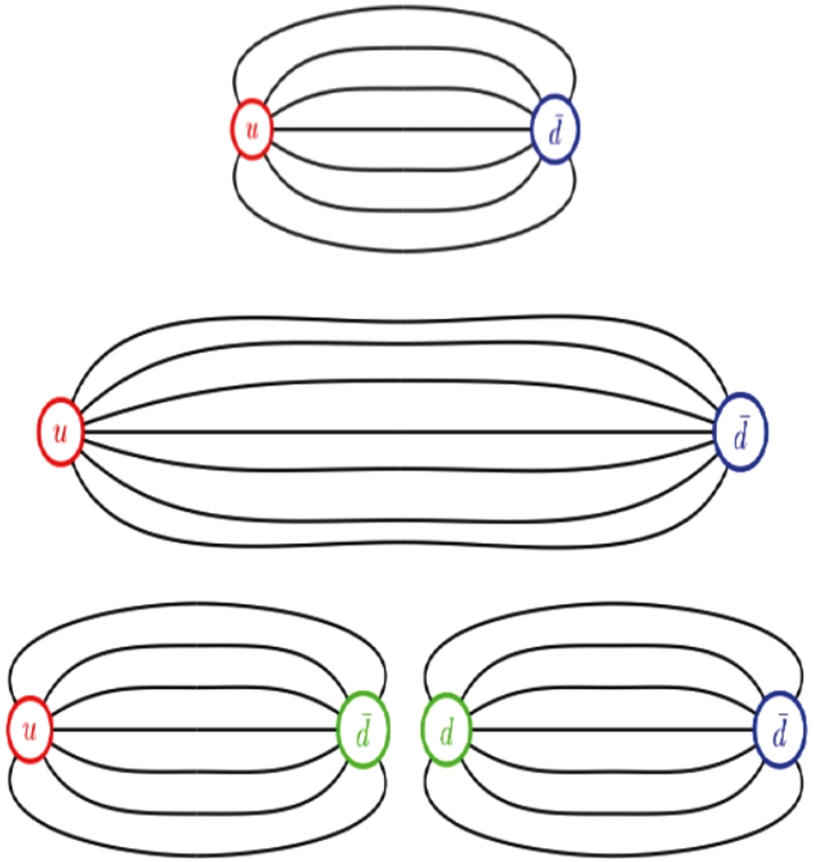



quarks
and
gluon field

Meson in Lattice QCD

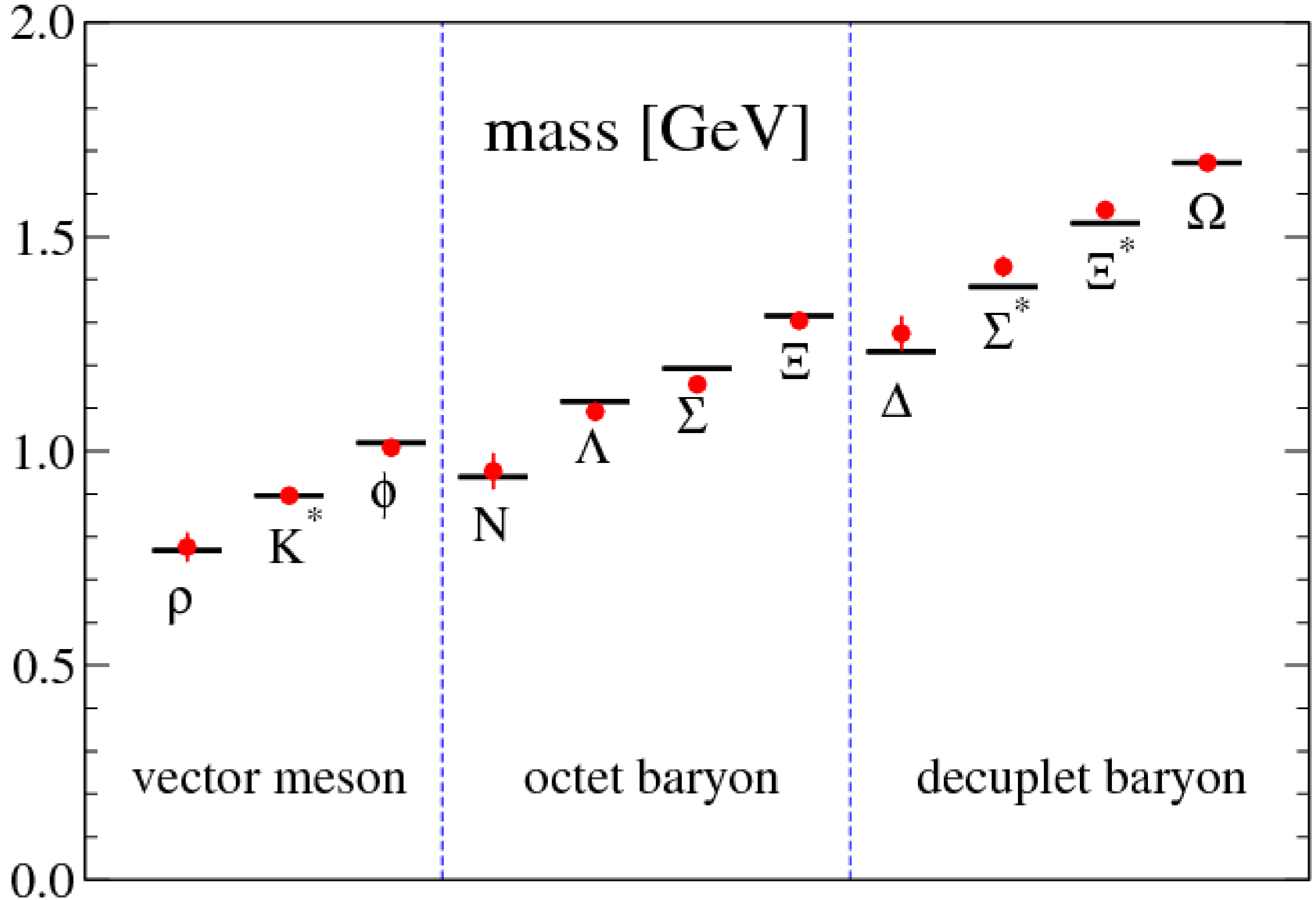


PRD81 (2010) 034504




- in electrostatics - field spreads over entire space
- in QCD - gluon field concentrates into narrow tube
- energy between two colors increases linearly with distance
- at some point, it is favorable to generate color anti-color pair from vacuum and break the string

# Hadron masses in lattice QCD




PACS-CS Coll., Phys. Rev. D79 (2009) 034503

Input: masses of $\pi$, $K$ a $\Omega$ (they determine current quark masses)

# Charm

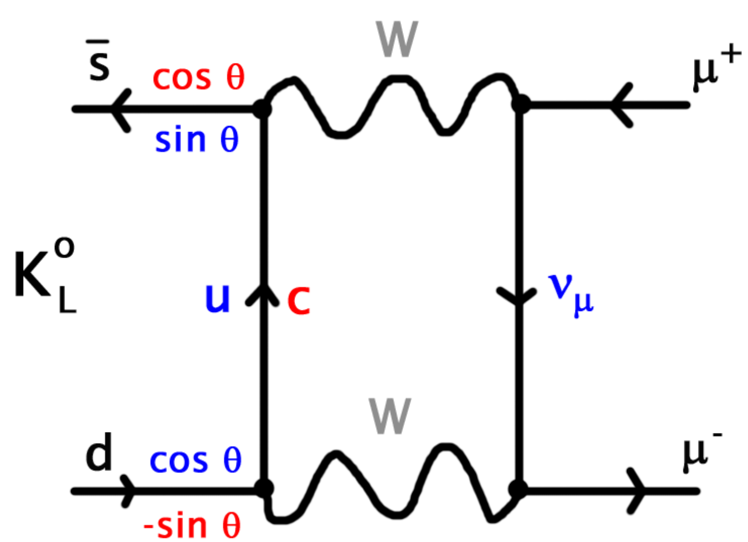
- speculations about $4^{\text{th}}$ quark, shortly after Zweig and Gell-Mann quark model (Björken, Glashow 1964)

- symmetry between quarks and leptons - discovery of $\nu_\mu$ in 1962 (Lederman, Schwartz, Steinberger - NP 1982)

- **GIM mechanism** (Glashow-Iliopulos-Maiami 1970) - $4^{\text{th}}$ quark provided elengant solution of **FCNC**

$$K^+ \to \pi^0 e^+ e^- \quad (\text{Br} = 0.048), \qquad K^+ \to \pi^+ e^+ e^- \quad (\text{Br} = 2.7 \cdot 10^{-7})$$

- another example of FCNC - rare decay $K_L^0 \to \mu^+\mu^-$ (Br $\approx 7 \cdot 10^{-9}$)




- 3-quarks weak current $J_\mu = \bar{u}\gamma_\mu(1+\gamma_5)(d\cos\theta + s\sin\theta)$ leads to Br $\approx 10^{-4}$
- 4-quarks weak current $J_\mu = U\gamma_\mu(1+\gamma_5)CD$

$$U = \begin{pmatrix} u \\ c \end{pmatrix} \qquad D = \begin{pmatrix} d \\ s \end{pmatrix} \qquad C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

