QCD: Lecture 1

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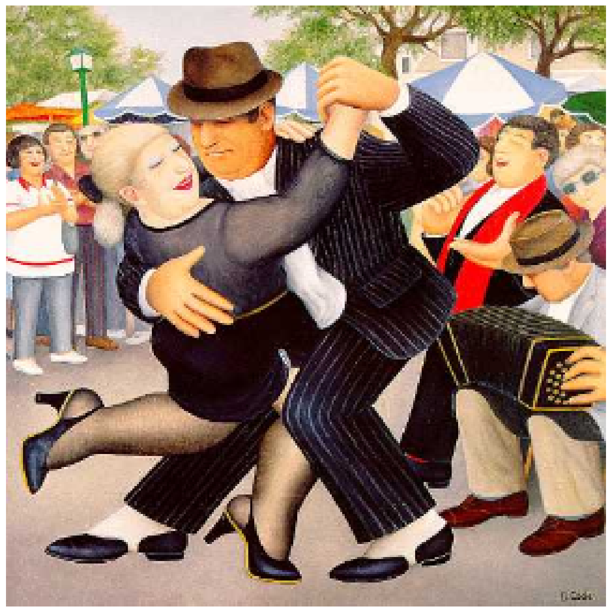
Jiri Chyla



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Výsledky zápočtové písemnky 1.6.
Termín další zápočtové písemky: pondělí 15.6. v 10.00 v seminární místnosti ÚČFF v Troji, 9. patro
Výsledky zápočtové písemnky 15.6.

Center for particle Physics
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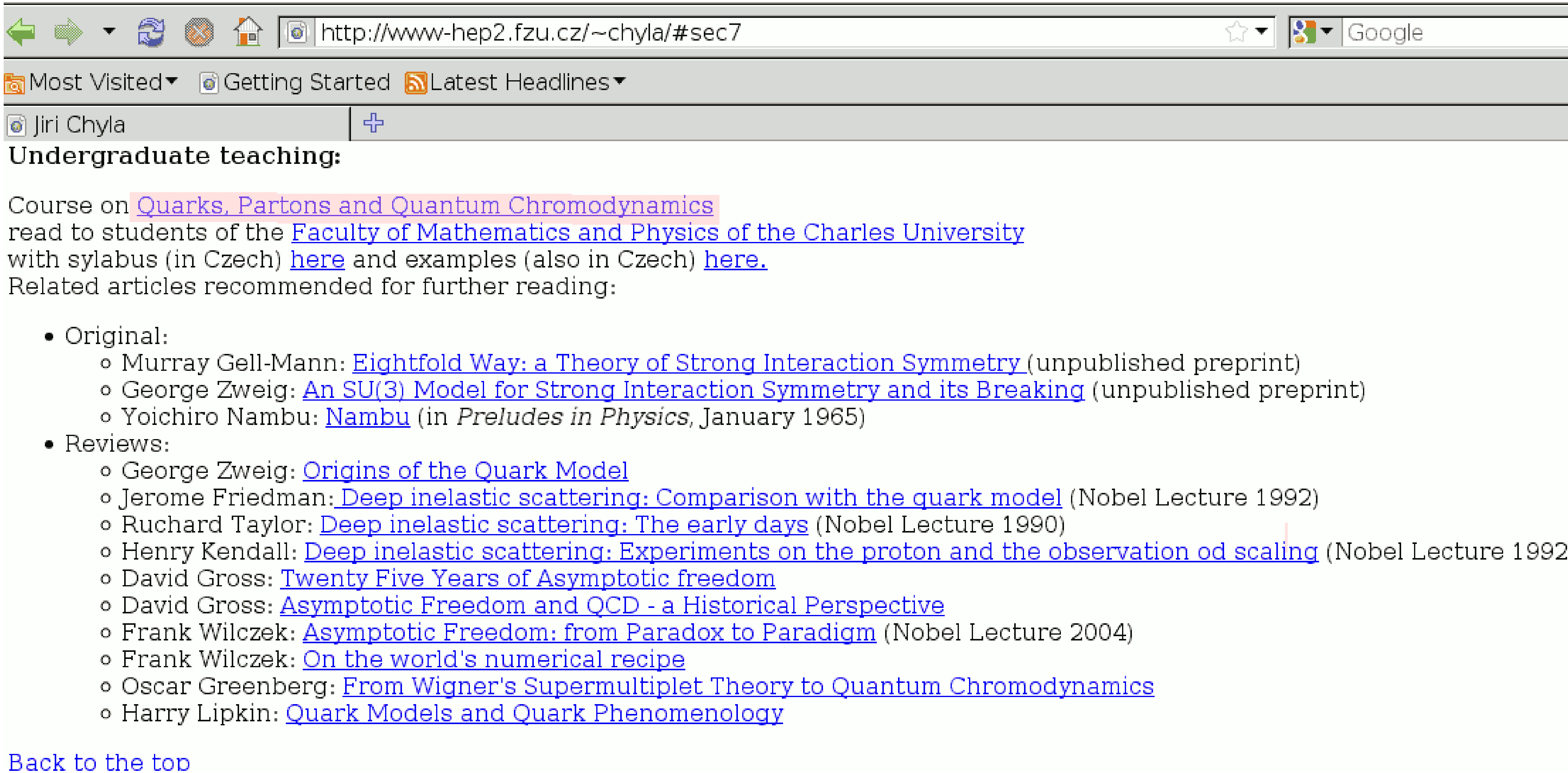


Undergraduate teaching:

Course on Quarks, Partons and Quantum Chromodynamics
read to students of the Faculty of Mathematics and Physics of the Charles University
with sylabus (in Czech) here and examples (also in Czech) here.
Related articles recommended for further reading:

- Original:
  - Murray Gell-Mann: Eightfold Way: a Theory of Strong Interaction Symmetry (unpublished preprint)
  - George Zweig: An SU(3) Model for Strong Interaction Symmetry and its Breaking (unpublished preprint)
  - Yoichiro Nambu: Nambu (in *Preludes in Physics*, January 1965)
- Reviews:
  - George Zweig: Origins of the Quark Model
  - Jerome Friedman: Deep inelastic scattering: Comparison with the quark model (Nobel Lecture 1992)
  - Ruchard Taylor: Deep inelastic scattering: The early days (Nobel Lecture 1990)
  - Henry Kendall: Deep inelastic scattering: Experiments on the proton and the observation od scaling (Nobel Lecture 1992)
  - David Gross: Twenty Five Years of Asymptotic freedom
  - David Gross: Asymptotic Freedom and QCD - a Historical Perspective
  - Frank Wilczek: Asymptotic Freedom: from Paradox to Paradigm (Nobel Lecture 2004)
  - Frank Wilczek: On the world's numerical recipe
  - Oscar Greenberg: From Wigner's Supermultiplet Theory to Quantum Chromodynamics
  - Harry Lipkin: Quark Models and Quark Phenomenology

Back to the top

- F. Halzen, A. Martin: “Quarks and leptons”

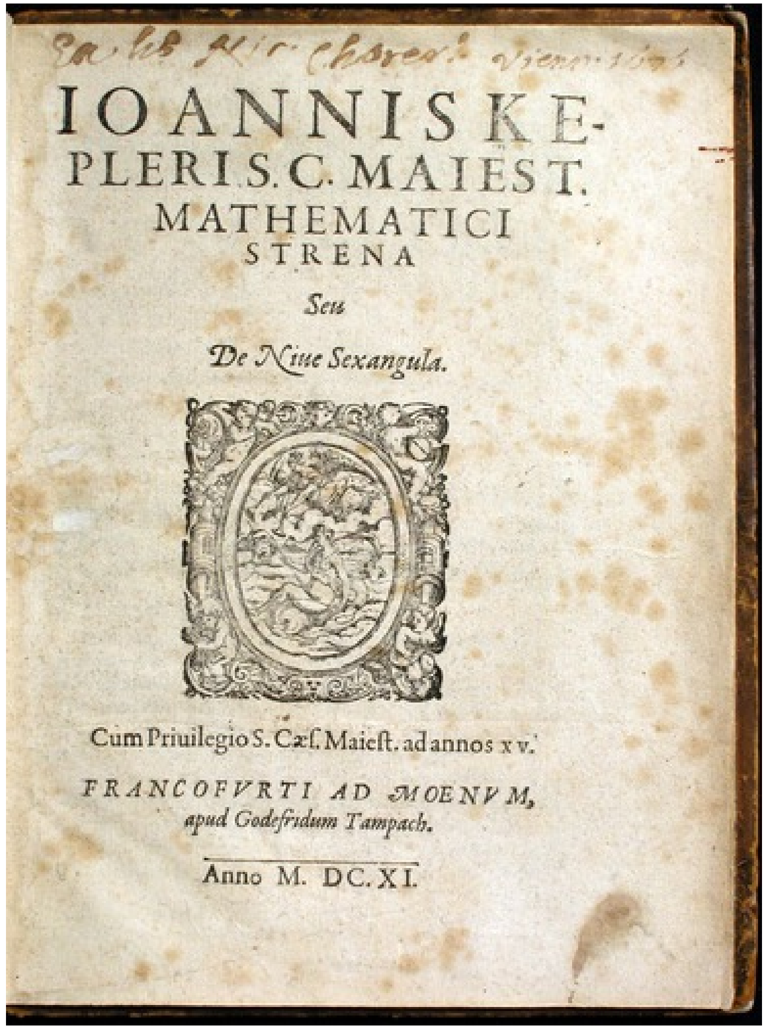
Quarks, partons and quantum chromodynamics

- from the discovery of proton structure to the theory of strong interaction and its applications

Lecture plan

- Scattering in classical and quantum mechanics (1 lecture)
  - basic tool for investigation the structure of matter
- Symmetry and Lie groups (2 lectures)
  - SU(3), SU(3) multiplets, decomposition to irreducible representations
- Quark model of hadrons (2 lectures)
- Feynman parton model (2 lectures)
  - deep inelastic scattering $e/\nu + p$, structure functions, PDF, fragmentation, other processes
- Quantum Chromodynamics (2 lectures)
  - QCD Lagrangian, running of α_S, color matrices and basic QCD processes
- IR divergence (2-3 lectures)
  - QCD splitting functions, parton showers, jets, DGLAP evolution equations

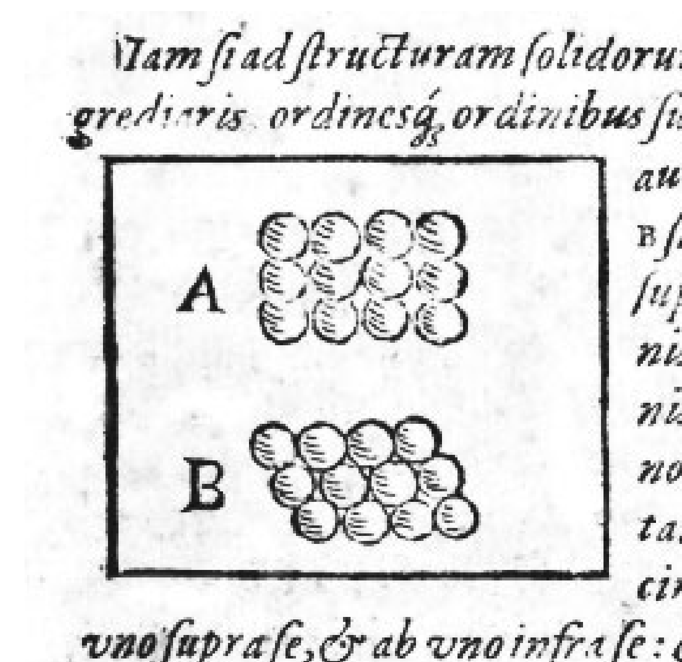
Symmetries and structure of matter


IOANNIS KE-
PLERI S.C. MAIEST.
MATHEMATICI
STRENA
Seu
De Niue Sexangula.
Cum Priuilegio S. Cæs. Maiest. ad annos xv.
FRANCOFVRTI AD MOENVM,
apud Godefridum Tampach.
Anno M. DC. XI.

