Quarks, Partons, and Quantum Chromodynamics

Lecture 4

The road to quarks

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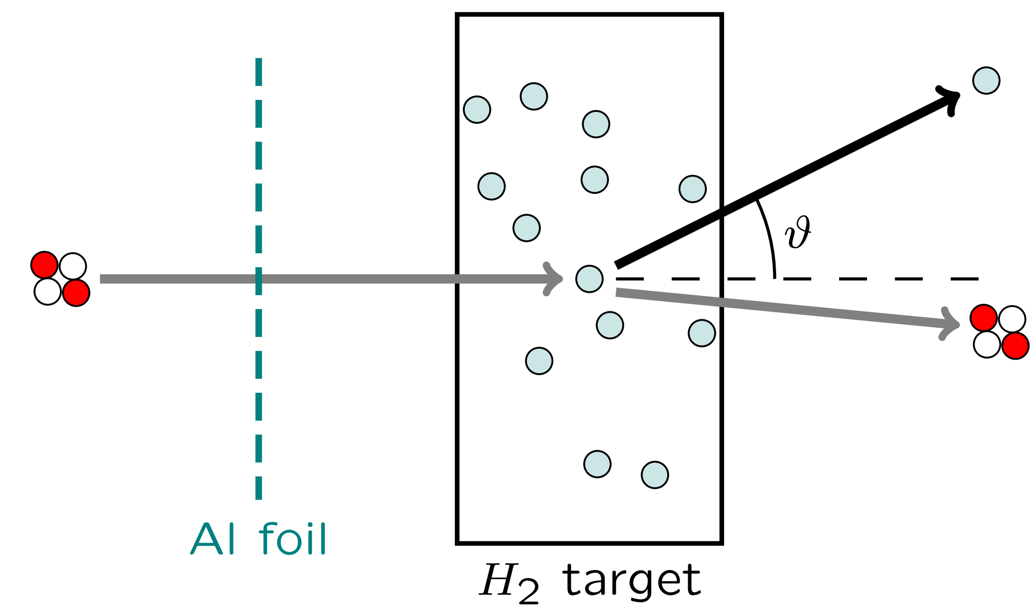
Historical topics for reading

J. Chyla: Quarks, partons and quantum chromodynamics, chapter 3 - The road to quarks

Strong interaction



- first observed in the Rutherford experiment with hydrogen
- Why hydrogen? The closest approach of incoming α particles

$$E_\alpha = 5\,\text{MeV} \;\; \text{implies} \;\; r_{\text{min}} \sim 3\,\text{fm} \quad \text{(ex. 2.3)}$$

$$r_{\text{min}} = \frac{\alpha Z_\alpha Z_p}{E_\alpha} \frac{m_\alpha + m_p}{m_p}$$

- energy E_α controlled by passing through Al foils of different thicknesses
- energy expressed in terms of length of flight of α in the air

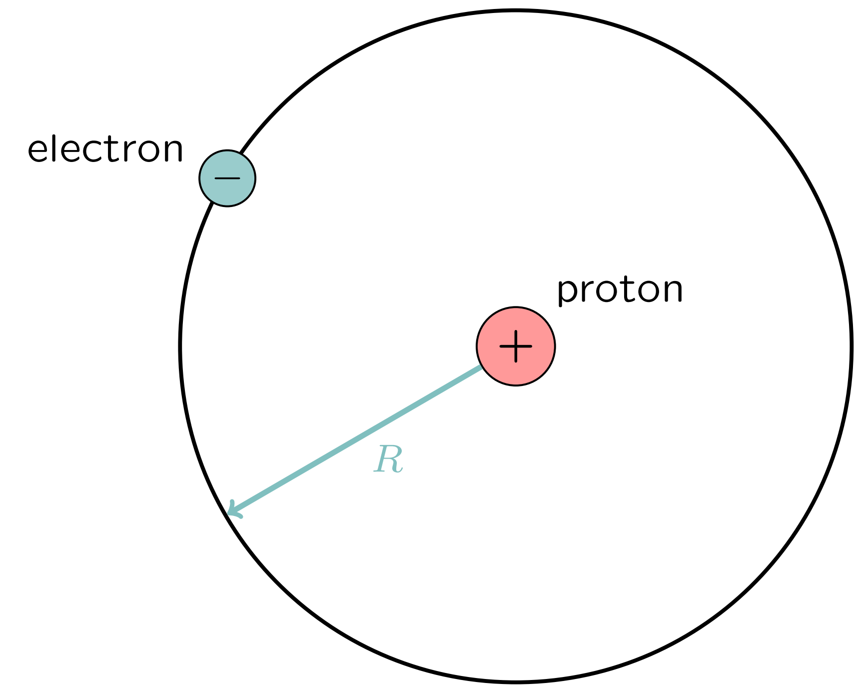
- for particles with $4\,$cm length of flight, the recoiled hydrogen nuclei followed Coulomb's law
- at full energy ($7\,$cm length of flight), an increase by factor of 30 in cross section observed
- Rutherford (wrongly) interpreted the result as the efect of nuclei deformations

- Chadwick and Beiler repeated the experiment and correctly interpreted the result as an effect of new

(strong) force

Why we call strong interaction strong?

Hydrogen atom size



- atom finite (non-zero) size is due to quantum effects
- we can get the size estimate from Heisenberg uncertainty relation

$$\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2}$$

- typical momentum of particle confined in sphere of radius R is

$$p \sim \frac{1}{R}$$

- corresponding electron kinetic energy is $E_k = \frac{p^2}{2m_e}$: $E_k \sim \frac{p^2}{m_e} \sim \frac{1}{m_e R^2}$
- note that kinetic energy rises faster with decreasing R than Coulomb's potential $E_p = \alpha/R$
- if electron is localized in too tiny space, Coulomb's force is not able to keep it there
  after a while electron must escape this confined space
  $\Rightarrow$ hydrogen size r_H is determined from condition $E_k \sim E_p$

$$\frac{\alpha}{r_H} \sim \frac{1}{m_e r_H^2} \quad \Rightarrow \quad r_H \sim \frac{1}{\alpha m_e} \quad \Rightarrow \quad r_H \sim \frac{137}{0.511\,\text{MeV}}\, 197\,\text{MeV}\cdot\text{fm} \sim \frac{\overset{-2\%}{140}}{\underset{-2\%}{0.5\,\text{MeV}}}\, \overset{-2\%}{200\,\text{MeV}\cdot\text{fm}} \sim \overset{-6\%}{56\,\text{pm}} \sim 53\,\text{pm}$$

Bohr radius

Why we call strong interaction strong?