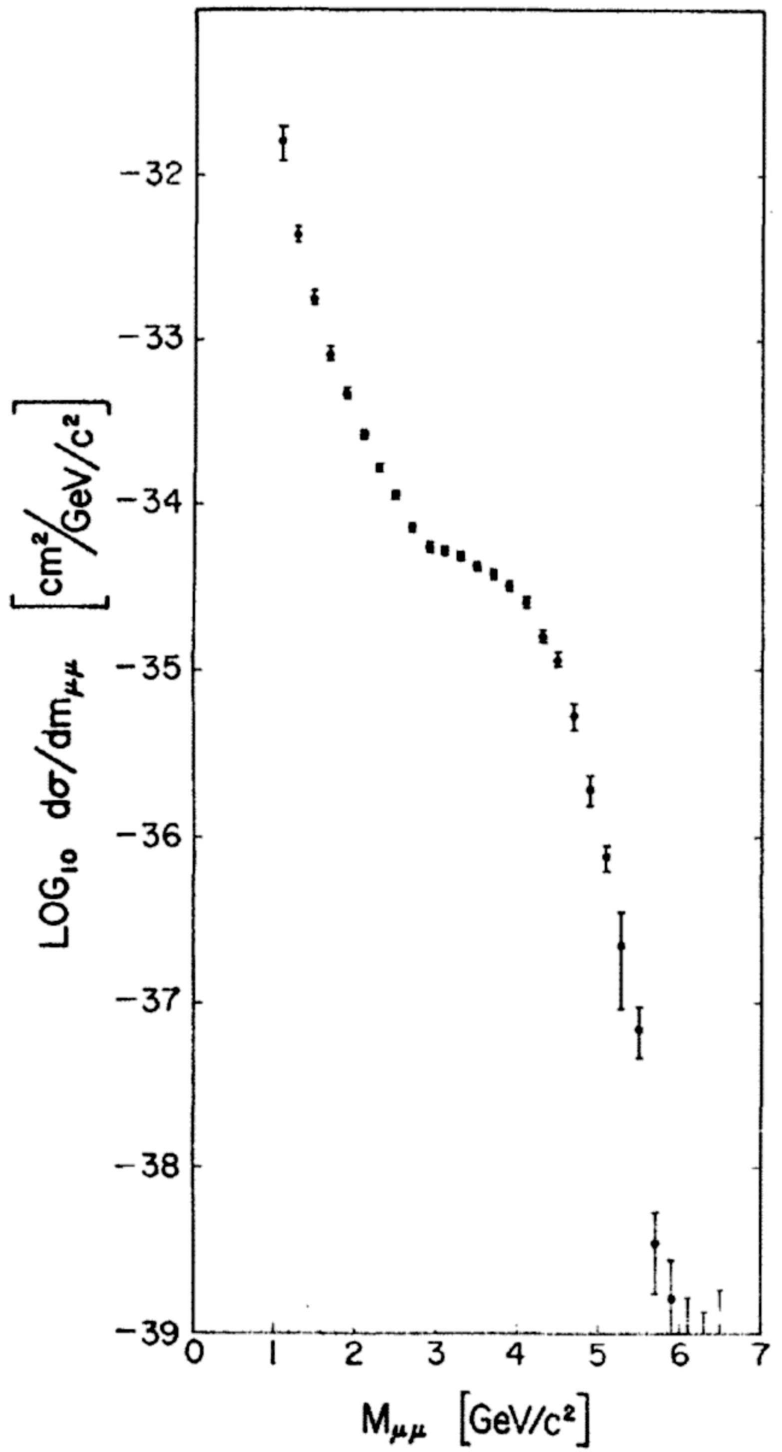
- same but opposite contribution for $c$ quark (if masses are neglected)

$$\text{Br} \approx \frac{m_c^2 - m_u^2}{m_W^2} 10^{-4} \approx \frac{m_c^2}{m_W^2} 10^{-4} \quad \Rightarrow \quad m_c \approx 0.01\, m_W \approx 0.8 \text{ GeV}$$

Historically, first estimates of $m_c$ even before the discovery of $c$

- cancelation of **EW axial anomalies** requires symmetry between leptons and quarks

# Prolog to discovery of $c$ quark




PRL25 (1970) 1523

**Lederman** in Brookhaven (1970)

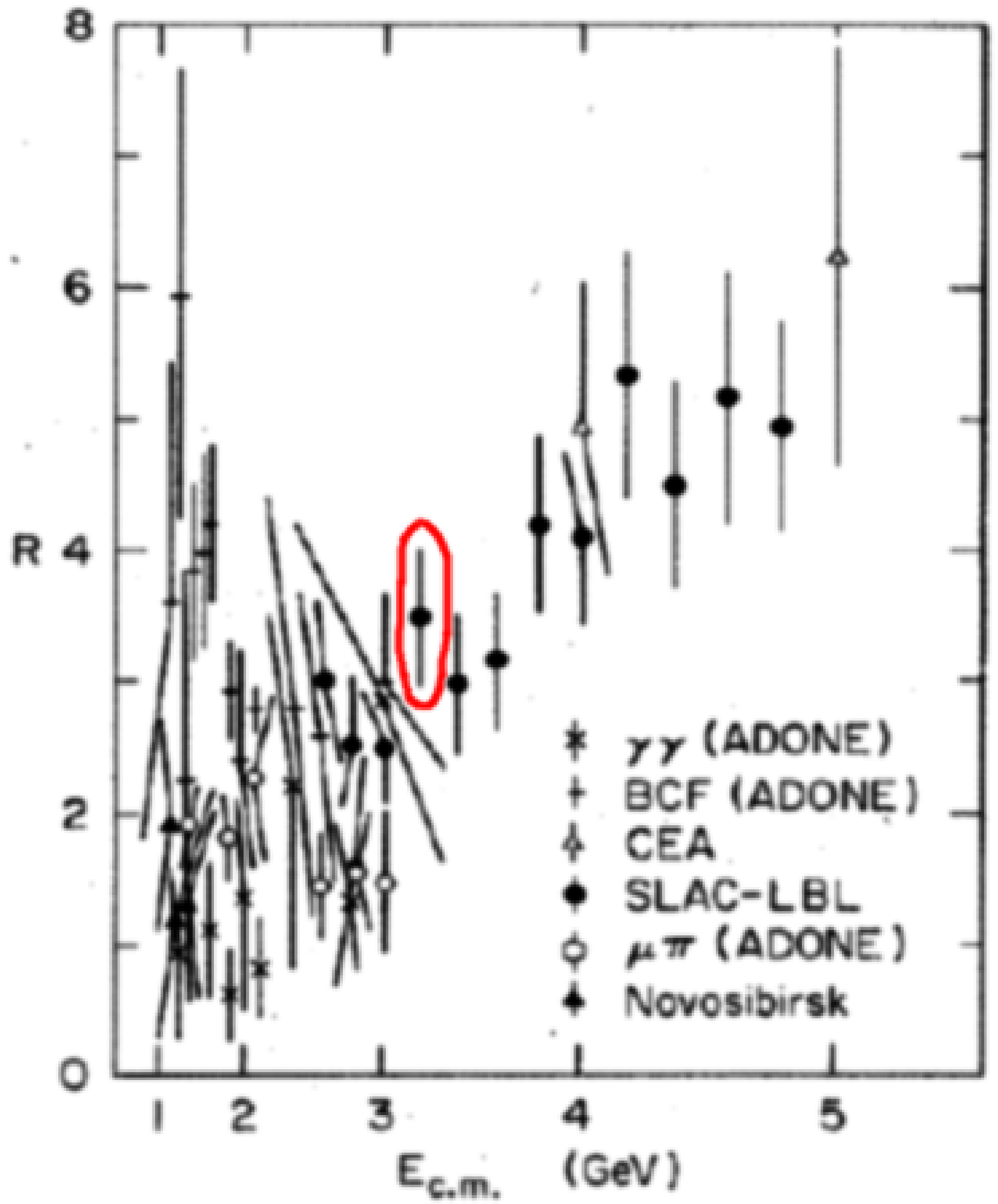
- 30 GeV proton beam on heavy target $p + U \to \mu^+\mu^- + X$
  - practicaly only muons could leave the target
  - bad energy and invariant mass resolution
- experiment was looking for "heavy fotons"
  - $c\bar{c}$ is such state (!)
  - wide resonances expected
  - this is exactly what was observed
- however

Muon pairs in the mass range $1 < m_{\mu\mu} < 6.7$ GeV$/c^2$ have been observed in collisions of high-energy protons with uranium nuclei. At an incident energy of 29 GeV, the cross section varies smoothly as $d\sigma/dm_{\mu\mu} \approx 10^{-32}/m_{\mu\mu}{}^5$ cm$^2$ (GeV$/c)^{-2}$ and exhibits no resonant structure. The total cross section increases by a factor of 5 as the proton energy rises from 22 to 29.5 GeV.