Bentley 1902

- Kepler's New Year's pamphlet on snowflakes

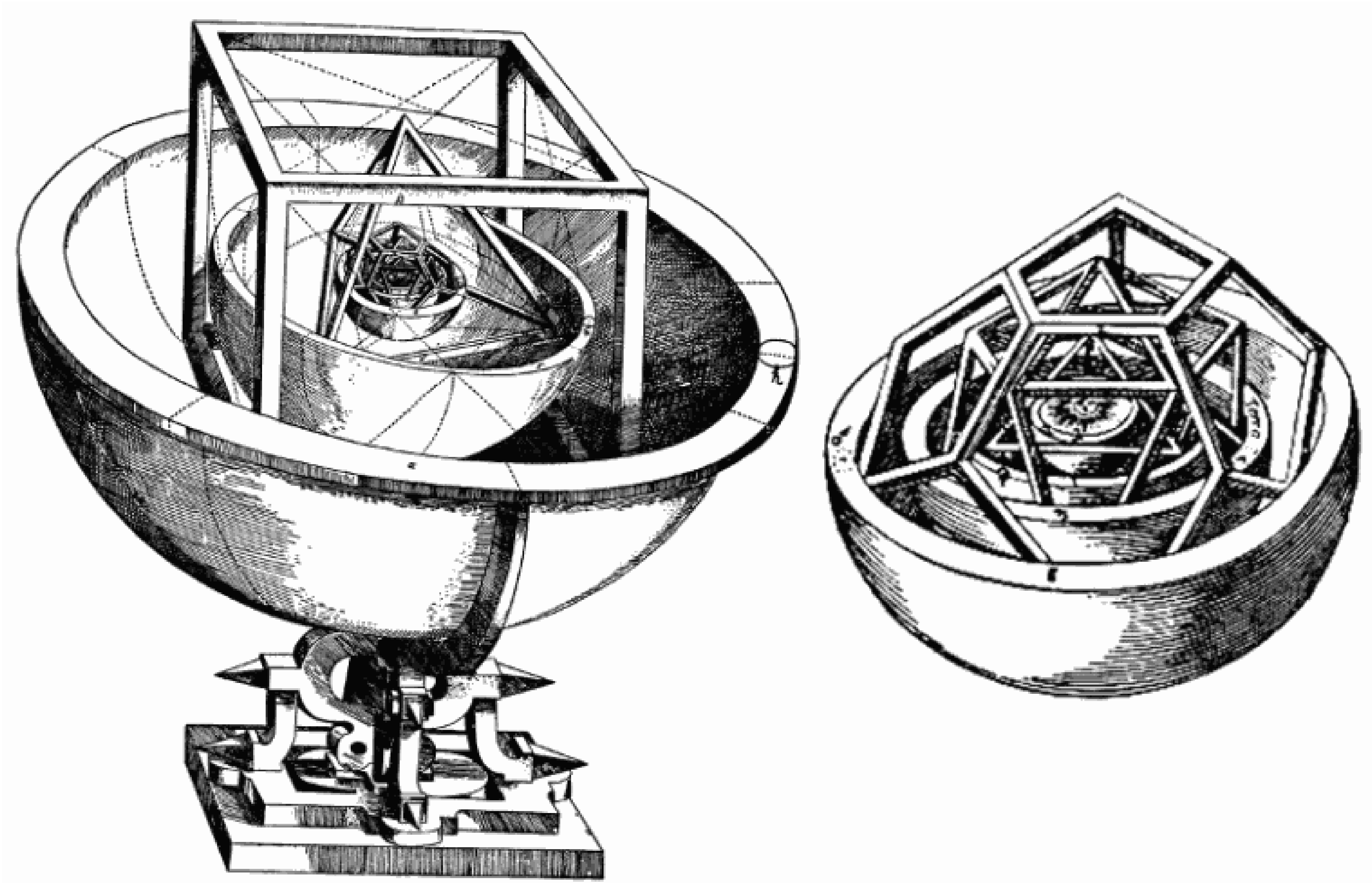
  Strena Seu De Nive Sexangula (1611)
- the first(?) scientific treatise on crystallography
- hexagonal symmetry of snowflakes is connected with the way how to arrange spheres in space
  - packing with hexagonal symmetry is the most dense one



- took almost 400 years to prove the Kepler's conjecture (1999, Hales)
  - first thoughts about connection between symmetry and structure of matter?
  - regularity/paterns - often a sign of a simpler internal structure

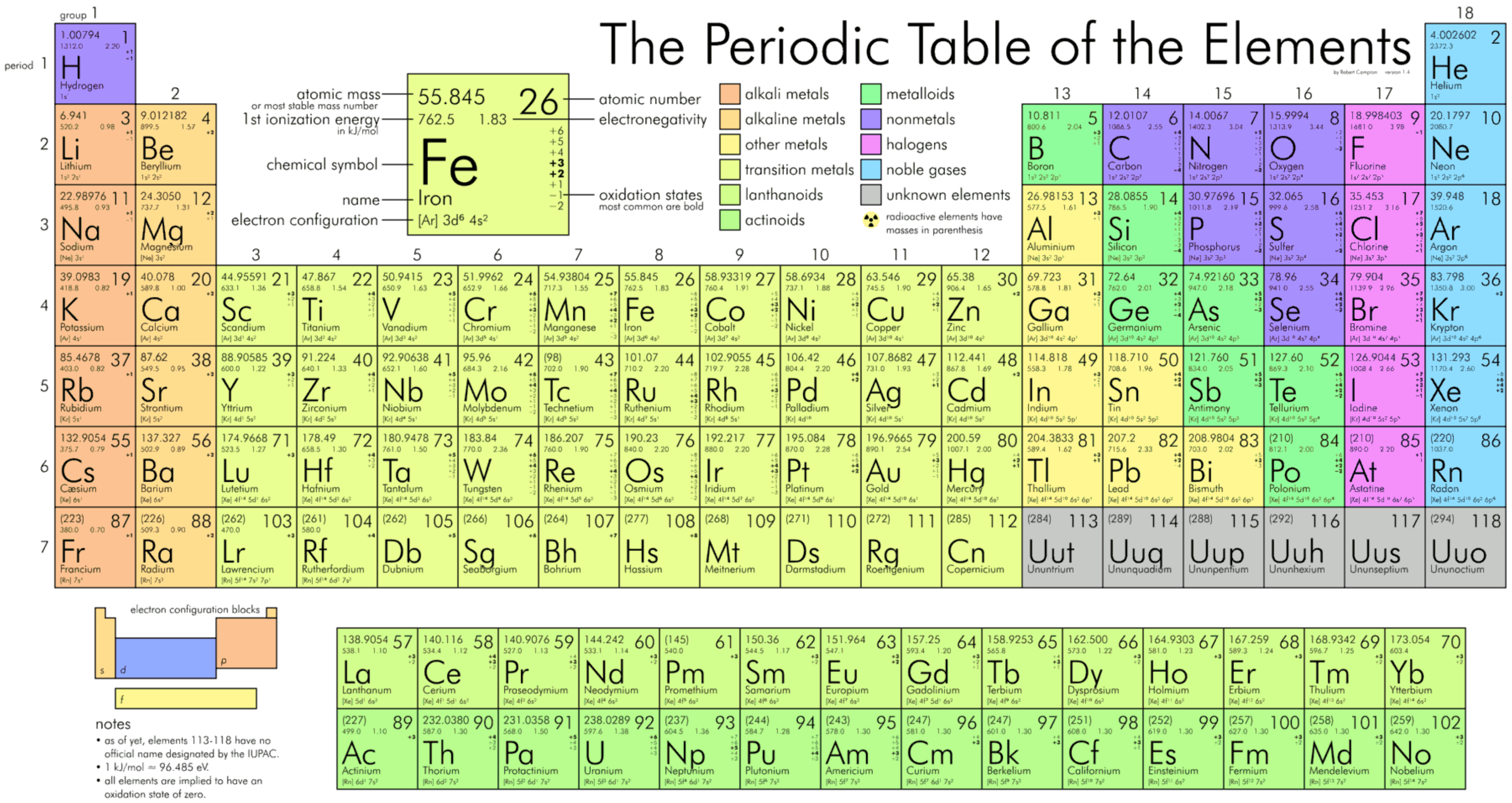
Symmetry arguments don't always work

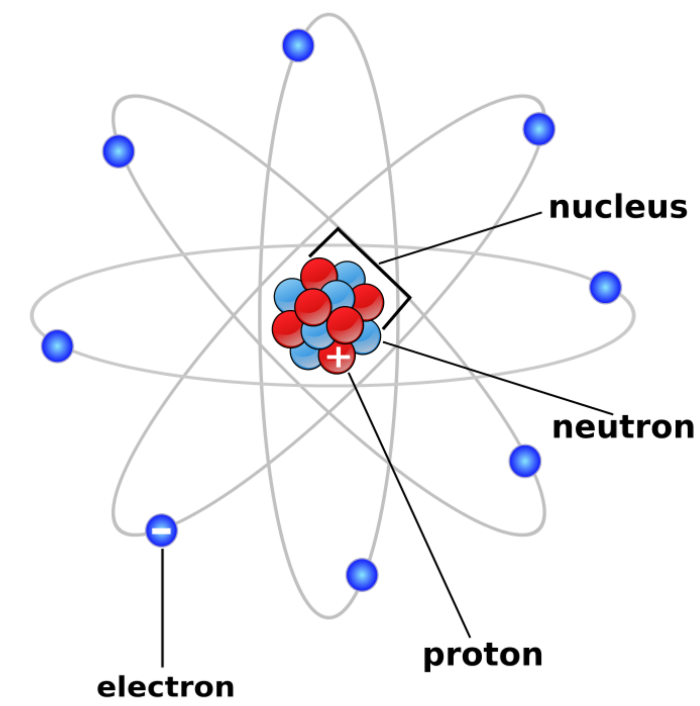
- six spheres separated by the five Platonic solids (regular polyhedra)