Strong force

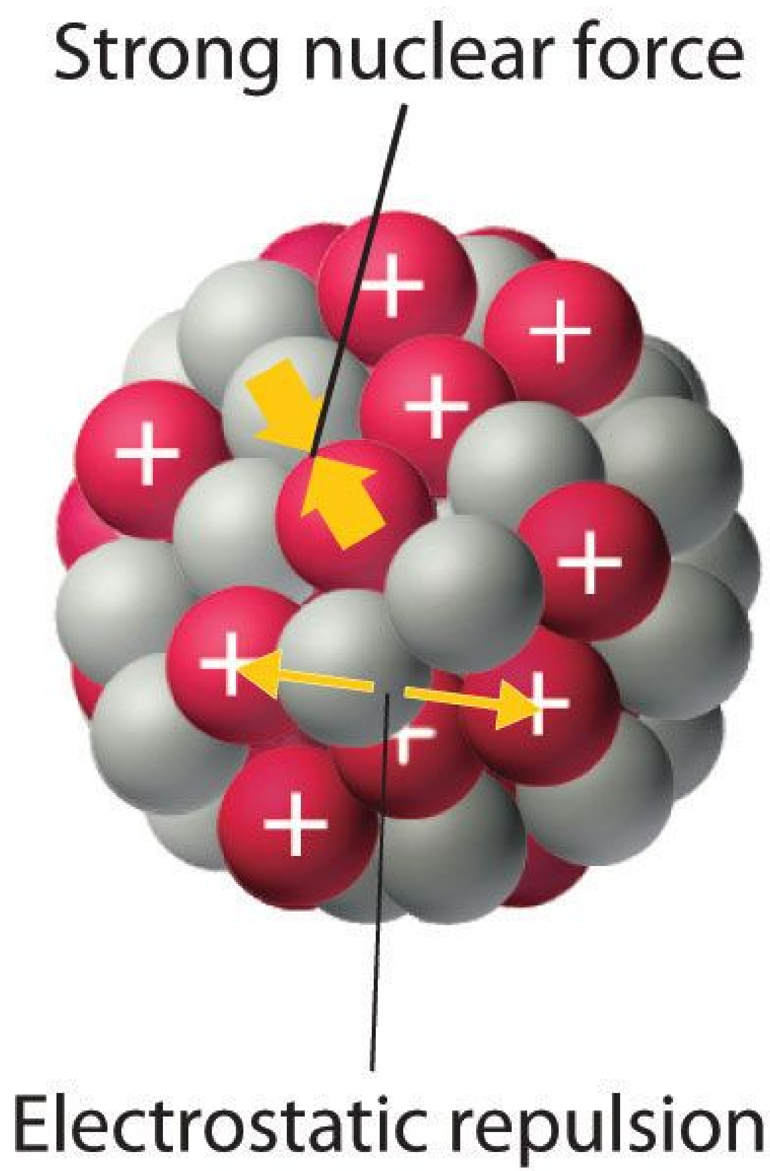
- proton is about $2000\times$ heavier than electron
- Coloumb force can keep it in a sphere of radius

$$\frac{53\,\text{pm}}{2000} \sim 30\,\text{fm}$$

- Rutherford experiment on hydrogen shows that

$$r_{He} \sim 1 - 2\,\text{fm}$$

- force that keeps proton in nucleus must be at least an order of magnitude stronger than electromagnetic force



Neutron / Neutrina - Pauli (1930)

Dear Radioactive Ladies and Gentlemen,

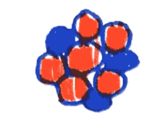
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the Nitrogen and Lithium 6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

I agree that my remedy could seem incredible because one should have seen these neutrons much earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's well better not to think about this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge.

Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you and also Mr. Back.

Your humble servant,
W. Pauli

^6Li



$6p^+ + 3e^-$

FERMION

$3p^+ + 3n$

BOSON

Chadwick: discovery of neutron in $\alpha + {}^9Be \rightarrow {}^{12}C + n$ (1932)

Anderson: discovery of positron (1932) - Dirac equation (QFT)

Fermi: 4-fermion (QF) theory of β-decay (1933)

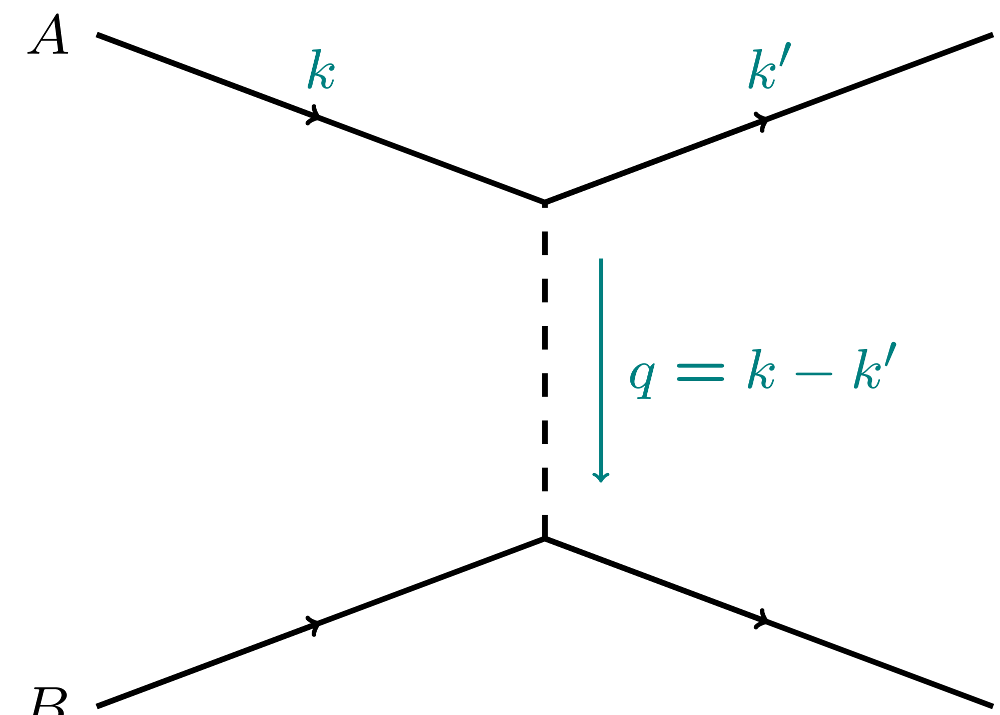
Strong interaction - Yukawa potential

- limited reach $\sim 2\,\text{fm}$, Yukawa potential $\quad V(r) = \frac{g}{r}\exp(-mr)$
- this is non-relativistic limit of QFT with interaction through massive scalar field ϕ
- derivation of Yukawa potencial
  - scattering amplitude in QFT: $\mathcal{M}_{if} \sim \frac{1}{q^2-m^2} \sim \frac{1}{q_0^2-|\vec{q}|^2-m^2}$
  - in non-relativistic limit $q_0 \ll m$: $\frac{1}{q_0^2-|\vec{q}|^2-m^2} \to \frac{-1}{|\vec{q}|^2+m^2}$
  - integral $V(\vec{r}) \sim g \int \frac{1}{|\vec{q}|^2+m^2} \exp(i\vec{q}\cdot\vec{r})\, \mathrm{d}^3\vec{q}$ quite difficult
  - we already know from exercise 2.4 the Fourier transform of

$$\frac{\alpha}{r}\exp(-\mu r) \quad \to \quad f(q) = \frac{2\mu\alpha}{|\vec{q}|^2+\mu^2}$$

$$\frac{1}{|\vec{q}|^2+\mu^2} \quad \to \quad V(r) \sim \frac{1}{r}\exp(-\mu r)$$

$$\mathcal{L} = g(\bar{\psi}_A\psi_B\phi^* + h.c.)$$



$$\mathcal{M}_{if} \sim \frac{1}{q^2-m^2}$$

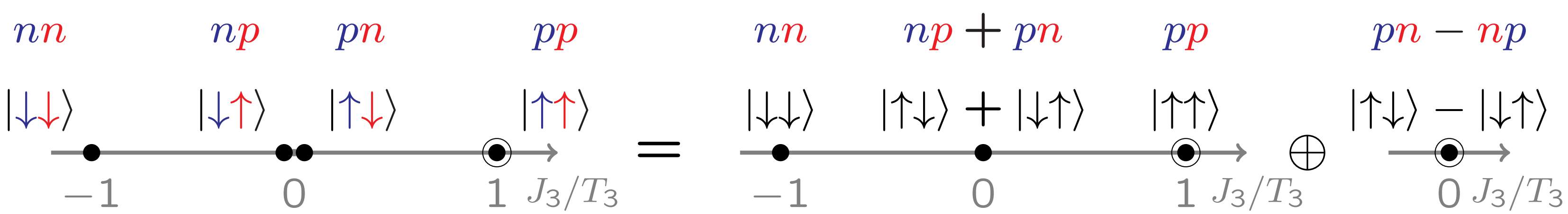
- interaction between proton and neutron - exchange of charged scalar $\pi^\pm$