PRL25 (1970) 1523

- Lederman missed the oportunity for the second Nobel Prize

# $e^+e^- \rightarrow$hadrons, summer 1974




$$R = \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-}$$

- in parton model (also LO QCD)

$$R = N_c \sum_i^{kvarky} e_i^2$$

- For 3 quarks (u,d,s): R=2
- For 4 quarks (u,d,s,c): R=10/3

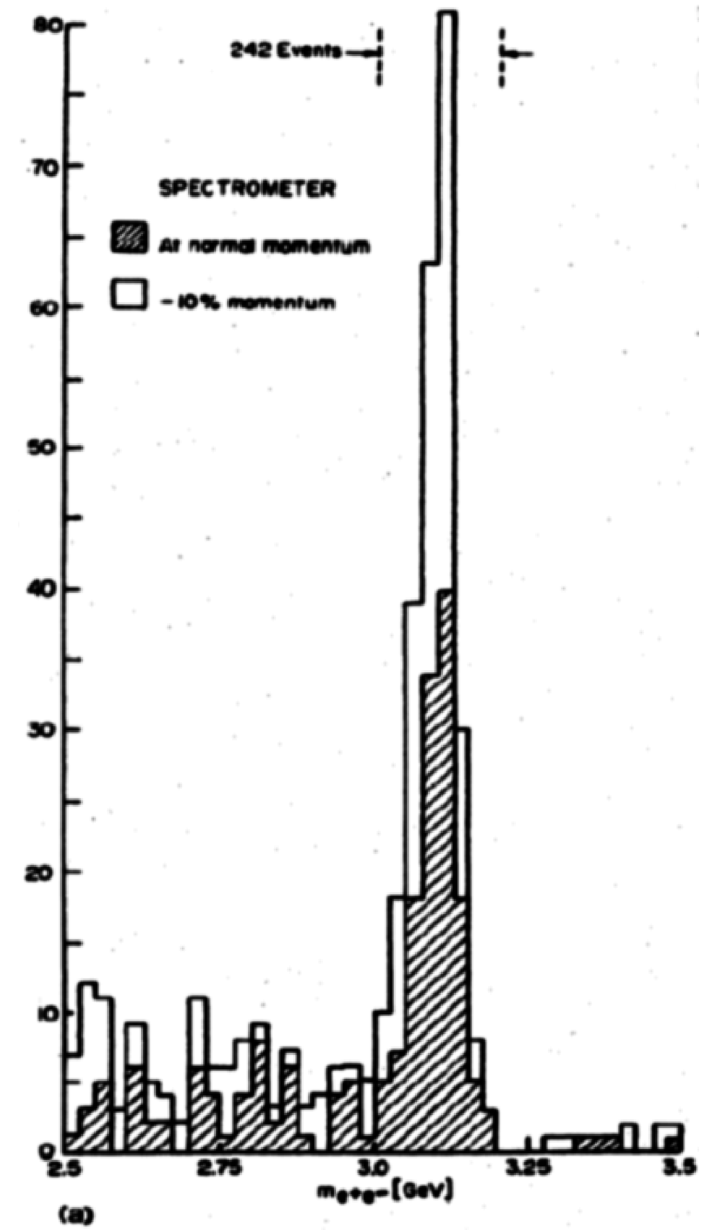
- No obvious resonance

# Ting - discovery of $J$ (Brookhaven)

$$p + Be \rightarrow e^+e^- + X$$

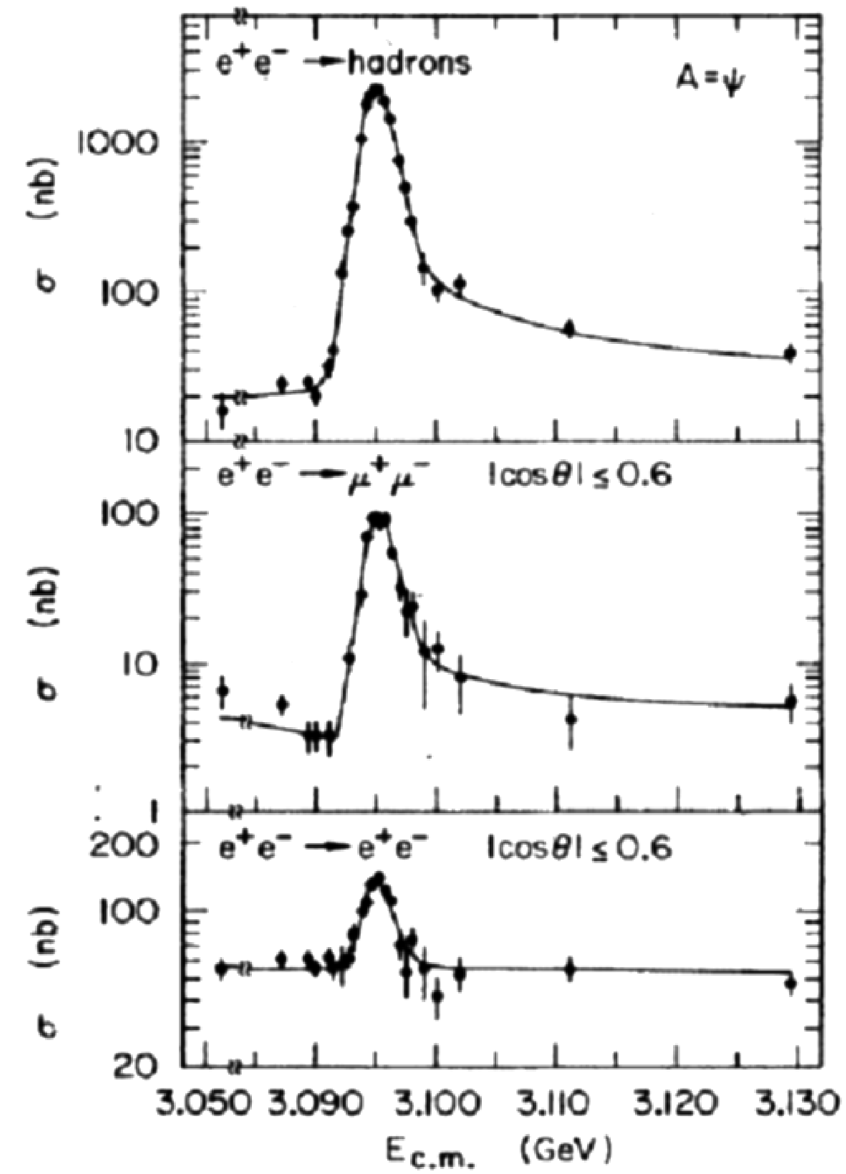
- magnetic 2-arm spectrometer with excellent resolution ($5$ MeV)
- electron identification with Čerenkov detector
- search for resonances in lepton pair invariant mass

- **August 1974**: Glashow visits Ting in Brookhaven to convice him to look for $c$ quark in pairs of strange kaons and pions, NOT IN $e^+e^-$
- **September 2**: Rhoades notices a cumulation of events at $3.1$ GeV mass (didn't tell Ting)
- **early October** - restart of experiment after accelerator shutdown
  - resonance starts to be obvious but no announcement
  - Ting wants an independent check
- **early November**: Ting knows that the resonance is new particle, probably $c\bar{c}$ state
  - still no publication, Ting wants to discover the whole family of $c$ hadrons
  - Ting under pressure from collaborators as the particle could be discovered meanwhile at SLAC in $e^+e^-$ collisions



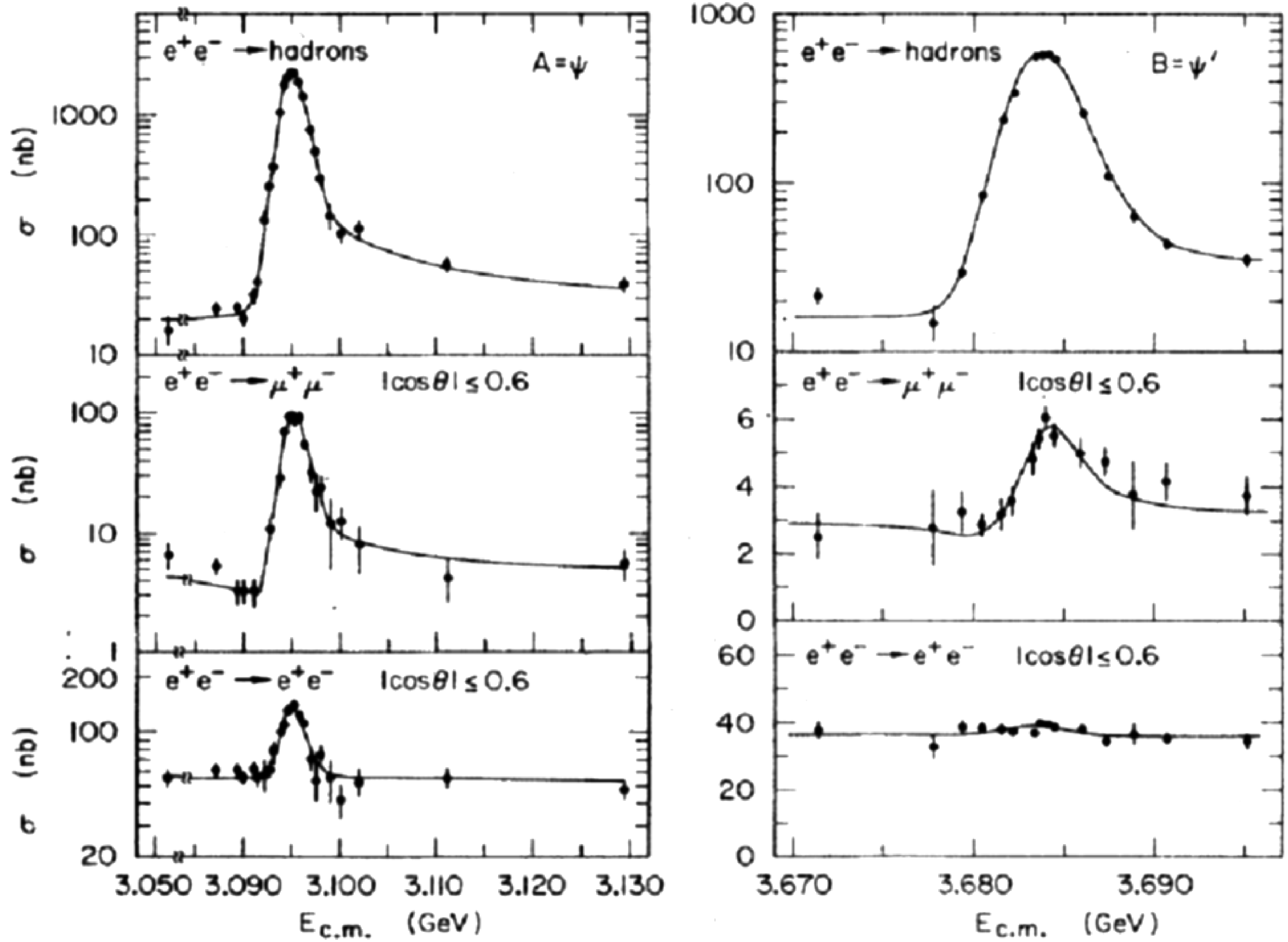
# Richter - discovery of $\Psi$ (SLAC)