J. Kepler, Mysterium Cosmographicum, 1596

Periodic table of elements (Mendeleev 1869)





http://www.wpclipart.com/

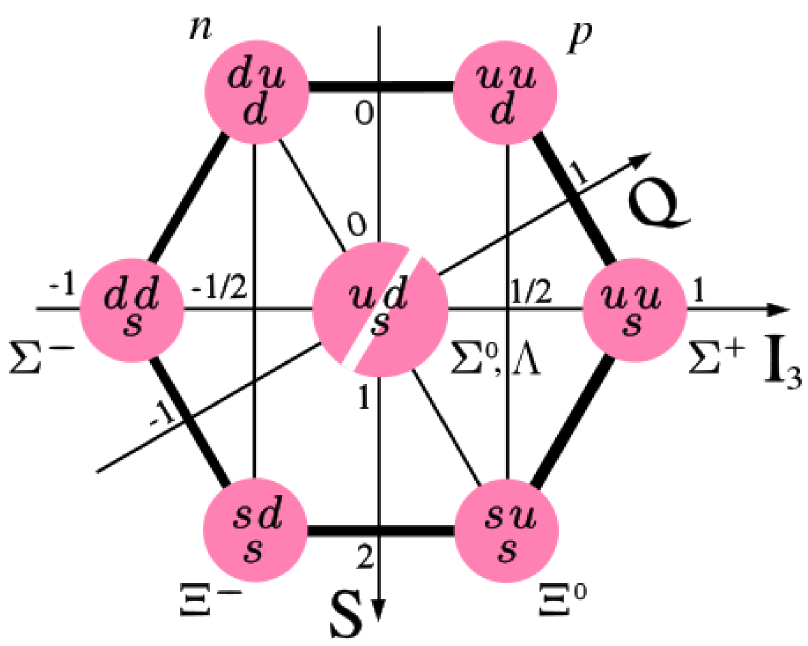
Took some time to understand what is behind the symmetries in the table

Experiment: discovery of electron (Thompson, 1897), atomic nucleus (Rutherford, 1910)

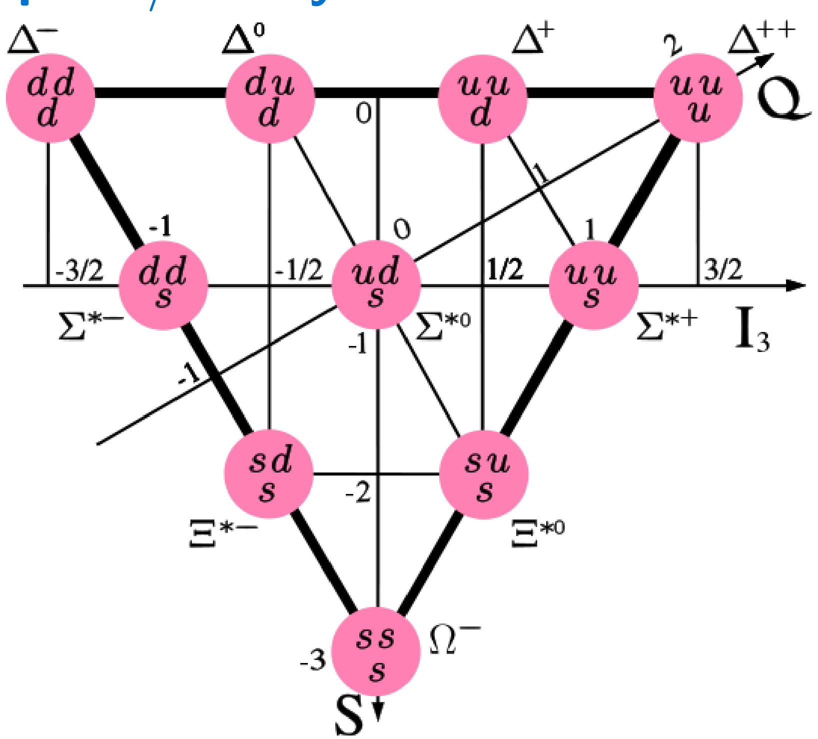
Theory: electrodynamics (Maxwell 1861-1862), quantum mechanics (Schrdinger 1926)

Periodic table of hadrons - quark model

Spin $1/2$ baryons



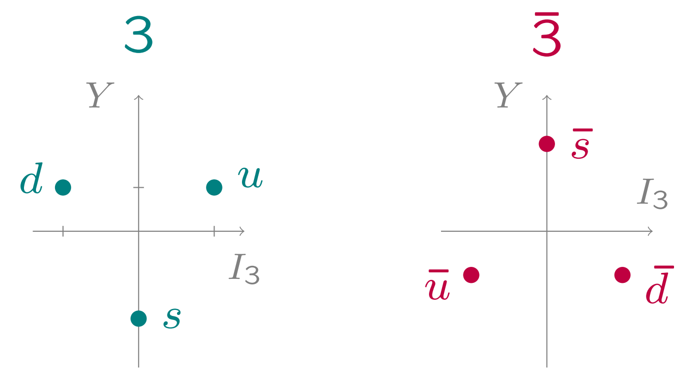
Spin $3/2$ baryons



Gell-Mann

Zweig

1961-1962



CERN, July 1962

- hadrons form SU(3) multiplets

  two conserved quantum numbers

  isospin (I_3) and **strangeness** (S)
- baryons (fermions) - qqq
  - proton: uud
- mesons (bosons) - $q\bar{q}$
- prediction of new particles and their properties $(m,\ \mu)$

Experiment: Is it possible to see individual quarks?

Theory: The nature of interactions between quarks?

Hint: quarks carry new quantum number, color (RBG)

baryon wave functions should be antisymmetric from Pauli's Exclusion Principle

Looking inside the proton

- we need light with high enough energy: Heisenberg relation of uncertainty $\Delta x \Delta p \geq \hbar/2$
- natural units where $\hbar = c = 1$. Connection with SI units through $\hbar c \sim 200$ MeV · fm

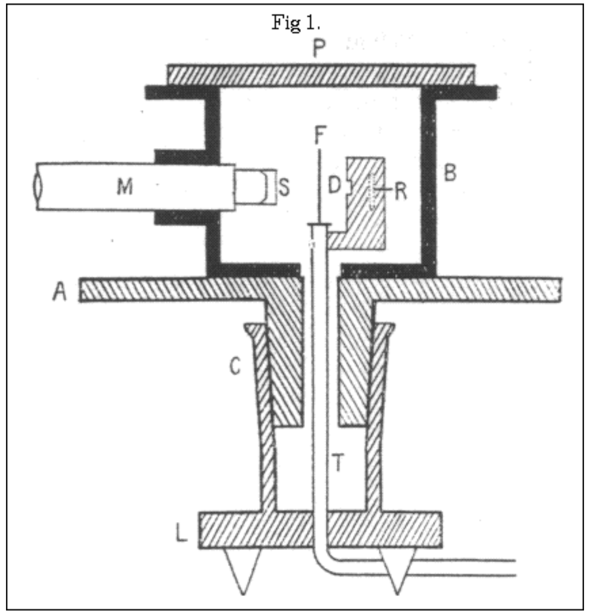
$E_\gamma \sim 2\,\text{eV}$ $E_\gamma \sim 2\,\text{MeV}$

100 nm 100 fm

light source: Sun α particles from ^{88}Ra

Science Museum, London (CC BY-SA)

Pasteur's microscope



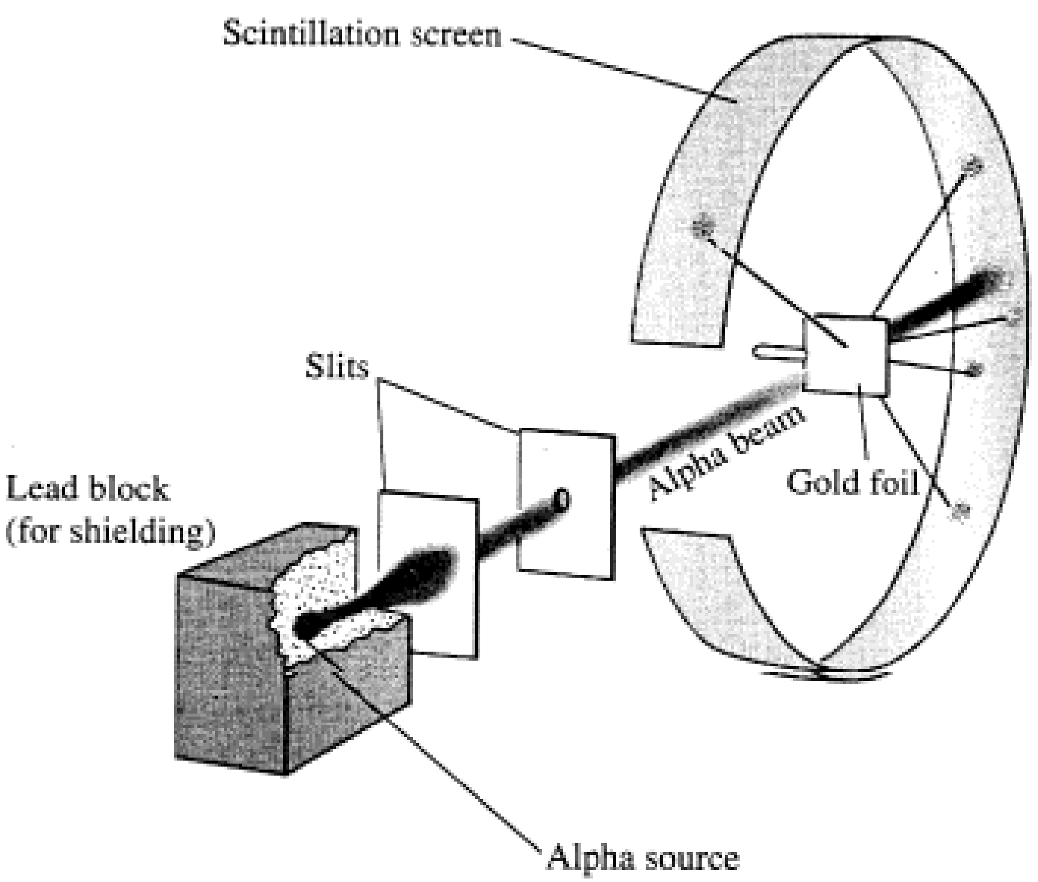
Geiger, Marsden (1913)