$$\mathcal{L}_\mathsf{Y} = g\left(\bar{p}n\phi^\star + \bar{n}p\phi\right)$$

- reach $\sim 2\,\text{fm}$ implies $m_\phi \approx 100\,\text{MeV}$

Historical topics for reading - Isospin

- 1935-38: ^3H and ^4He bounding energies + first scattering exp.: $pp \sim nn \sim (pn + np)$
- **Isospin** doublet $\begin{pmatrix} p \\ n \end{pmatrix}$ in analogy with spin $\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$



$$\boxed{\mathbf{2} \otimes \mathbf{2} = \mathbf{3}_s \oplus \mathbf{1}_a}$$

- Kemmer (1938) - isospin formulation of strong interaction

$$\mathcal{L}_{\mathsf{Y}} = g\left(\bar{p}n\phi^\star + \bar{n}p\phi\right) = g\bar{\psi}\left(\sigma_1\phi_1 + \sigma_2\phi_2\right)\psi, \quad \text{with} \quad \psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}, \quad \begin{array}{l} \phi_1 = \phi + \phi^\star \\ \phi_2 = \phi - \phi^\star \end{array}$$

- generalization to $\mathcal{L}_{\text{int}} = g\bar{\psi}\left(\sigma_1\phi_1 + \sigma_2\phi_2 + \sigma_3\phi_3\right)\psi,$ ϕ_3 ... neutral 'meson'
- Pauli matrices σ_i are generators of SU(2)
- Lagrangian SU(2) invariant for field transformations (see ex. 3.17)

$$\psi \to \psi' = \exp(i\alpha_i\sigma_i)\psi, \quad \phi \to \phi' = \exp(i\alpha_i F_i)\phi, \quad \text{with } (F^i)_{jk} = -if_{ijk} \text{ (adjoint repr.)}$$

- strong interaction preserve **isospin**
- $\pi^\pm$ (1947, Powell, in cosmic rad.), π^0 (1950, in $p + N$ in Brookhaven)

Strangeness (1947-1955)

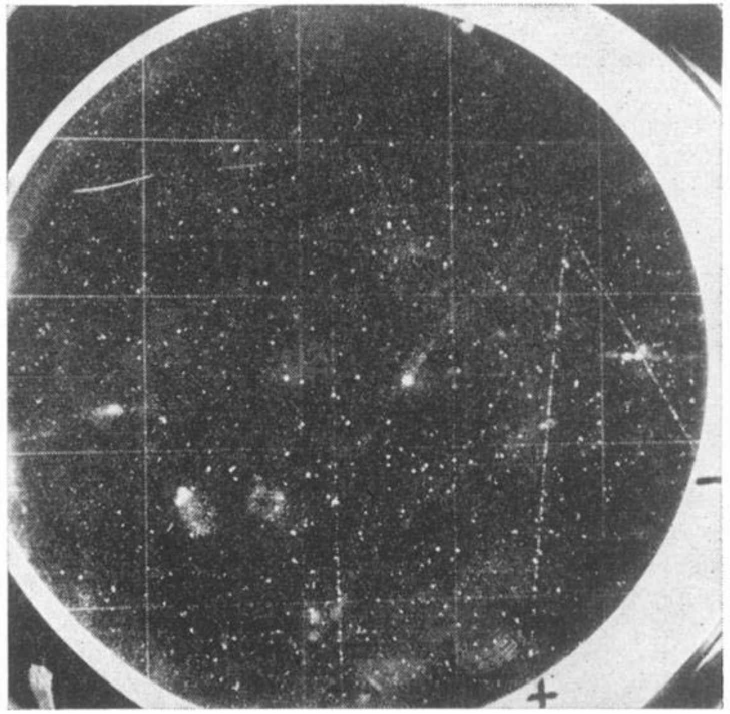


FIG. 2. V^0-disintegration in which the positive particle is probably a π-meson.

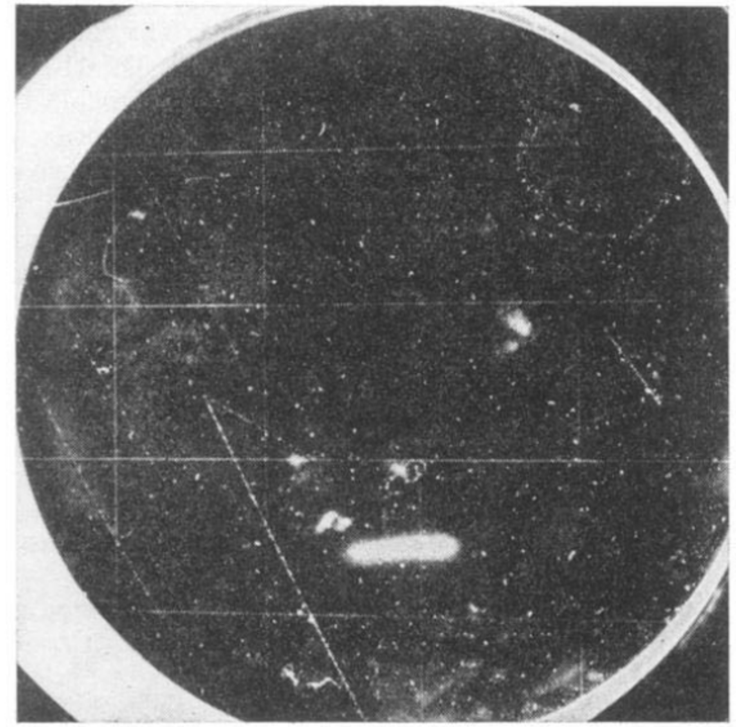


FIG. 1. V^0-disintegration resulting in a heavily ionizing proton.

Figure 3.2: Two events showing the presumable decay of K^0 (left) and Λ (right) observed in a cloud chamber placed in magnetic field. Taken from [30].