- **November 4,1974** - Richter back at SLAC
  - his team refers about small cross section increase at $\sqrt{s} \sim 3.1$ GeV
  - proposal to reconfigure SPEAR from $5$ GeV back to this energy
- **Nov 8** - Goldhaber convinces Richter to tune SPEAR back to $3.1$ GeV (he showed an increased production of $K$ mesons in the same run)
- **Nov 9** - hadron event rate: $1$ per min. ($3\times$ above normal)
- **Nov 10** - $7\times$ increase in rates at $3.11$ GeV; Goldhaber starts to write discovery paper; an hour later the team is reporting increase of $70\times$ at $3.094$ GeV; the peak was found at $3.095$ GeV (rate 100$\times$)
- **Nov 10** - Ting flights to meeting at SLAC; at deep night from Sunday to Monday he got a message in the hotel from colleagues at Brookaven about the discovery at SLAC (TWA did not passed down the message - they considered a message at 1:00 AM as a joke)
  - Ting writes during the night a discovery paper




- **Nov 11** - Ting meets with Richter in the morning - they report to each other about their discoveries
- 1976 - Nobel Prize for Ting and Richter *"for their pioneering work in the discovery of a heavy elementary particle of a new kind."*

# SPEAR, Richter



# $J/\Psi$

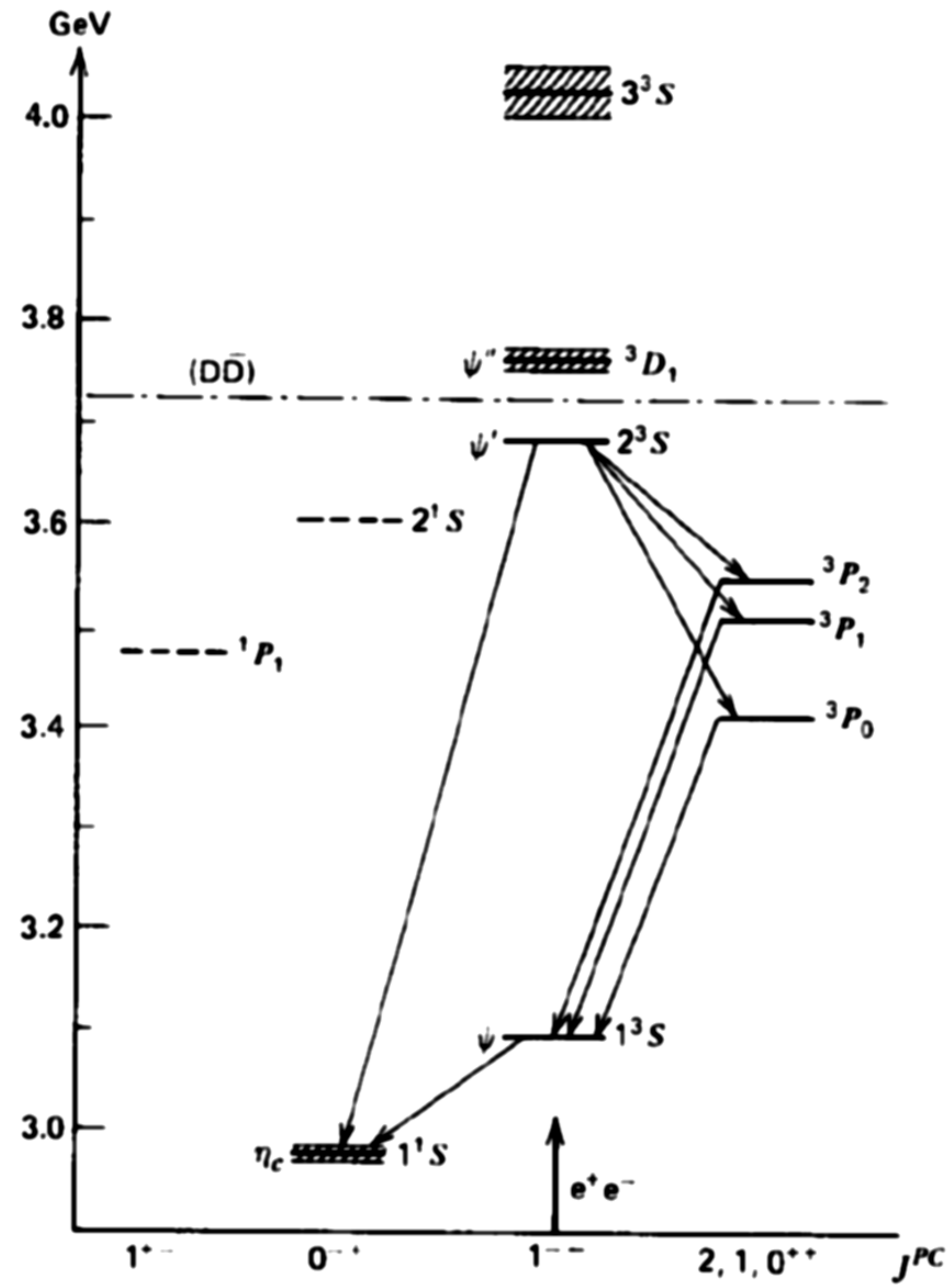
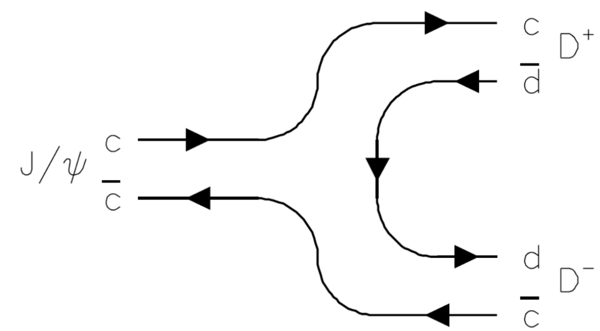



**Fig. 2.13 The observed charmonium spectrum. The**

- mass $m_{J/\Psi} = 3.16$ GeV
- very narrow, $\Gamma = 88$ keV

Zweig allowed
kinematically forbiden




Zweig forbiden
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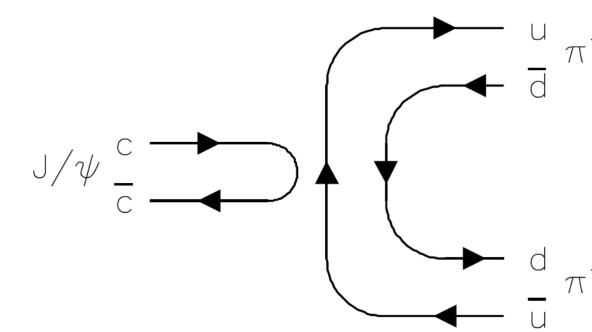



$m_{D^+} = 1.86$ GeV $\Rightarrow$ $2m_D > m_{J/\Psi}$

- $c$ quark quite heavy $m_c \approx 1.6$ GeV

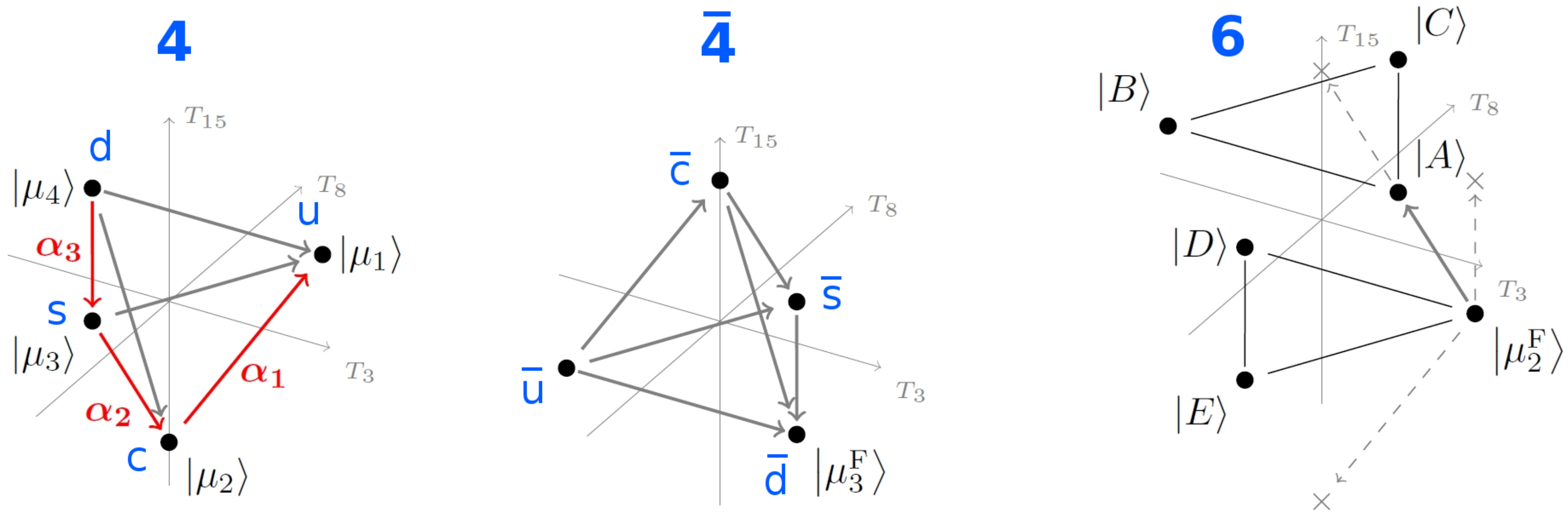
⇒ charmonium spectrum ($c\bar{c}$ bound states) can be calculated in non-relativistic QM