Rutherford experiment

$R_{Au} < 30$ fm

Rutherford experiment - atomic nucleus

- scattering of α particles on gold foil
- angular distribution corresponded to a scattering on a point-like charge
- $E_\alpha = 7.86$ MeV $\Rightarrow$ positive charge is localized in a sphere with $R < 30$ fm

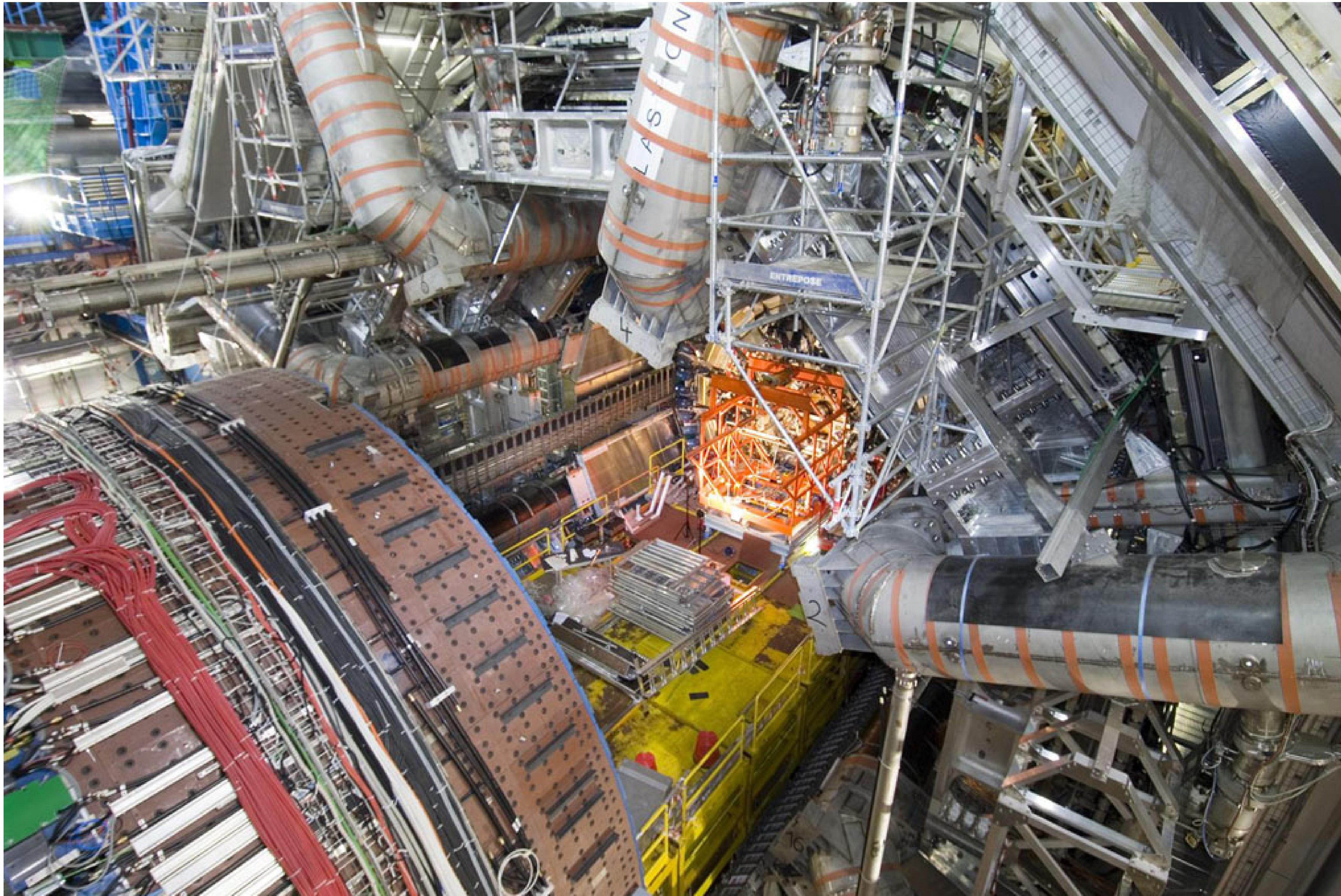


Marsden and Rutherford, Manchester University

- first particle physics experiment
- experimental setup could fit on a table
- nowadays collider experiments are basically of the same nature

  beam - target (second beam) - detector
- just a little bit sophisticated and a little bit larger

Detector ATLAS at LHC



Construction of detector ATLAS

Looking inside the proton

- we need light with high enough energy: Heisenberg relation of uncertainty $\Delta x \Delta p \geq \hbar/2$
- natural units where $\hbar = c = 1$. Connection with SI units through $\hbar c \sim 200$ MeV $\cdot$ fm

$E_\gamma \sim 2\,\mathrm{eV}$	$E_\gamma \sim 2\,\mathrm{MeV}$	$E_\gamma \sim 200\,\mathrm{MeV}$	$E_\gamma \sim 20\,\mathrm{GeV}$
100 nm	100 fm	1 fm	0.01 fm

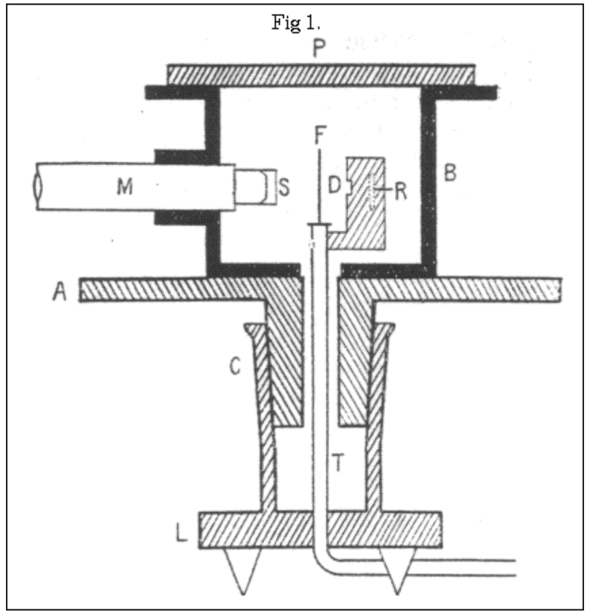
light source: Sun

α particles from ^{88}Ra

electrons from linear accelerators

Science Museum, London (CC BY-SA)

Pasteur's microscope



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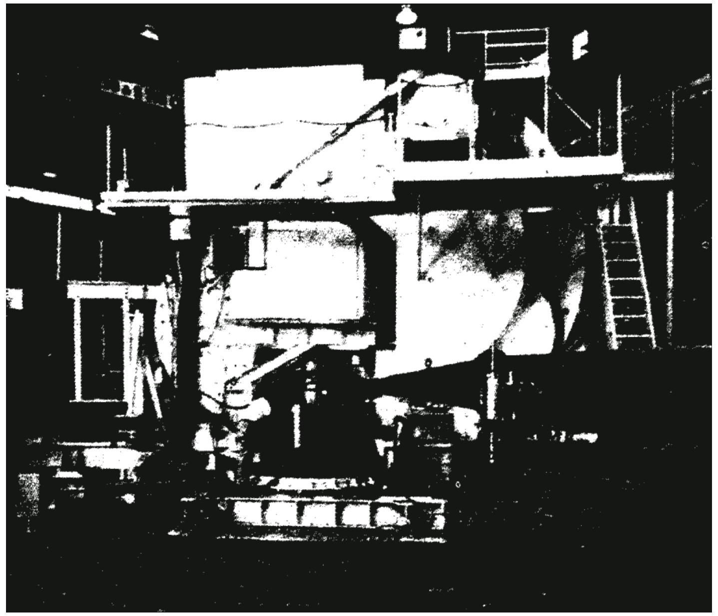
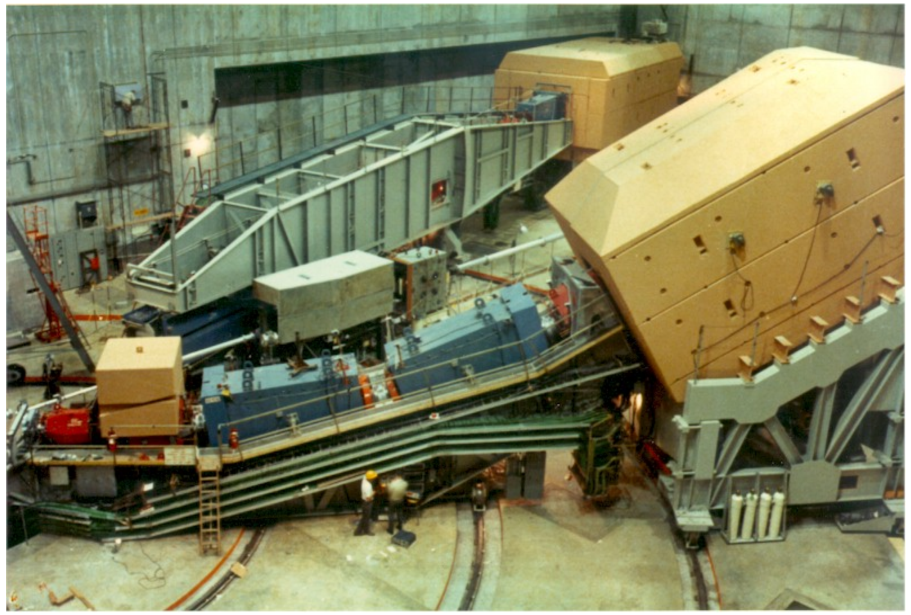