- 1947 - discovery of strange hadrons in cosmic rays

  (Butler and Rochester)

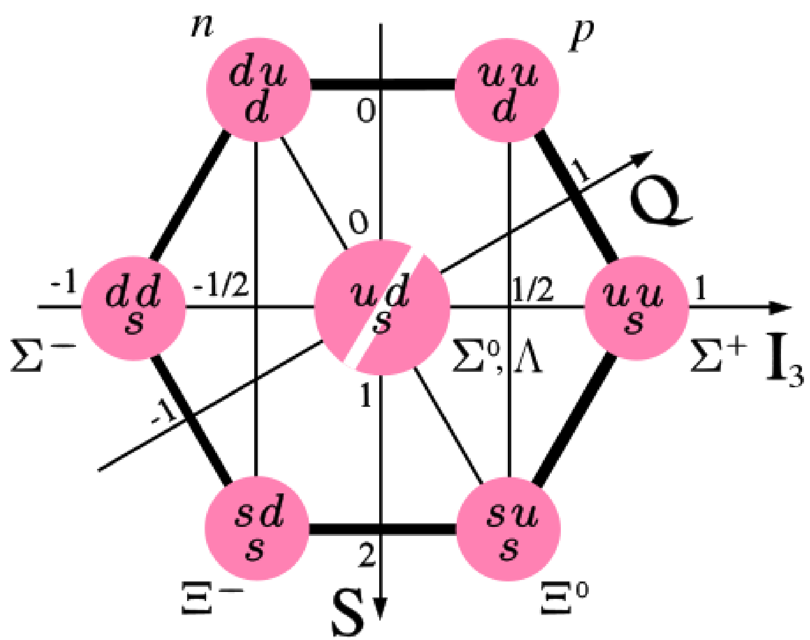
- strange behaviour - production rates similar to π^0 but much slower decay

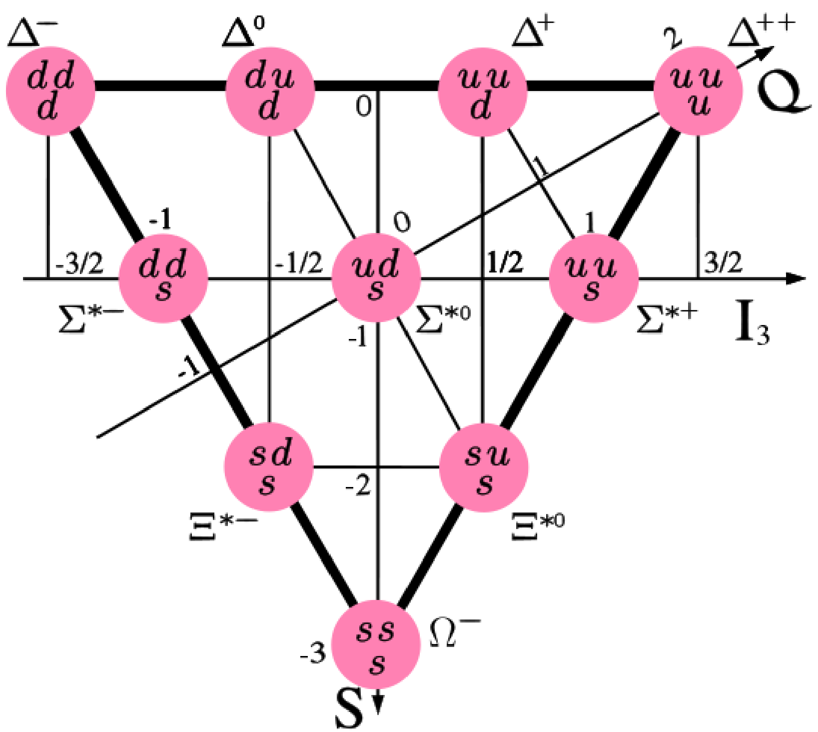
$$\pi^0: \ 25\,\text{nm},\ K^0_S: \ 2.7\,\text{cm},\ K^0_L: \ 15\,\text{m},\ \Lambda^0: 7.9\,\text{cm}$$

- experiments on pion beams at BNL confirmed the suspicion that the strange hadrons are born in pairs
- new quantum number, **strangeness** (S), conserved by strong interaction (Gell-Mann, Pais, Nishijima)

$$\begin{array}{ccc} \pi^- p & \rightarrow & K^0 \ \ \Lambda^0 \\ & & S: +1 \ \ -1 \end{array} \qquad \begin{array}{ccc} \pi^- p & \rightarrow & K^+ \ \ \Sigma^- \\ & & S: +1 \ \ -1 \end{array} \qquad \begin{array}{ccc} \pi^- p & \not\rightarrow & K^- \ \ \Sigma^+ \\ S: 0+0 & = & -1+1 \\ I_3: -1+\frac{1}{2} & \neq & -\frac{1}{2}+1 \end{array}$$

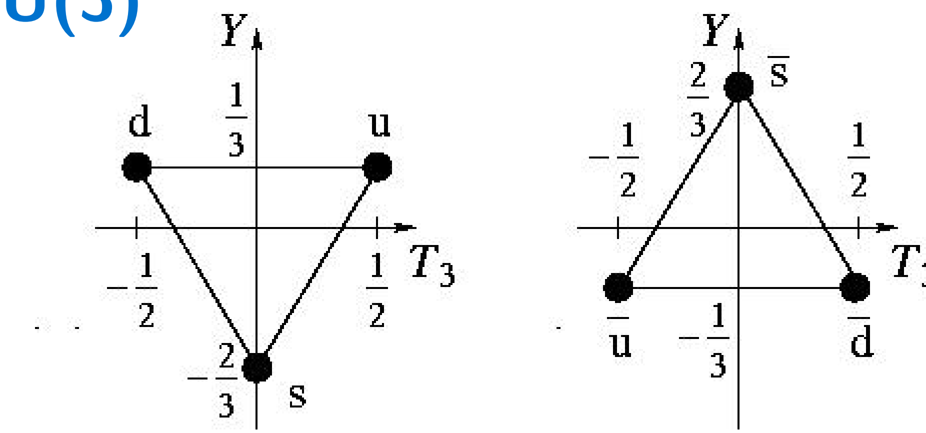
Eightfold way (1961-1962)





CERN, July 1962

SU(3)



http://ati.tuwien.ac.at

Ne'eman

Gell-Mann

Zweig

→ discovery of $\Xi^\star$

→ Gell-Mann's predicion of Ω^-

→ Ω^- discovered in 1964 (Samios)

$$K^- p \to K^+ K^0 \Omega^-$$

- baryons (qqq): $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$
- mesons ($q\bar{q}$): $3 \otimes \bar{3} = 8 \oplus 1$