- effective potential that leads to observed spectrum

$$V_{\text{eff}}(r) = -\frac{4}{3}\frac{\alpha}{r} + kr$$

# Classification of charm mesons - SU(4)

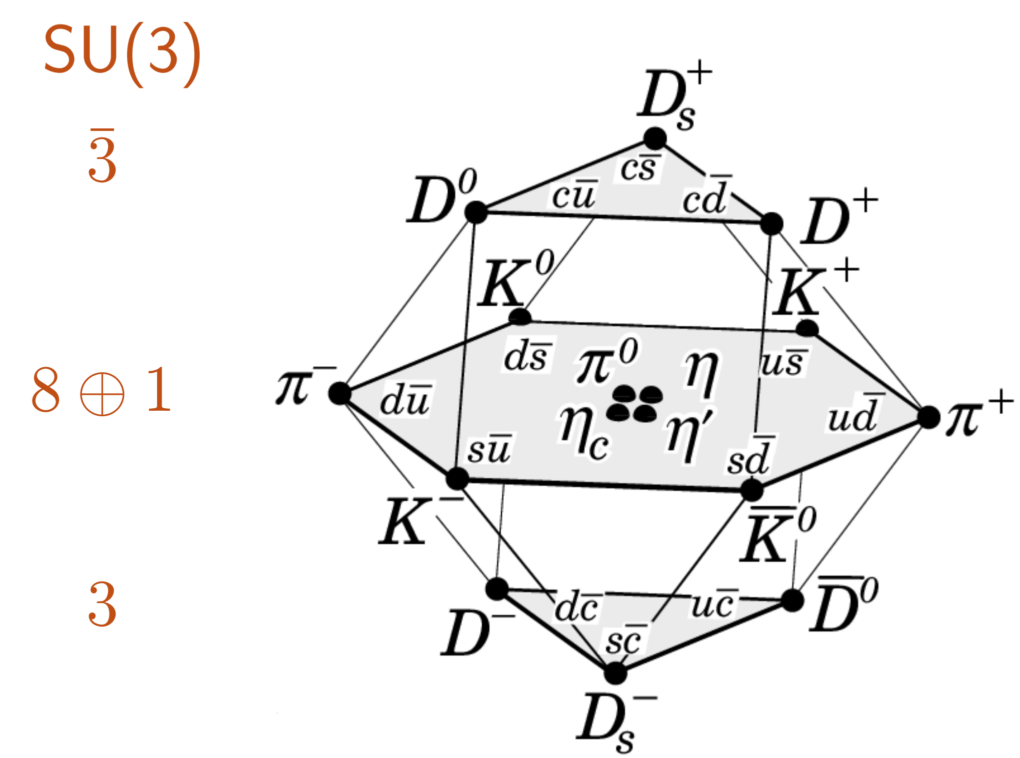
- SU(4) - 15 generators, rank 3 $\Rightarrow$ 3 simple roots and 3 fundamental representations (ex. 3.15)




- mesons $4 \otimes \bar{4} = 15 \oplus 1$

  ex. 3.16: for SU(N), $N \otimes \bar{N} = A \oplus 1$

pseudoscalar multiplet $\rightarrow$




PDG, Phys. Rev. D98 (2018) 030001

# Baryon SU(4) multiplets

$$4 \otimes 4 \otimes 4 = 20_s \oplus 20_{m,s} \oplus 20_{m,a} \oplus 4_a$$

- spin $1/2$ baryons wave function

$$20_{m,s} \otimes 2_{m,s} + 20_{m,a} \otimes 2_{m,a}$$

- spin $3/2$ baryons wave functions

$$20_s \otimes 4_s$$

  - contains only symmetric SU(3) multiplets

$\Xi_{cc}^{+}$ discovered by LHCb in 2026
with mass of about $3620$ MeV

$SU(3): 3$

$6 \oplus \bar{3}$

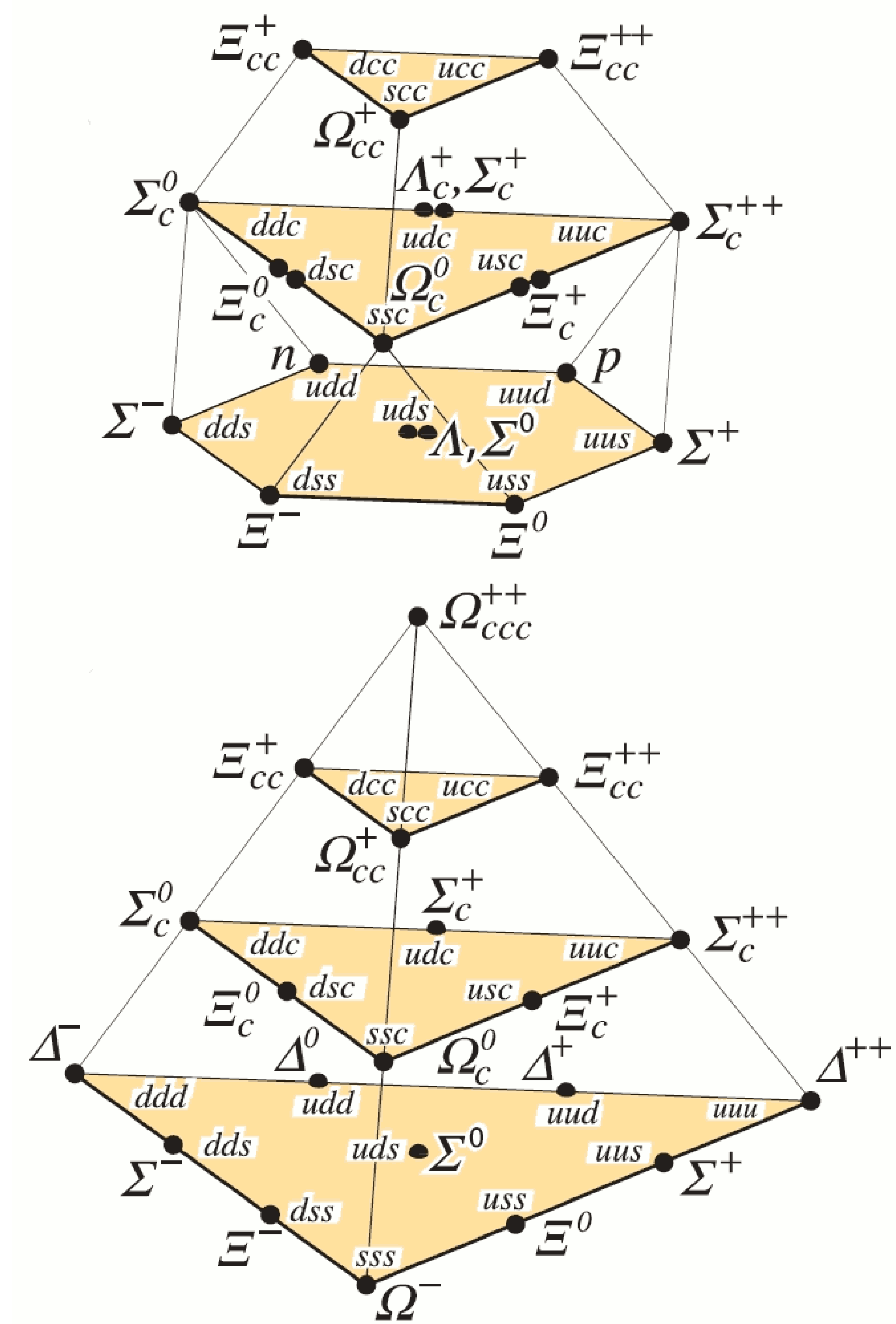
$8$

$1$

$3$

$6$

$10$

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# $b$ quark

- History repeates
  - $\tau$ lepton discovered in 1975 in SLAC (Perl - BN in 1995)
  - implied and existence of two more (3rd generation) quarks (**bottom** and **top**)
    ($\nu_\tau$ was discovered in 2000 by DONUT in Fermilab as the last fundamental fermion)
- Disovery of $b$ quark
  - Leon Lederman - 1977 in Fermilab ($400$ GeV proton beams)

$$p + B \to e^+e^- + X \qquad \text{or} \qquad p + B \to \mu^+\mu^- + X$$

  - magnetic spectrometer with resolution $\sigma \sim 2\%$ (still much worse than Ting)