Hofstadter's Nobel Prize lecture (1961)

ep elastic scattering

$R_{\text{proton}} \sim 0.8$ fm



Experimental area at SLAC (1968)

ep deep inelastic scattering (DIS)

proton structure

DIS at SLAC - proton structure

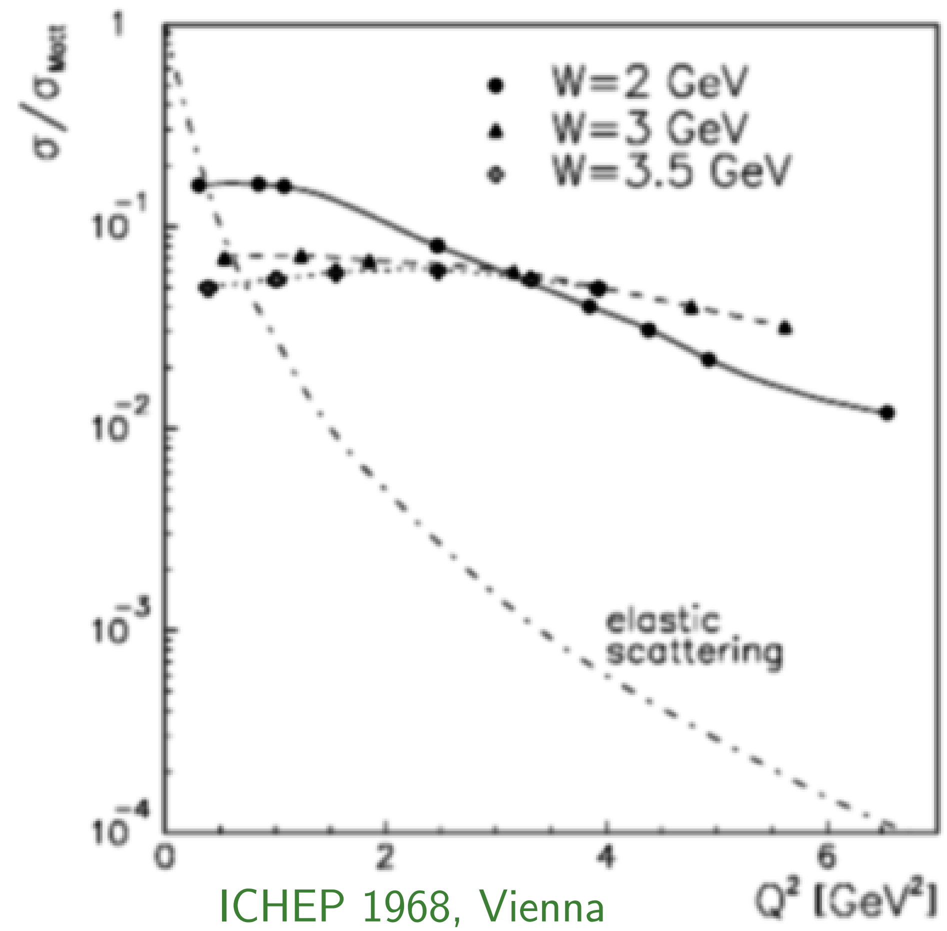
SLAC founded in 1962

3km long linear accelerator

electrons up to 20 GeV

Stanford Linear Accelerator Center

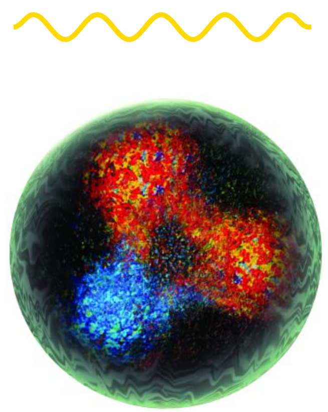
Ratio to the scattering on a point-like proton

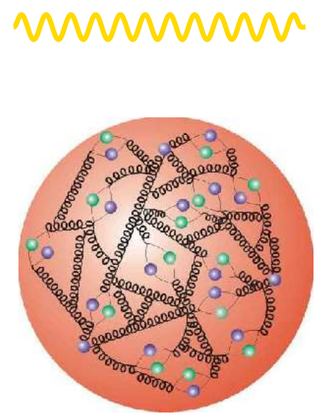


ICHEP 1968, Vienna

W - measure of inelasticity

Photon virtuality $Q^2 = -E_\gamma^2 + |\vec{p}_\gamma|^2$





Parton model (Feynman 1968)

- proton behaves in DIS as a bag of almost free point-like particles - **partons**
- **parton distribution functions** $f_A(x)$ - number of partons of type A carrying longitudinal fractional momentum of proton x;
  not calculable, determined from data, universal

Quantum Chromodynamics - theory of interactions between quarks

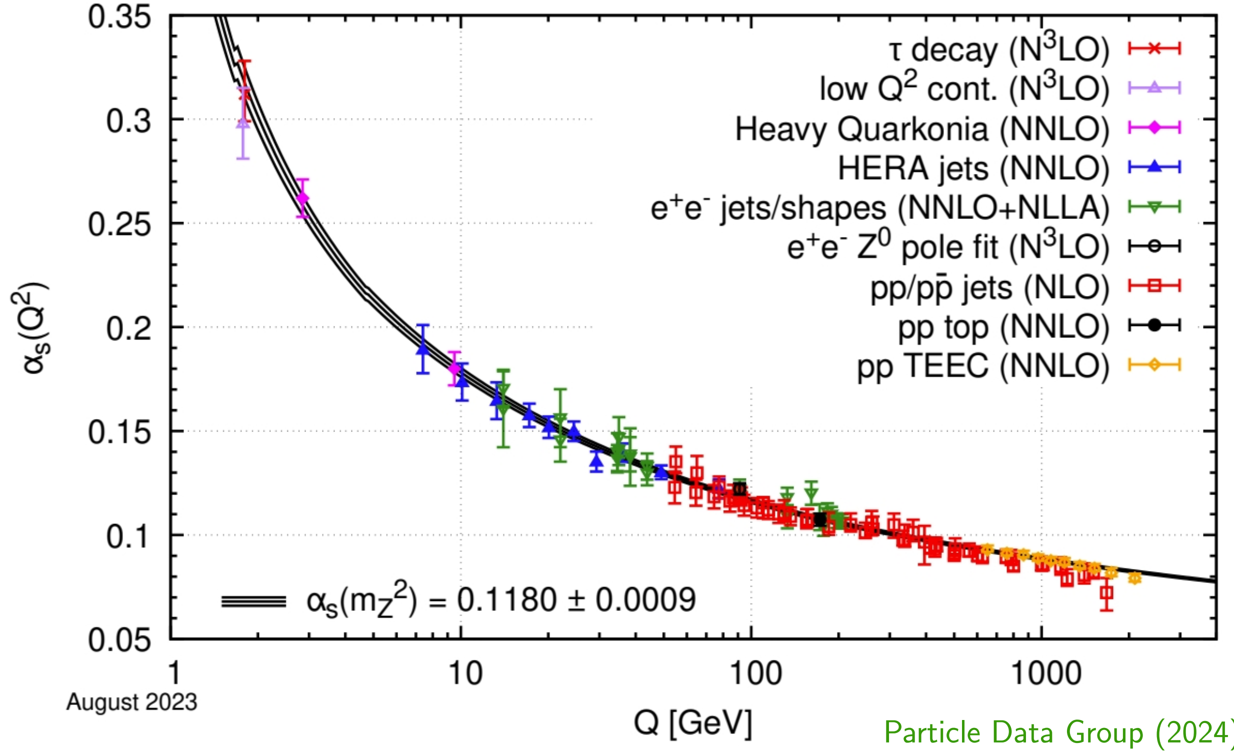
- Theory that gives confinement at 1 fm scale while almost free at smaller distances
- Gross, Wilczek, Politzer (1973)

  QFTs with local internal non-abelian symmetries can have this behaviour

Quantum Chromodynamics (QCD)

- theory of strong interaction between quarks
- group of gauge invariance: SU(3)
- preserving charge - **color**

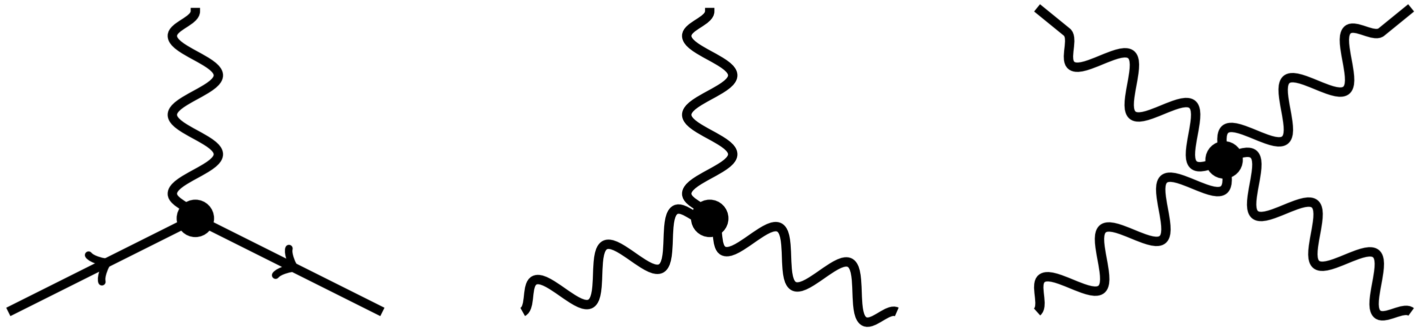
Running of strong coupling α_S



Particle Data Group (2024)