Quark models of hadrons

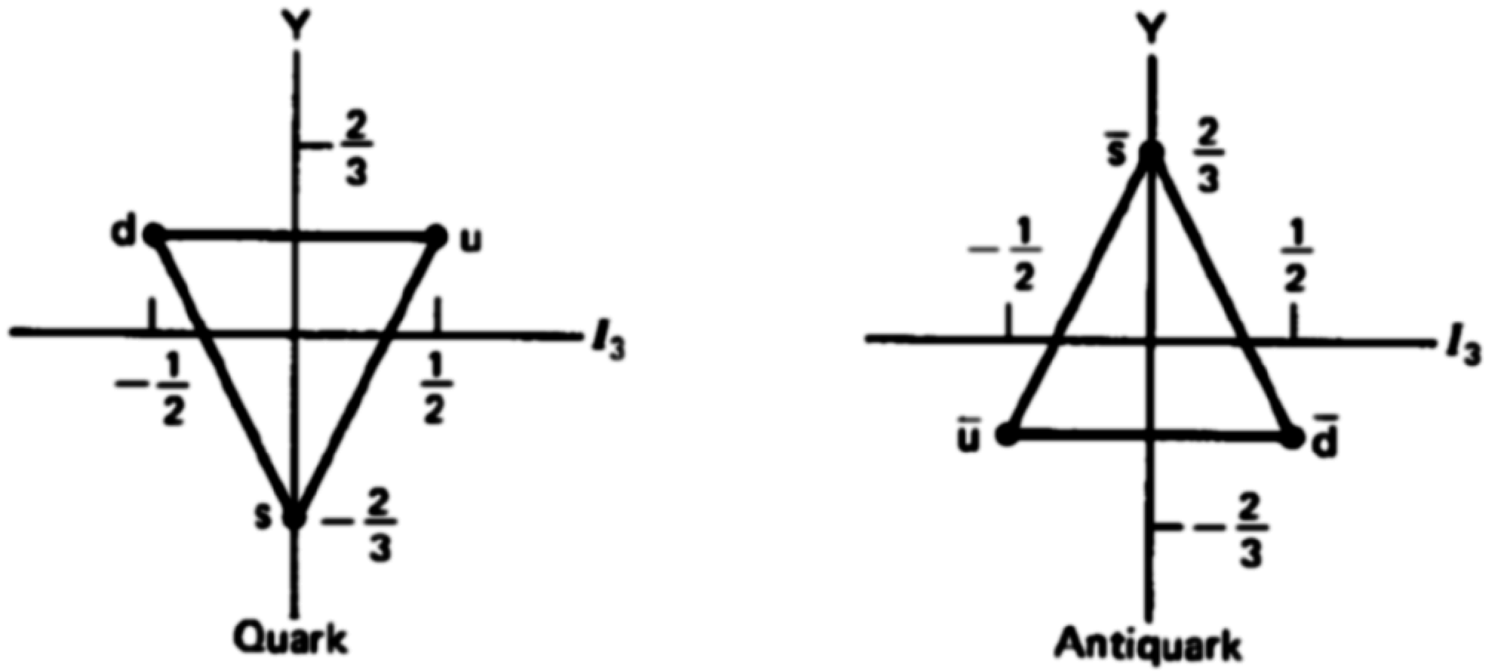


Fig. 2.4 $SU(3)$ quark and antiquark multiplets; $Y \equiv B + S$.

Hypercharge Y $(T_8 = \frac{\sqrt{3}}{2} Y)$

TABLE 2.1

Quantum Numbers of the Quarks ($Y = B + S$, $Q = I_3 + Y/2$)[a] Nishijima (1954), Gell-Mann (1955)

Quark	Spin	B	Q	I_3	S	Y
u	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	0	$\frac{1}{3}$
d	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$\frac{1}{3}$
s	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	-1	$-\frac{2}{3}$

[a] Here, S denotes the strangeness.

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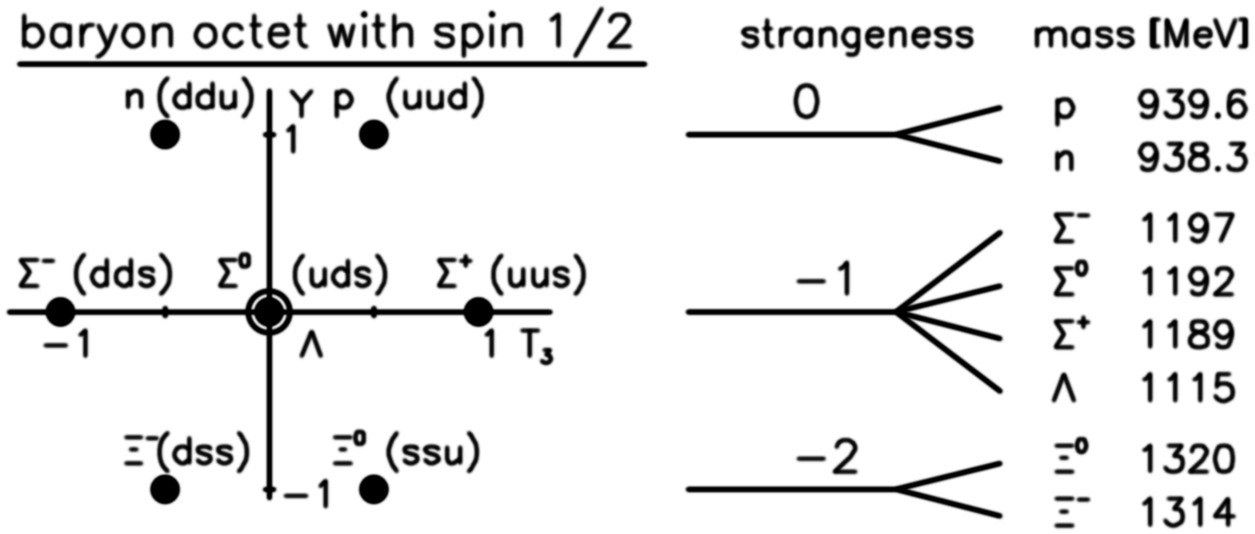
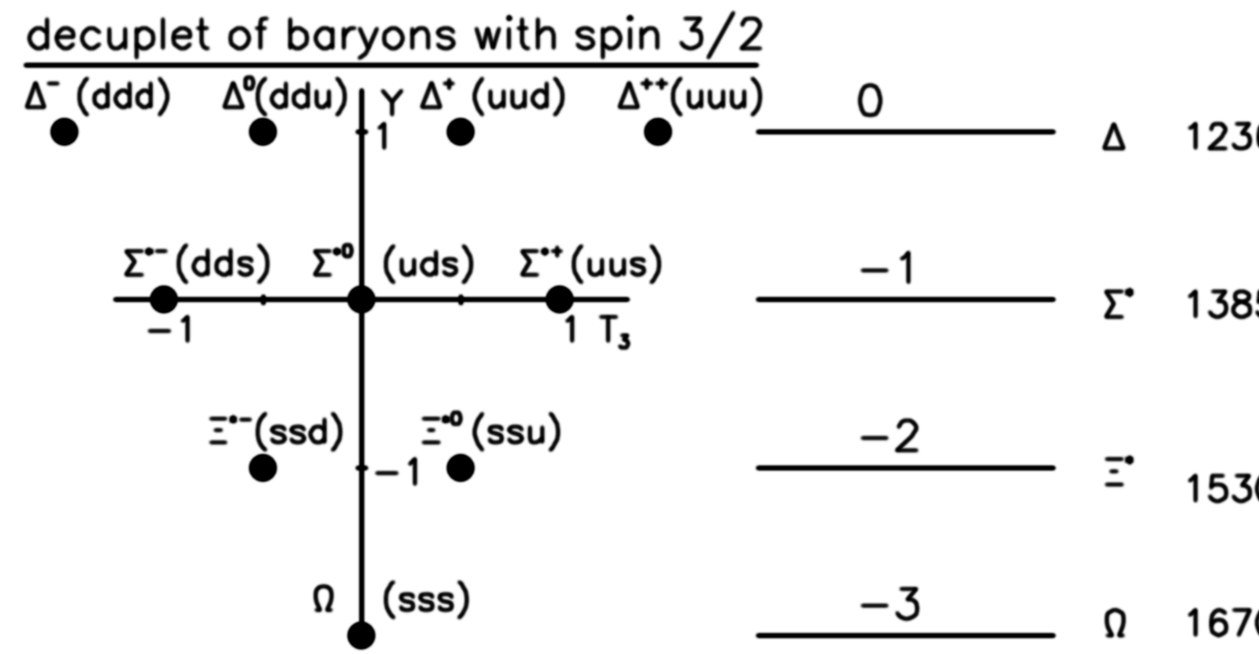
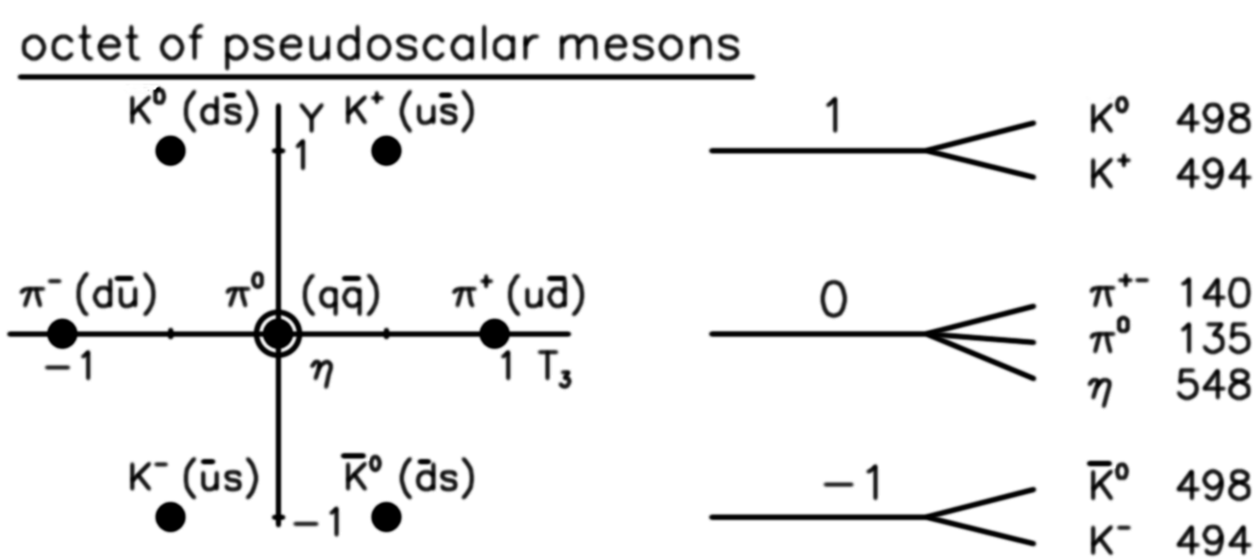
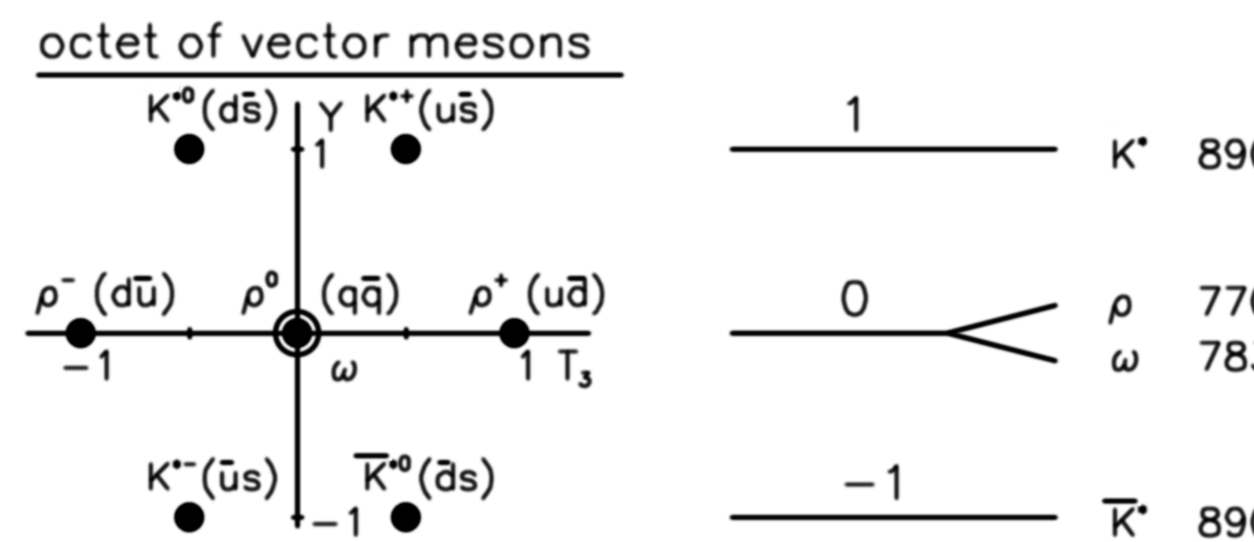
Hadron SU(3) multiplets