# $\Upsilon$ discovery - Fermilab (Lederman)

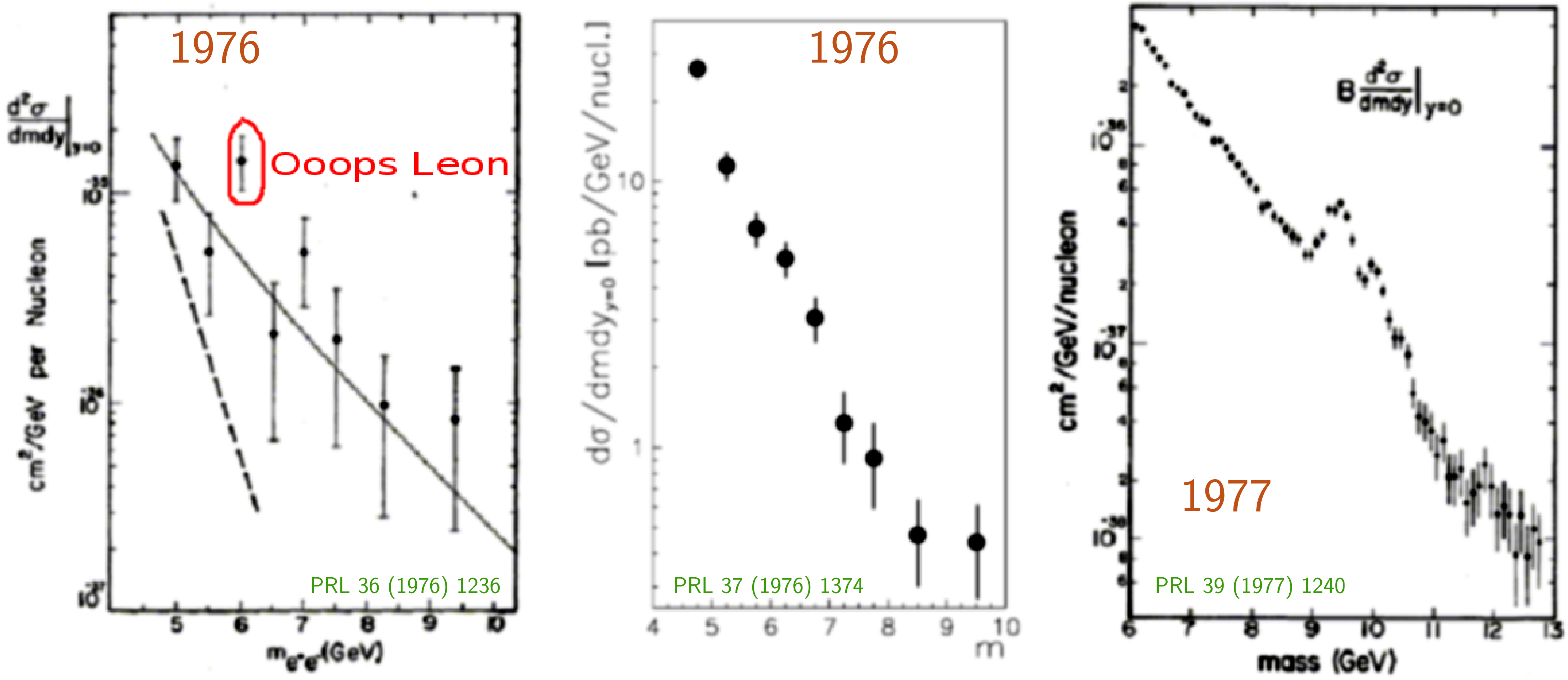



**Figure 4.8: Twists and turns of the discovery of the bottom quark. Evidence of "a new resonance at 6 GeV" [81] observed in the distribution of invariant mass of $e^+e^-$ pairs (left), but shortly afterwards ruled out by the data [82] on $\mu^+\mu^-$ pairs (middle), and replaced by the observation of the dimuon resonances at 9.44 and 10.06 GeV in [84] (right).**

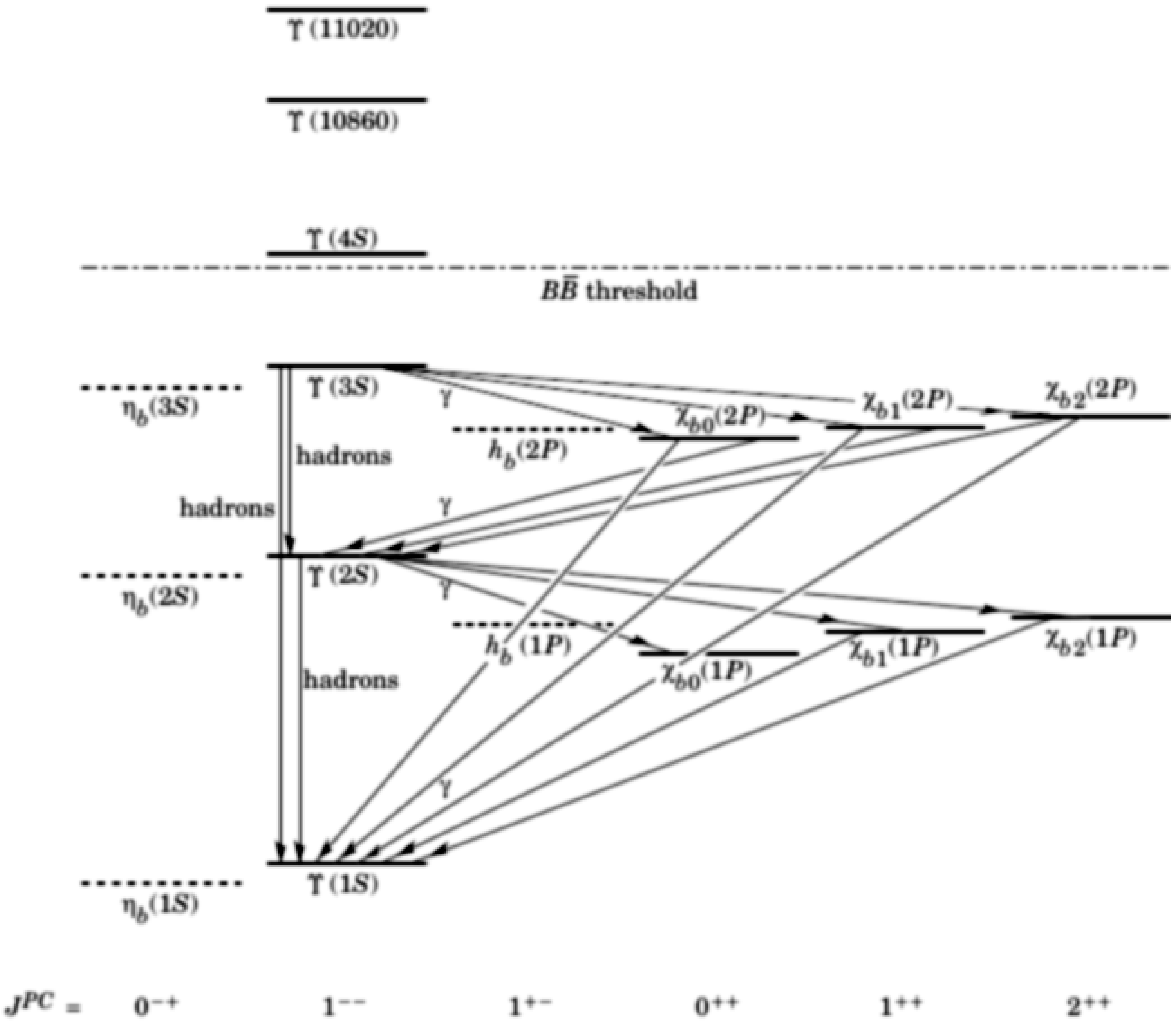
We report preliminary results on the production of electron-positron pairs in the mass range 2.5 to 20 GeV in 400-GeV $p$-Be interactions. 27 high-mass events are observed in the mass range 5.5–10.0 GeV corresponding to $\sigma = (1.2 \pm 0.5) \times 10^{-35}$ cm$^2$ per nucleon. Clustering of 12 of these events between 5.8 and 6.2 GeV suggests that the data contain a new resonance at 6 GeV.

PRL 36 (1976) 1236

We report results on the production of muon pairs in the mass range 2.5 to 20 GeV in 400-GeV proton-nucleus collisions. A total of 159 events are observed in the mass range 5.5 to 11 GeV with a cross section which is in agreement with the previous observation of a high-mass dielectron continuum signal in this interval. Details on the production dynamics and comparisons with parton-model predictions are presented. Within limitations of resolution and continuum uncertainty, the dimuon mass spectrum provides no evidence for fine structure above 5 GeV.

PRL 37 (1976) 1374

# $b\bar{b}$ spectroscopy - bottomonium states




J. Chyla, Quark, partons and Quantum Chromodynamics

- three states below $B\bar{B}$ threshold

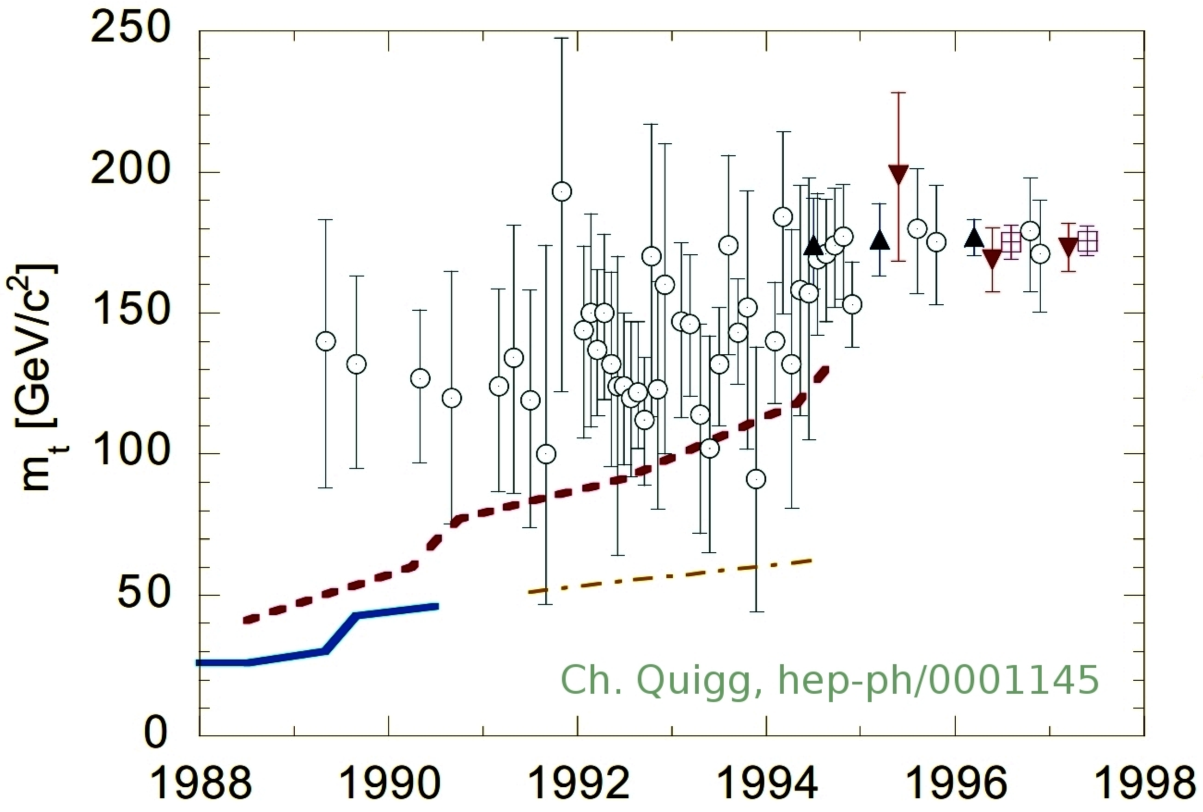
  $\Upsilon(1S): m \approx 9.46$ GeV, $\Gamma \approx 54$ keV
  $\Upsilon(2S): m \approx 10.02$ GeV, $\Gamma \approx 32$ keV
  $\Upsilon(3S): m \approx 10.36$ GeV, $\Gamma \approx 20$ keV