- **gluons** - mediators of strong interaction

  massless, carry the charge of strong interaction

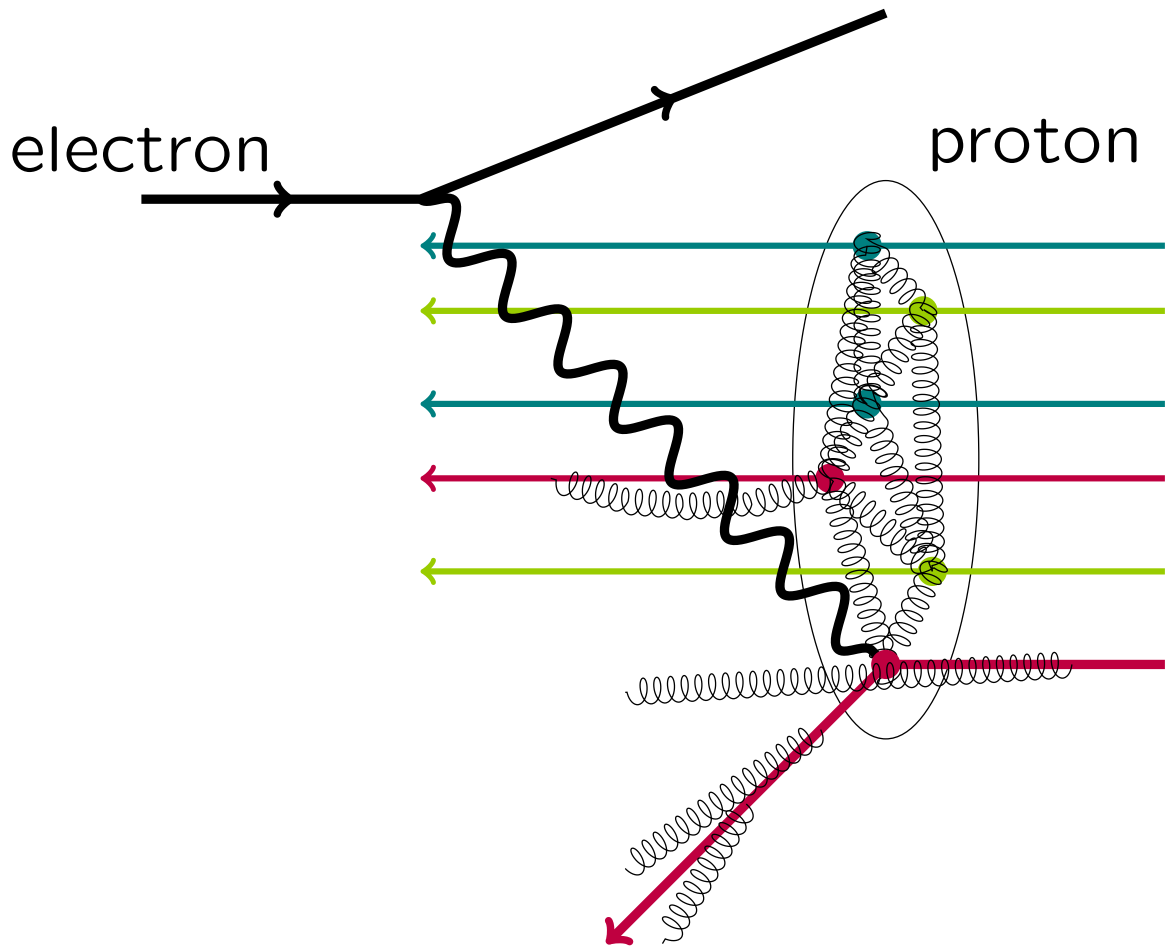


Standard Model (SM)

gauge symmetry: $U(1) \otimes SU(2) \otimes SU(3)$
electro-weak strong

$+$ Higgs mechanism

Quark confinement and jets

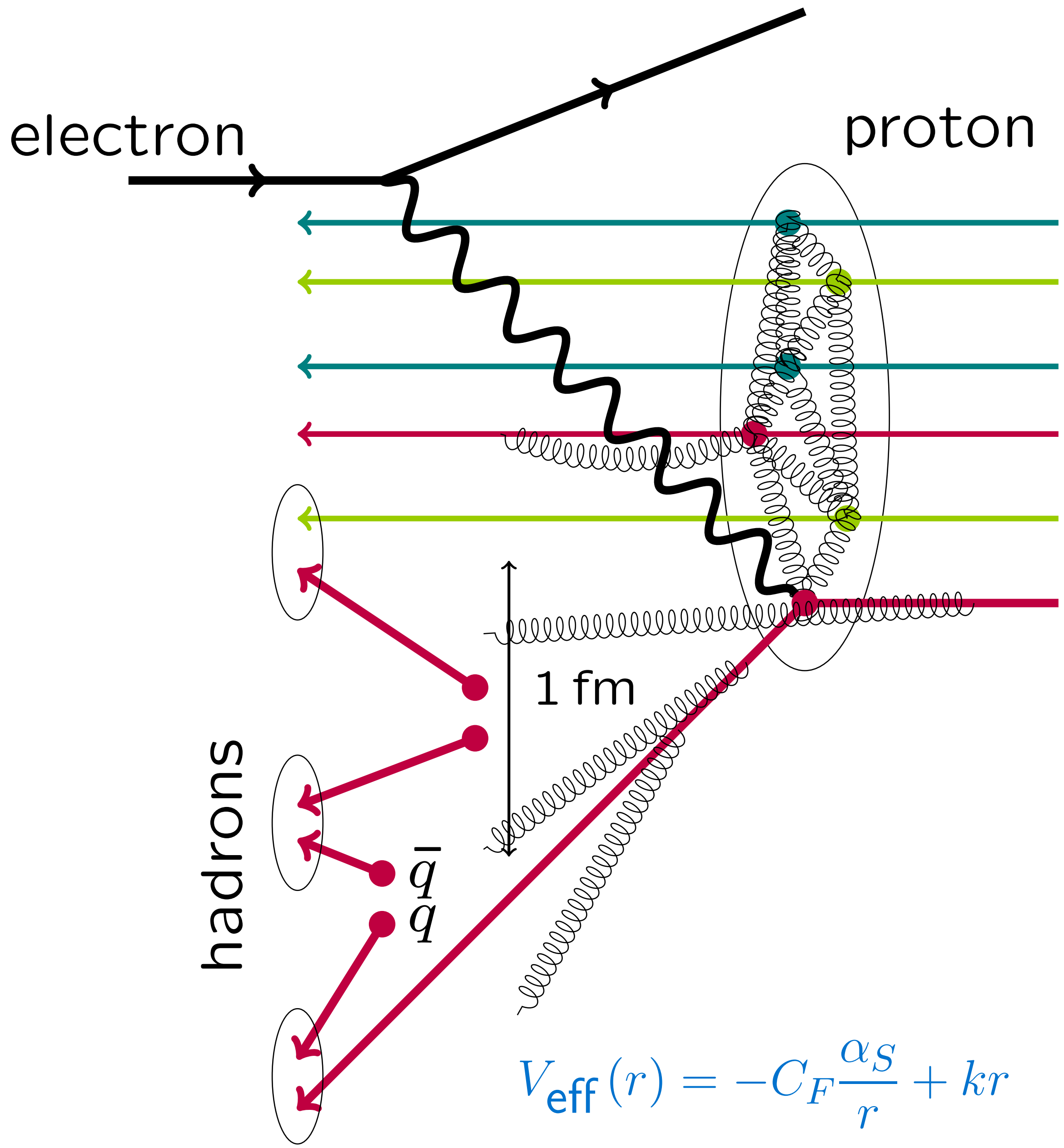


What happens with the kicked-out parton?

Parton shower

- as in QED, accelerated strong charge radiates
- as in QED, radiation mostly in the direction of motion
- pQCD can be applied to describe this radiation

Quark confinement and jets



What happens with the kicked-out parton?

Parton shower

- as in QED, accelerated strong charge radiates
- as in QED, radiation mostly in the direction of motion
- pQCD can be applied to describe this radiation

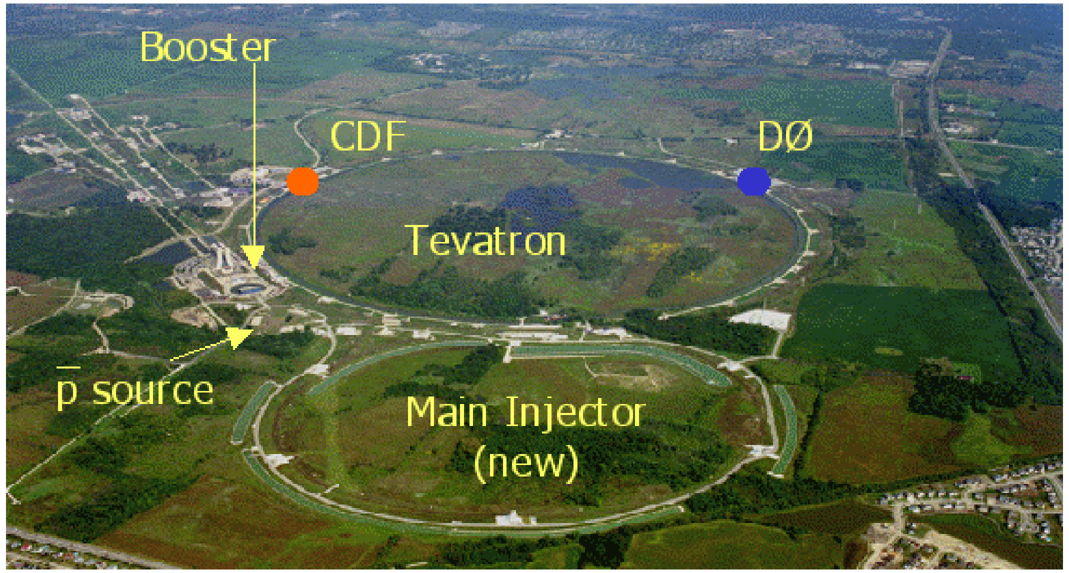
Hadronization

- once the distance between charges reaches 1 fm
- enough energy in the field to create quark anti-quark (color anti-color) pairs
- neutral (colorless) clusters are formed
- process continues until there is no free color left
- non-perturbative process - we have only models

  `Pythia`, `Herwig`, ...

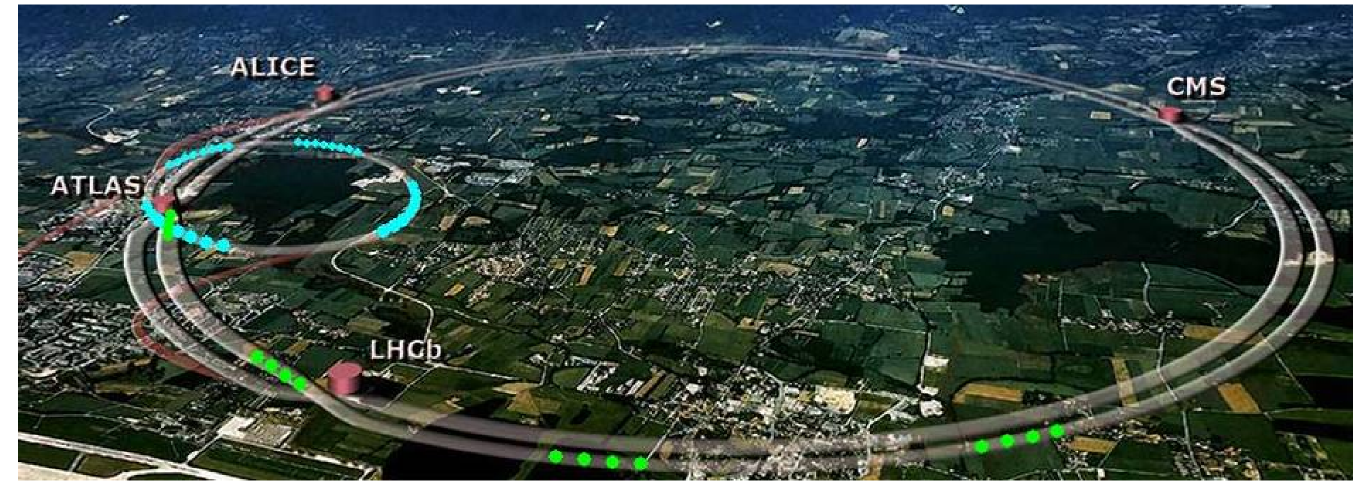
- we observe strongly collimated spray of particles in the direction of kicked-out parton - **jets**