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Meson multiplets

TABLE 2.2
Quantum Numbers of Observed Mesons Composed of u, d, and s Quarks

$q\bar{q}$ Orbital Ang. Mom.	$q\bar{q}$ Spin	J^{PC}	Observed Nonet $I = 1$	$I = \frac{1}{2}$	$I = 0$	Typical Mass (MeV)
$L = 0$	$S = 0$	0^{-+}	π	K	η, η'	500
	$S = 1$	1^{--}	ρ	K^*	ω, ϕ	800
$L = 1$	$S = 0$	1^{+-}	B	Q_2	H, ?	1250
	$S = 1$	2^{++}	A_2	K^*	f, f′	1400
	$S = 1$	1^{++}	A_1	Q_1	D, ?	1300
	$S = 1$	0^{++}	δ	κ	ε, S^*	1150

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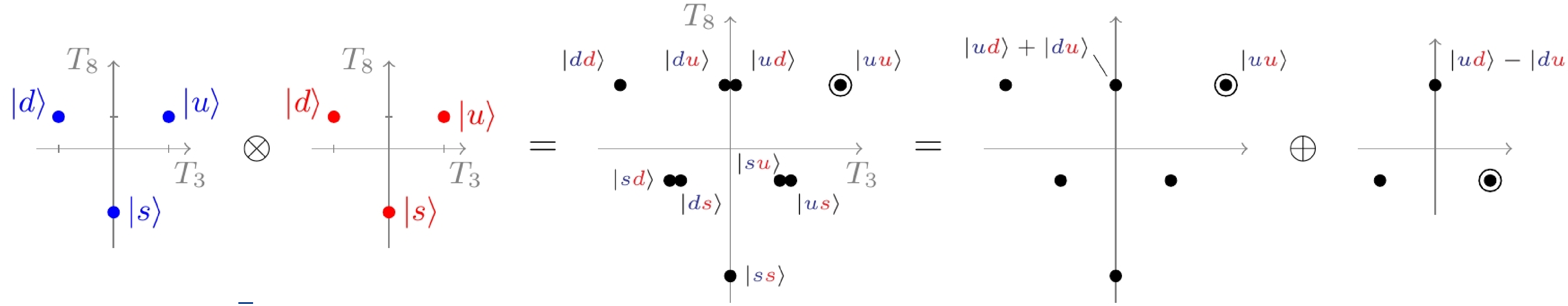
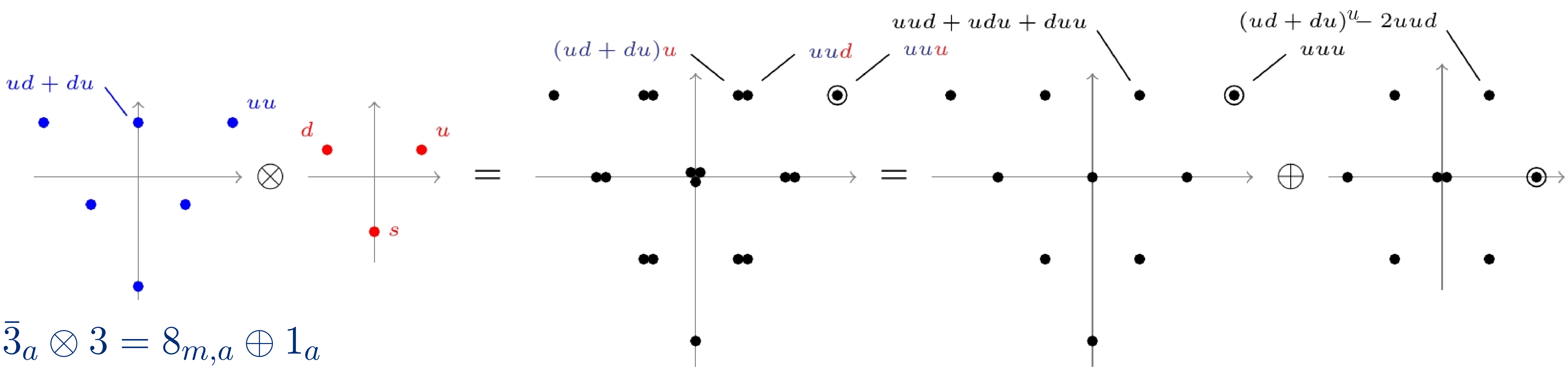
- $P = -(-1)^L$ (q and $\bar{q}$ have opposite intrinsic parity)
- neutral mesons: $C = -(-1)^{S+1}(-1)^L$ (exchange of $q \leftrightarrow \bar{q}$, positions and spins)
  - $(-)$ - fermion exchange
  - $(-1)^{S+1}$ - symetry of spin state ($2 \otimes 2 = 3_S \oplus 1_A$)
  - $(-1)^L$ - space symetry