- one state just above $B\bar{B}$ threshold

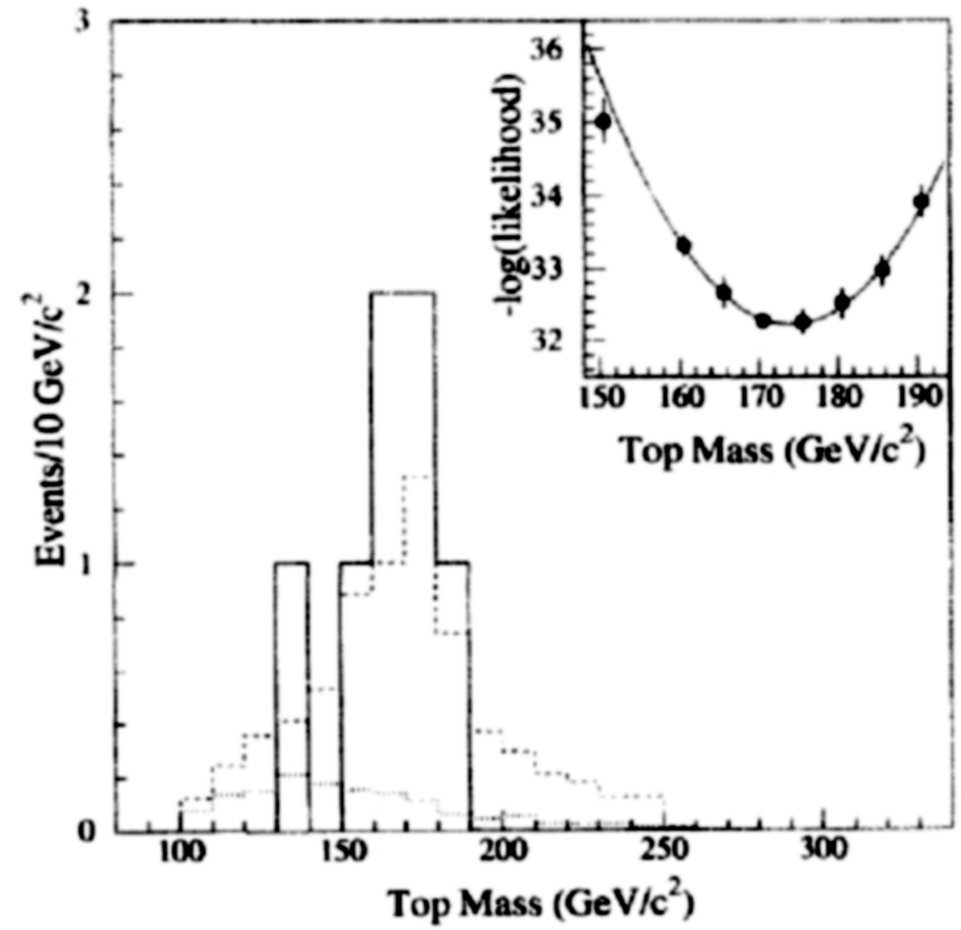
  $\Upsilon(4S): m \approx 10.58$ GeV, $\Gamma \approx 21$ MeV

- $b$ quark mass: $m_b \approx 5$ GeV
  - non-relativistic approximation (description with effective potential) works even better than in case of charmonia

# Top quark discovery



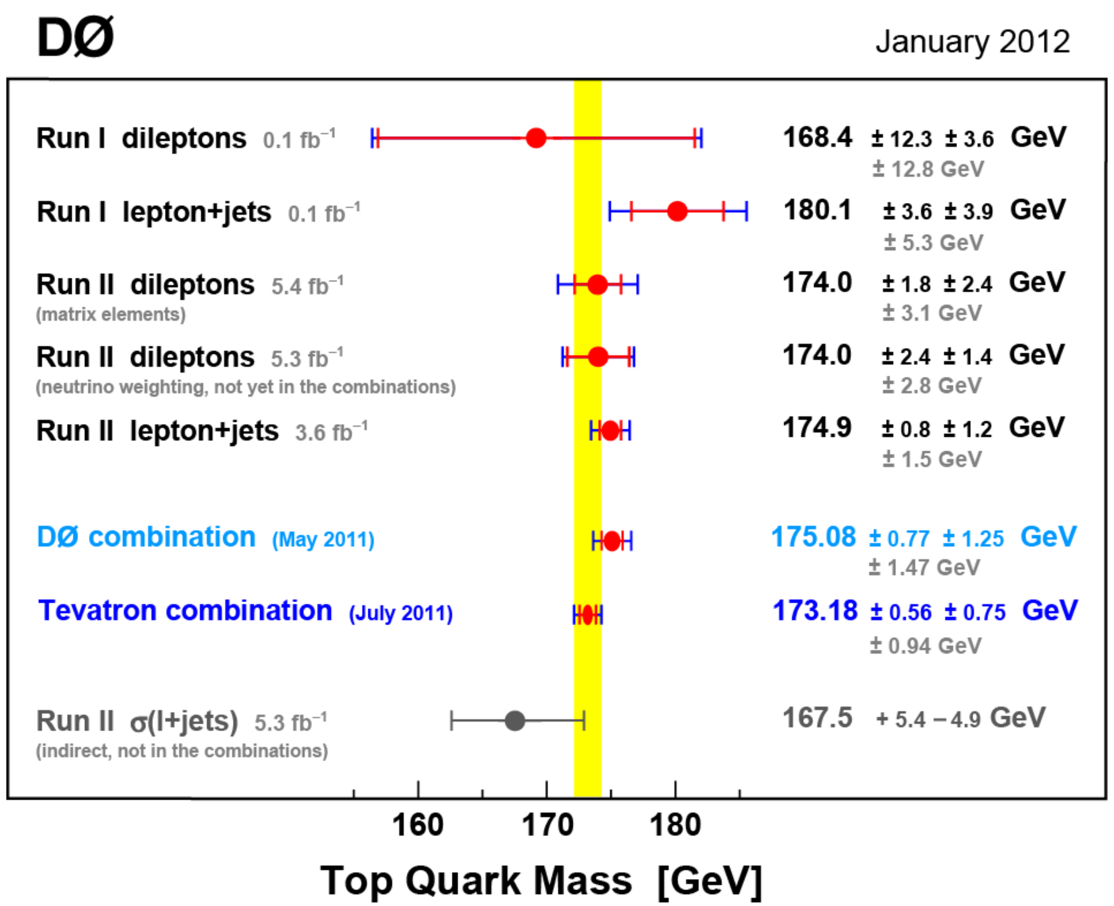

- inderect estimates from LEP
- for example Z decay width is, thanks to radiation corrections, sensitive to top quark mass