Hadron colliders - the largest microscopes in the world



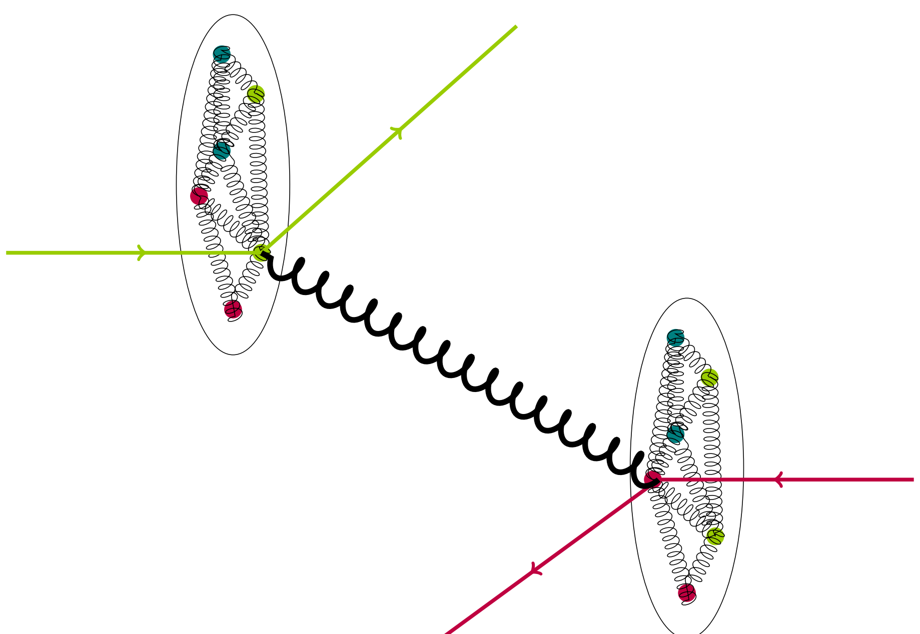
TEVATRON at Fermilab (Run2: 2001-2011)

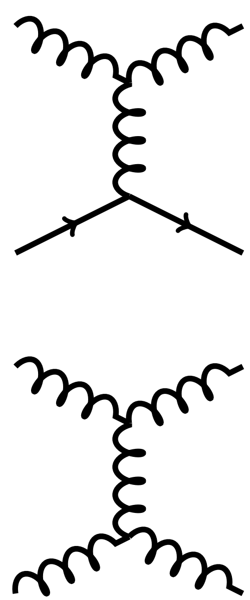
- $p + \bar{p}$ at $\sqrt{s} = 1.96$ TeV



LHC at CERN (from 2010)

- $p + p$ at $\sqrt{s} = 13.6$ TeV



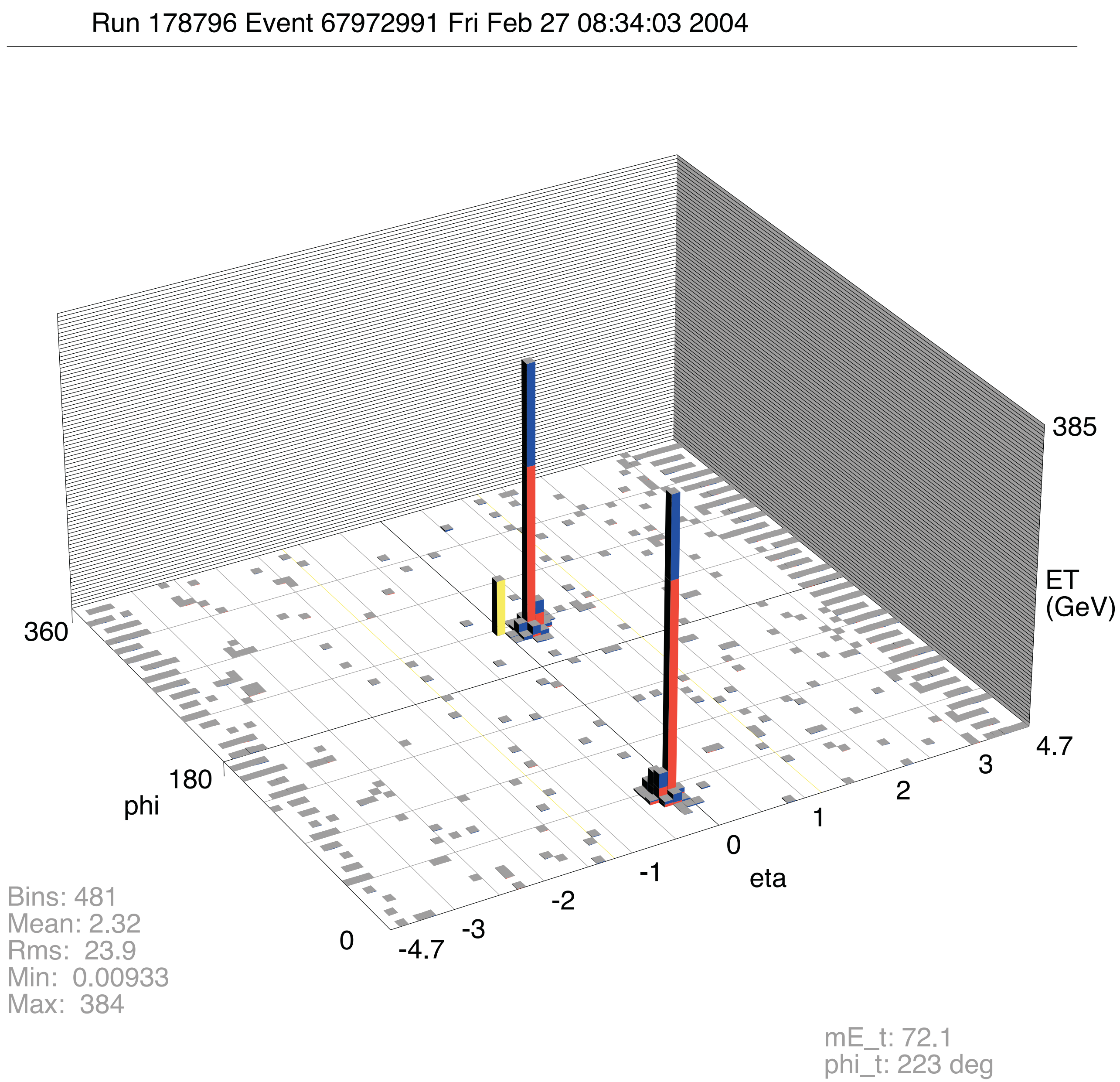


- gluon instead of photon
- more fundamental processes $(qq \to qq,\, qg \to qg,\, gg \to gg)$
- production rate (cross section), $[\sigma] = \mathrm{m}^2$, $1\,\mathrm{b} = 10^{-28}\mathrm{m}^2$

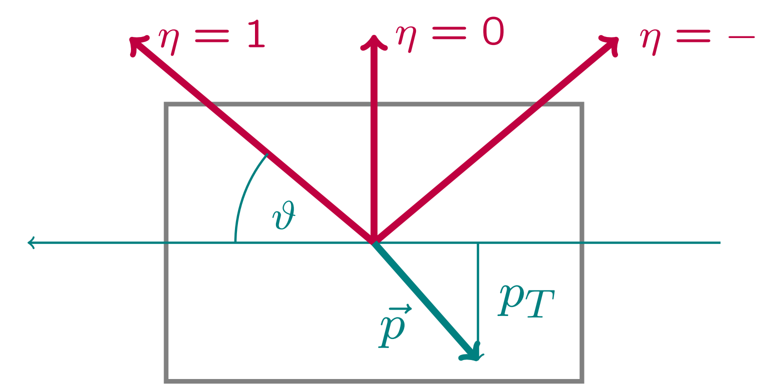
$$\sigma_X = \sum_{abcd}^{flavors} \int \mathrm{d}x_1 \int \mathrm{d}x_2 f_a(x_1) f_b(x_2) \sigma(ab \to cd) D(cd \to X)$$

$N = \sigma\mathcal{L}$, luminosity (particle flux) $[\mathcal{L}] = \mathrm{m}^{-2}$

Jets at DØ experiment



Dijet event at DØ, $p_{Tjet} \sim 600$ GeV



Coordinates

- beam - z-axis
  - polar angle ϑ

Boost invariant quantities

- CMS of parton collision moves along beam axis
- azimuthal angle φ, transverse momentum p_T
- **rapidity** (y)

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

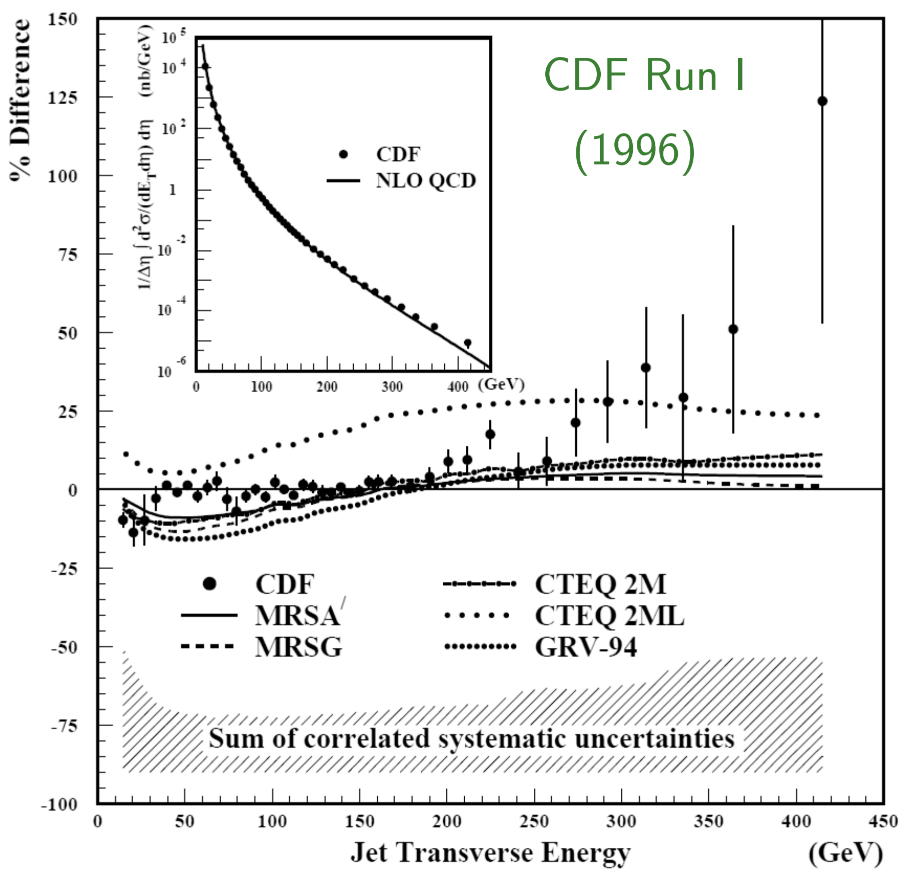
- **pseudorapidity** $\eta = -\ln \tan(\vartheta/2)$
  $y \approx \eta$ for $m \to 0$