Baryon wave functions

- $3 \otimes 3 \otimes 3 = ?$ (exercise 3.12)
- $3 \otimes 3 = 6_s \oplus \bar{3}_a$



- $6_s \otimes 3 = 10_s \oplus \bar{8}_{m,s}$

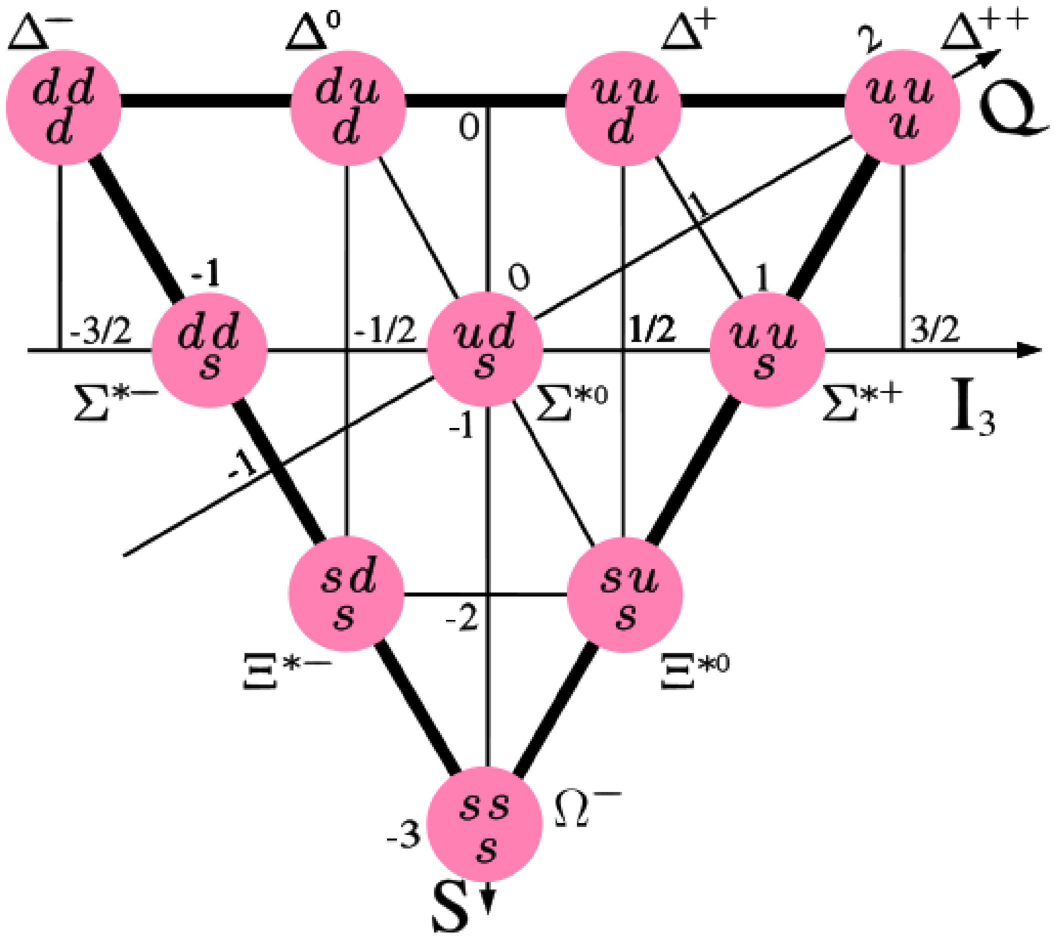


- $\bar{3}_a \otimes 3 = 8_{m,a} \oplus 1_a$

$|1_a\rangle = uds + dsu + sud - dus - usd - sdu$ (follows from $E_\alpha |1\rangle = 0$)

Baryon wave functions

- $3 \otimes 3 \otimes 3 = 10_s \oplus 8_{m,s} \oplus 8_{m,a} \oplus 1_a$ in analogy with $2 \otimes 2 \otimes 2 = 4_s \oplus 2_{m,s} \oplus 2_{m,a}$



- baryons with spin $3/2$ are forming decuplet
- their wave function is symmetric (!)

$$10_s(\text{flavour}) \otimes 4_s(\text{spin}): \; |\Delta^{++}, 3/2\rangle = |uuu\rangle \, |\uparrow\uparrow\uparrow\rangle$$

- contradition with Pauli exclusion principle
- solution - quarks are in three (color) states
- hadrons are colorless - SU(3) singlets

- symmetric octet: $|8_s\rangle = 8_{m,s} \otimes 2_{m,s} + 8_{m,a} \otimes 2_{m,a}$
- or guessed as $|2_{m,s}\rangle = (\uparrow\downarrow + \downarrow\uparrow)\uparrow - 2\uparrow\uparrow\downarrow$ and $|2_{m,a}\rangle = (\uparrow\downarrow - \downarrow\uparrow)\uparrow$

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}} \left[|uud\rangle \, |(\uparrow\downarrow + \downarrow\uparrow)\uparrow - 2\uparrow\uparrow\downarrow\rangle + \text{cycl. per.} \right]$$

Baryon magnetic momens (ex. 4.1)

Additive quark model assumes that hadron magnetic moment comes just from magnetic moments of its constituents, quarks, and that there is no additional magnetic moment connected with the orbital momenta of quarks. Assigning μ_q to a magnetic moment of quark q in state $|q\uparrow\rangle$, and $-\mu_q$ in state $|q\downarrow\rangle$, we can write for the proton magnetic moment

$$\mu_p = \frac{3}{18}\left[(\mu_u - \mu_u + \mu_d) + (-\mu_u + \mu_u + \mu_d) + 4(\mu_u + \mu_u - \mu_d)\right] = \frac{4\mu_u - \mu_d}{3}. \tag{3.1.2}$$

Neutron wave function and magnetic moment can be quickly obtained from the isospin symmetry (u $\rightleftharpoons$ d)

$$\mu_n = \frac{4\mu_d - \mu_u}{3}. \tag{3.1.3}$$

Magnetic moment of a point-like spin 1/2 fermion is given in the leading order of perturbative theory by *Bohr magneton*

$$\mu_q = \frac{e_q e}{2m_q}, \tag{3.1.4}$$

where e_q is charge of quark q expressed in units of elemental charge e, and m_q is its mass. Assuming the same masses for u and d quarks, we get

$$2\mu_d = -\mu_u.$$

And for the ratio of magnetic moments of proton and neutron we find

$$\frac{\mu_p}{\mu_n} = \frac{4\mu_u - \mu_d}{4\mu_d - \mu_u} = \frac{8+1}{-4-2} = -\frac{3}{2}.$$