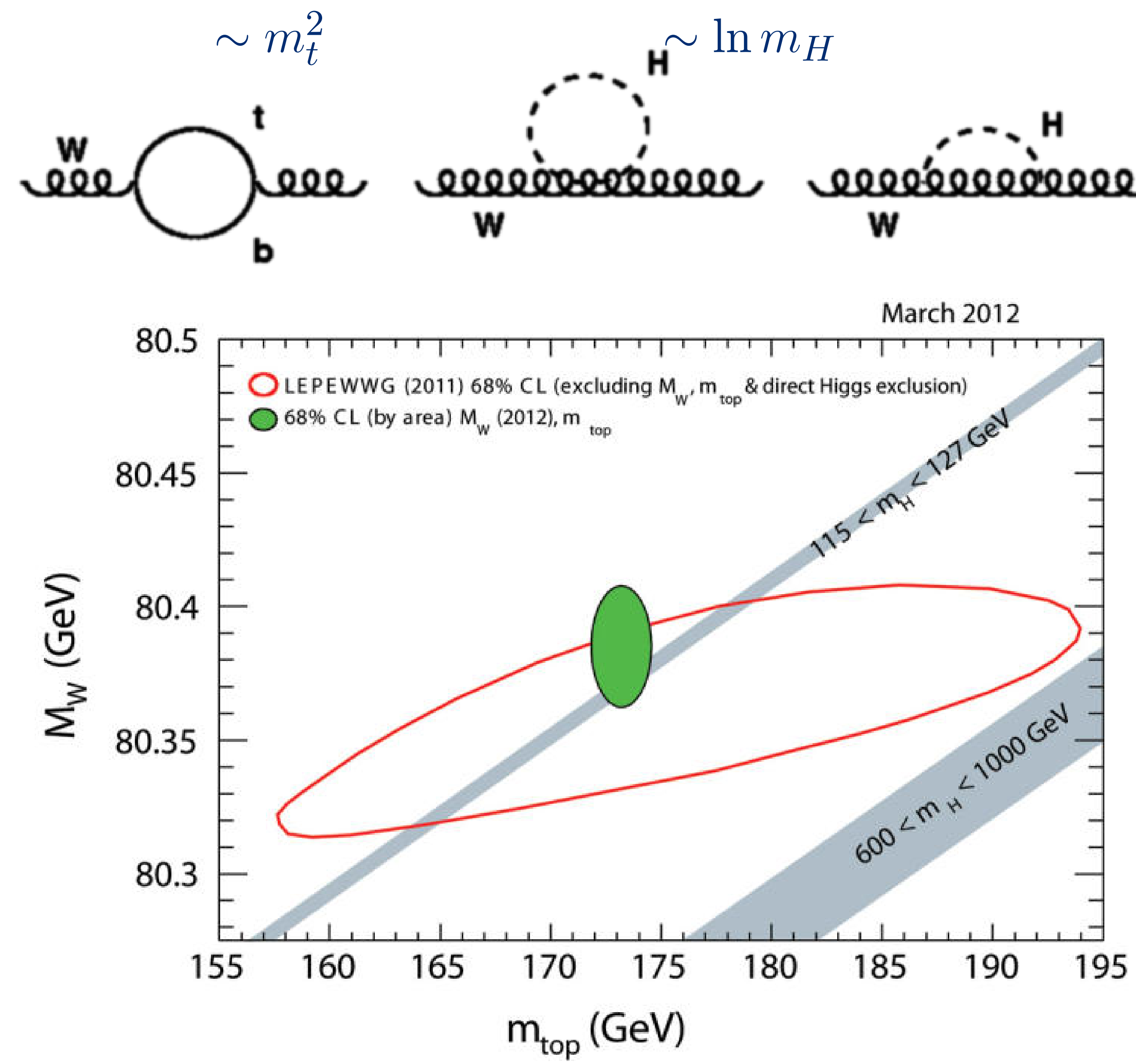



- Tevatron 1994 (CDF and DØ experiments)

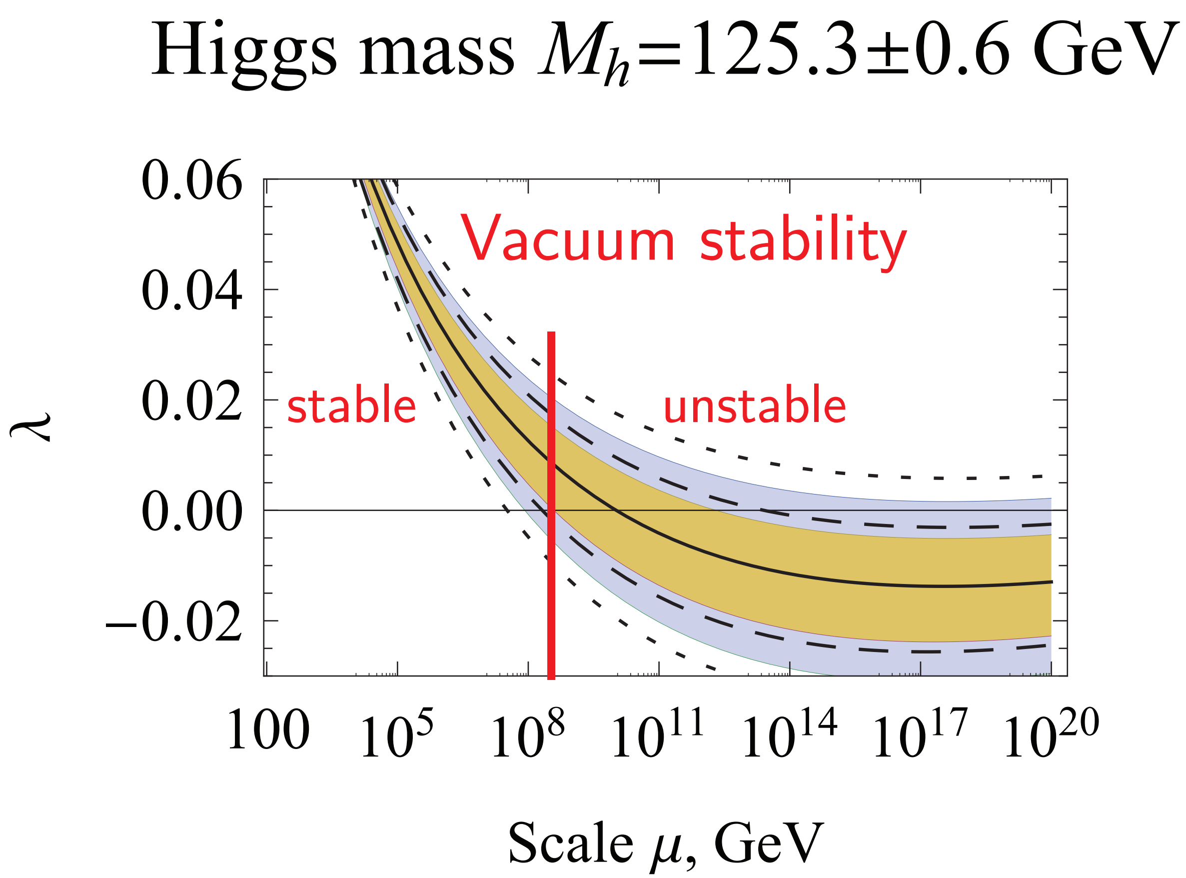
# Top quark mass

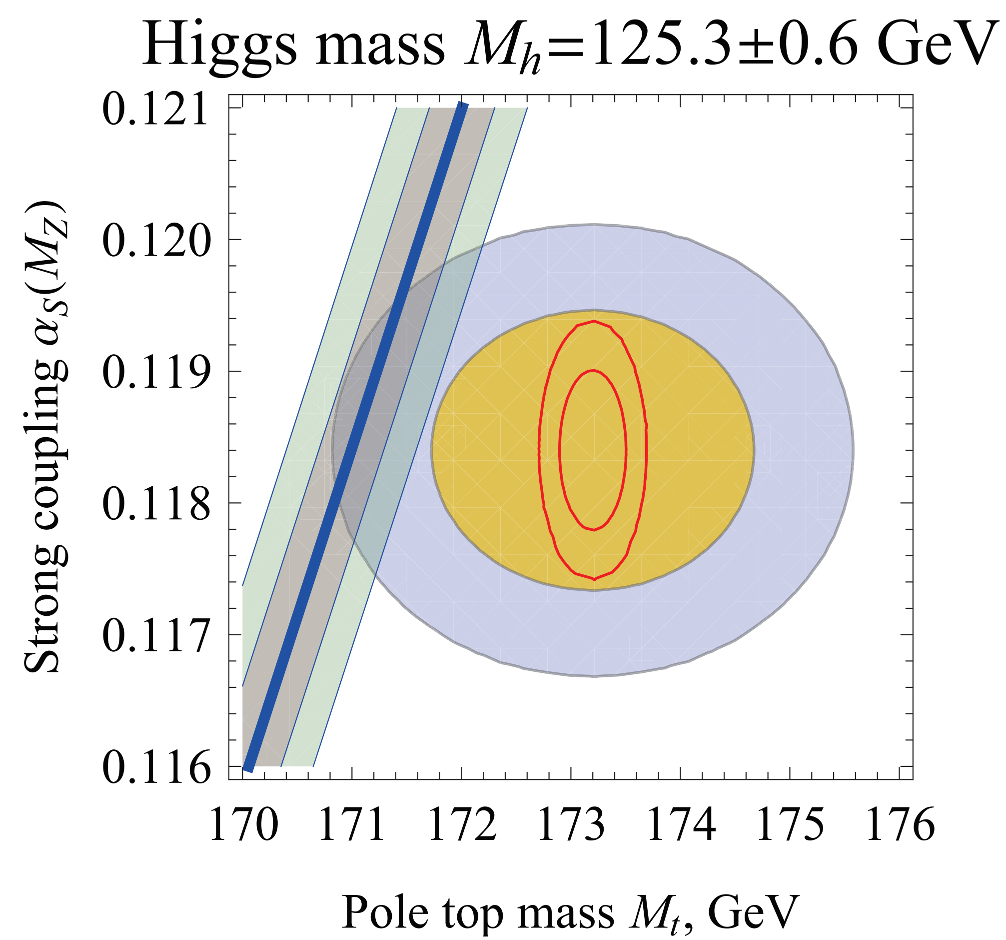
### Correction to $m_W$



# We do not know which possibility is realised!




errors in $y_t$: theory + experiment

Tevatron: $M_t = 173.2 \pm 0.51 \pm 0.71$ GeV $V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$

ATLAS and CMS: $M_t = 173.4 \pm 0.4 \pm 0.9$ GeV

$\alpha_s = 0.1184 \pm 0.0007$

Stockholm, July 23, 2013 – p. 9

Shaposhnikov, EPS 2009

# For reading and for exercises

- Chapter 4 from J. Chyla's scripts
- Exercises
  - ex. 3.14 (Casimir operators)
  - ex. 3.17 (invariance of Lagrangian)
  - ex. 4.1 and 4.2 (baryon magnetic moments)
  - ex. 4.3 (baryon masses)
  - ex. 4.4 (application of isospin)
  - ex. 4.5 (Decay modes of $\phi$ - application of Zweig rule)
  - ex. 4.6 (application of quark model on radiative decays)
- Extra exercises for further study
  - ex. 3.13 (general SU(3) multiplets)
  - ex. 3.15 and 3.16 (SU(4) and SU(N))
  - ex. 4.7 (rotating string)
  - ex. 6.5 (glueballs)