Measured values of proton and neutron magnetic moments in terms of *nuclear magneton* $\mu_N = e/(2m_p)$, where m_p is proton mass, are [27]

$$\mu_p = 2.793\mu_N, \qquad \mu_n = -1.913\mu_N. \tag{3.1.5}$$

The prediction of additive quark model is quite close to the measured ratio

$$\frac{\mu_p}{\mu_n} \approx -\frac{2.793}{1.913} \approx \frac{2.793 + 3\cdot 0.087/2}{2} \approx -\frac{2.793+0.131}{2} = -\frac{2.924}{2} \approx -1.46.$$

Baryon magnetic momens

$$\begin{array}{llllll} \mu_p &= (4\mu_u - \mu_d)/3, & \mu_n &= (4\mu_d - \mu_u)/3, & \mu_\Lambda &= \mu_s, \\ \mu_{\Sigma^+} &= (4\mu_u - \mu_s)/3, & \mu_{\Sigma^-} &= (4\mu_d - \mu_s)/3, & \mu_{\Sigma^0} &= (2\mu_u + 2\mu_d - \mu_s)/3, \\ \mu_{\Xi^0} &= (4\mu_s - \mu_u)/3, & \mu_{\Xi^-} &= (4\mu_s - \mu_d)/3, & \mu_{\Omega^-} &= 3\mu_s. \end{array} \tag{4.20}$$

Solving the first three equations for μ_u, μ_d, μ_s we get (in nuclear magnetons)

$$\mu_u = +1.852; \quad \mu_d = -0.972; \quad \mu_s = -0.613, \tag{4.21}$$

corresponding to $m_u = 338$ MeV, $m_d = 332$ MeV and $m_s = 510$ MeV. Quark masses obtained in this way are called **constituent** masses. In the next Chapter we shall encounter the so called **current** quark masses,

p	n	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	Ω
2.793	-1.913	-0.613±0.004	2.458±0.010	-1.160±0.025	-1.25±0.014	-0.651±0.003	-2.02±0.05
input	input	input	2.674	-1.092	-1.435	-0.494	-1.839

Table 4.2: Current status of experimental determination (first row) of and theoretical predictions (second row) for baryon magnetic moments in units of μ_N. Magnetic moments of the proton and neutron are known with large accuracy (2.10^{-9}% and 2.10^{-8}% respectively).

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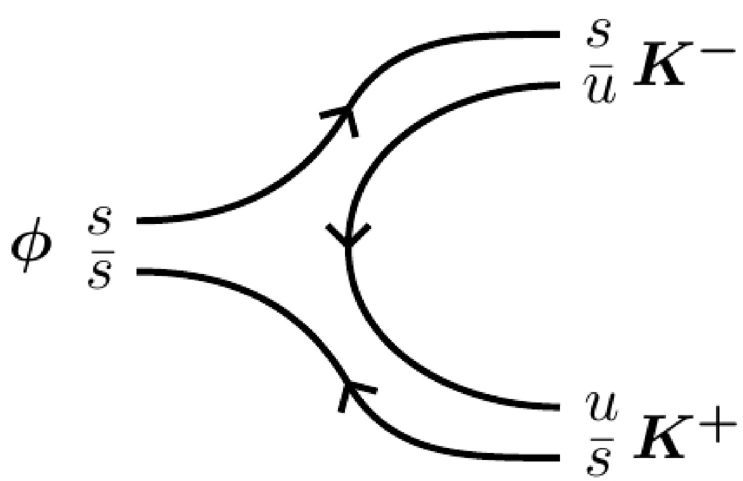
Forbiden decays - Zweig rule (ex. 4.5)

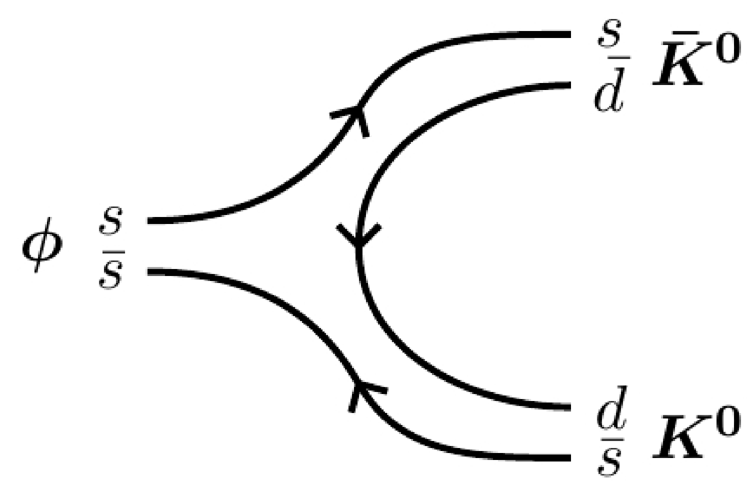
- consider decays $\phi \to K^+K^-$, $\phi \to K^0\bar{K}^0$, and $\phi \to \varrho\pi$

  $m_\phi = 1020\,\text{MeV}$, $m_{K^\pm} = 494\,\text{MeV}$, $m_{K^0} = 498\,\text{MeV}$, $m_\varrho = 770\,\text{MeV}$, $m_\varrho = 140\,\text{MeV}$
- $\phi \to \varrho\pi$ should be kinematically favoured, yet

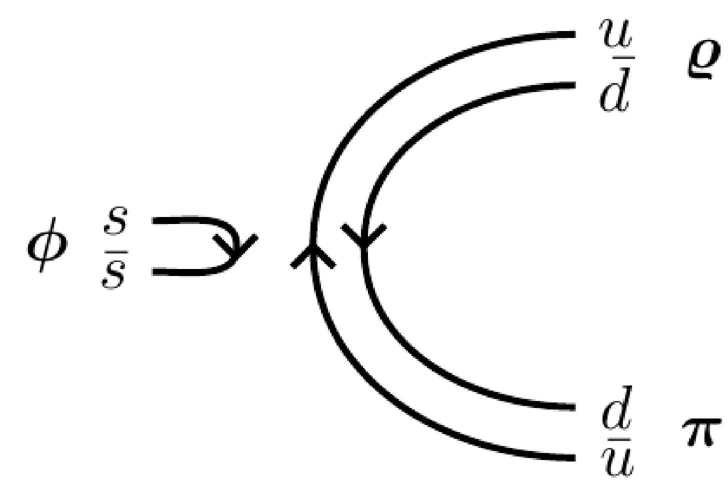
  $Br(\phi \to K^+K^-) \approx 49\%$, $Br(\phi \to K^0\bar{K}^0) \approx 34\%$, $Br(\phi \to \varrho\pi) \approx 15\%$
- Zweig developed quark model to understand this peculiarities



$\phi \to K^-K^+$



$\phi \to K^0\bar{K}^0$



$\phi \to \varrho\,\pi$

Zweig rule (phenomenological observation)

Only process with connected planar flow diagrams are allowed, disconnected diagrams are forbiden.

For reading and for exercises

- For reading (J. Chyla textbook)
  - Whole chapter 3 (The road to quarks)
  - Chapter 4 - Quark Model: sections 4.1-4.4
- Exercises 3.12, 3.17, 4.1, 4.2, and 4.5