Jet algorithm

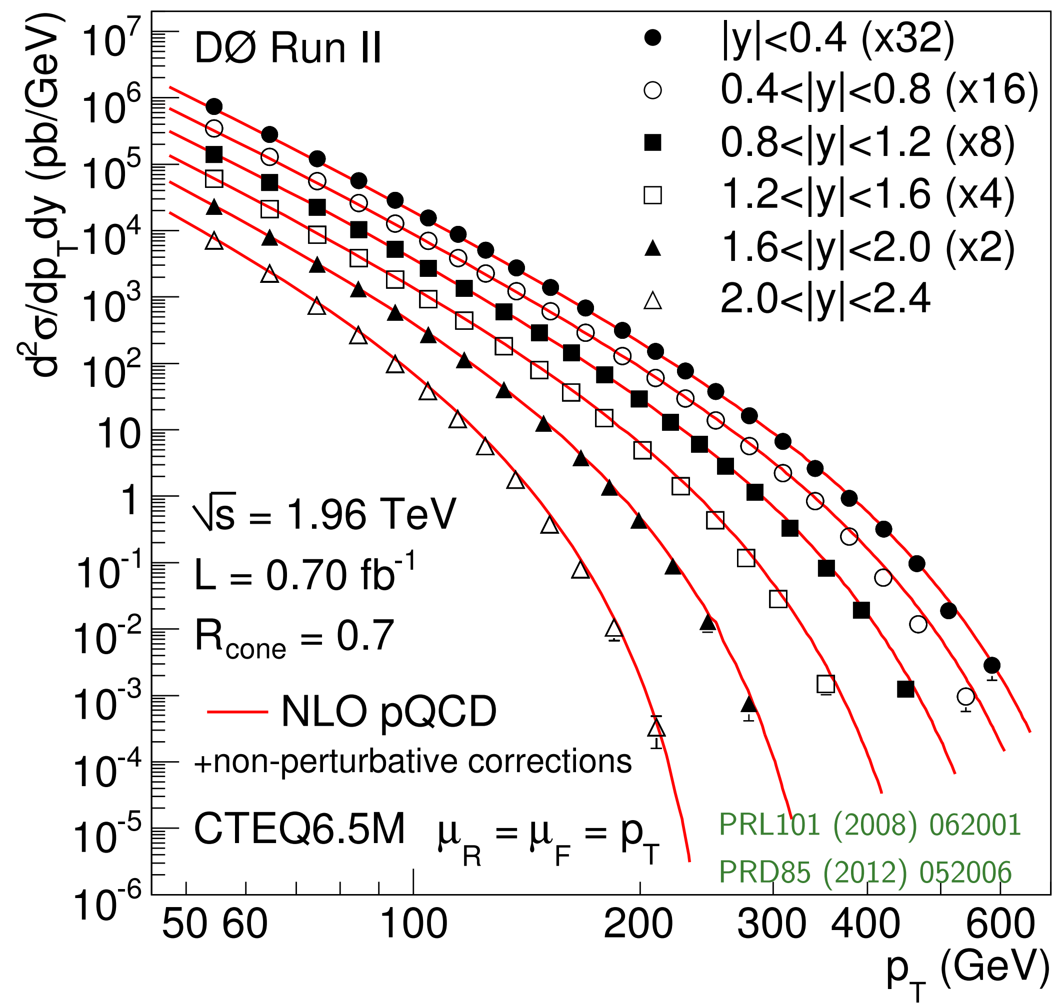
- DØ: cone in $R = \sqrt{\Delta^2\varphi + \Delta^2\eta} < 0.7$

Jet production at the Tevatron

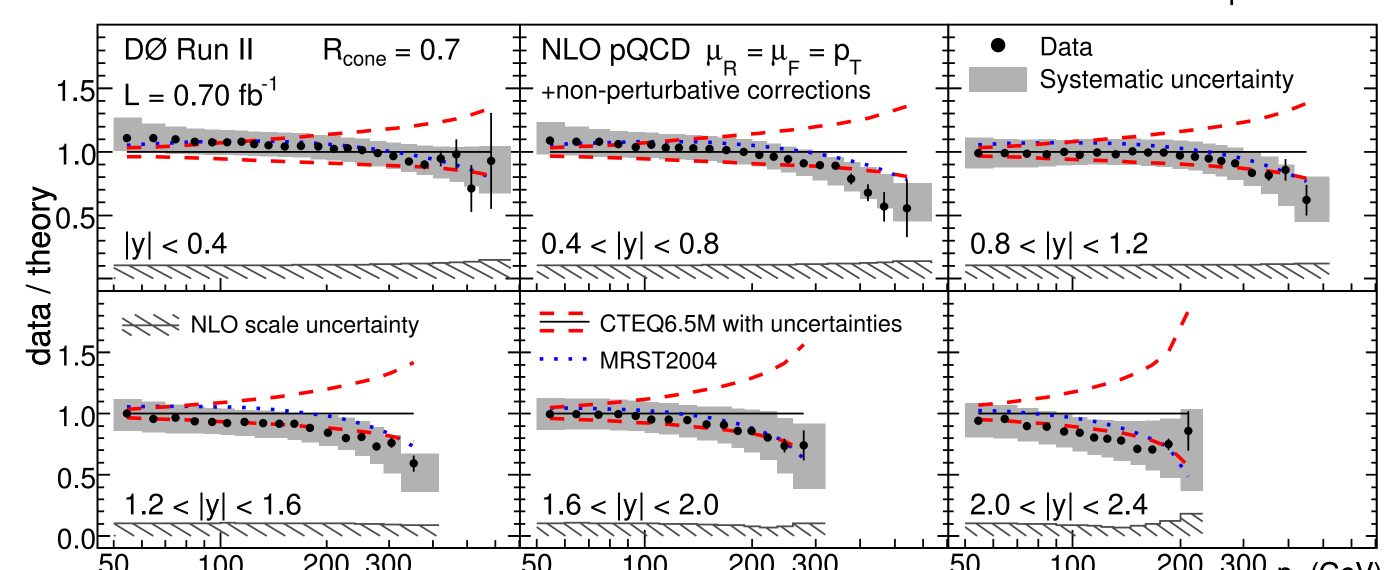
- two times better than the Run I precision
- $7\times$ more data (luminosity)
- improved jet algorithm
- improved PDFs (mostly from HERA)
- excellent agreement with QCD
- no sign of quark substructure down to scale of $\sim 10^{-3}$ fm
- data used in the following PDF fits (gluon PDF)



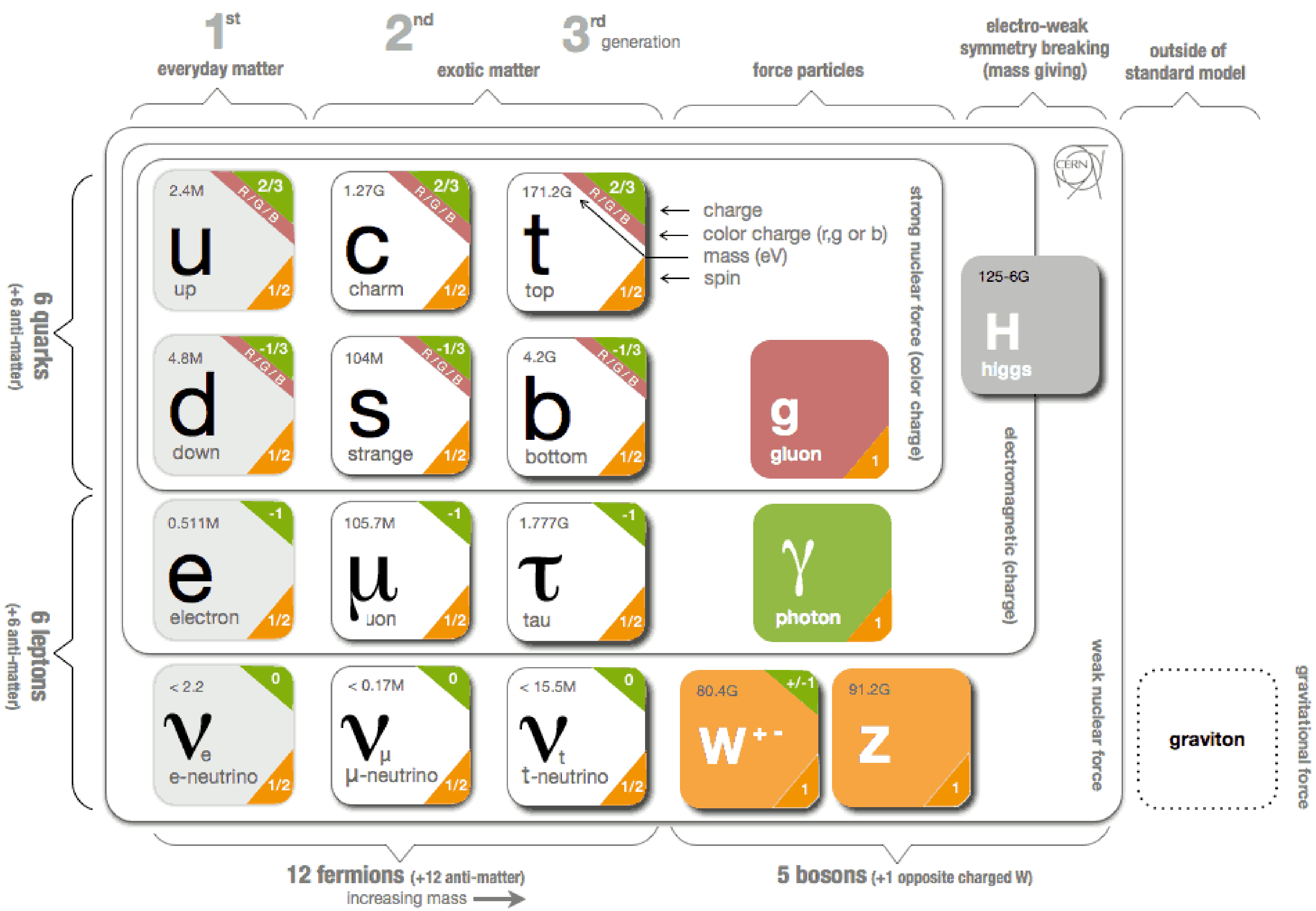
PRL77 (1996) 438



PRL101 (2008) 062001
PRD85 (2012) 052006



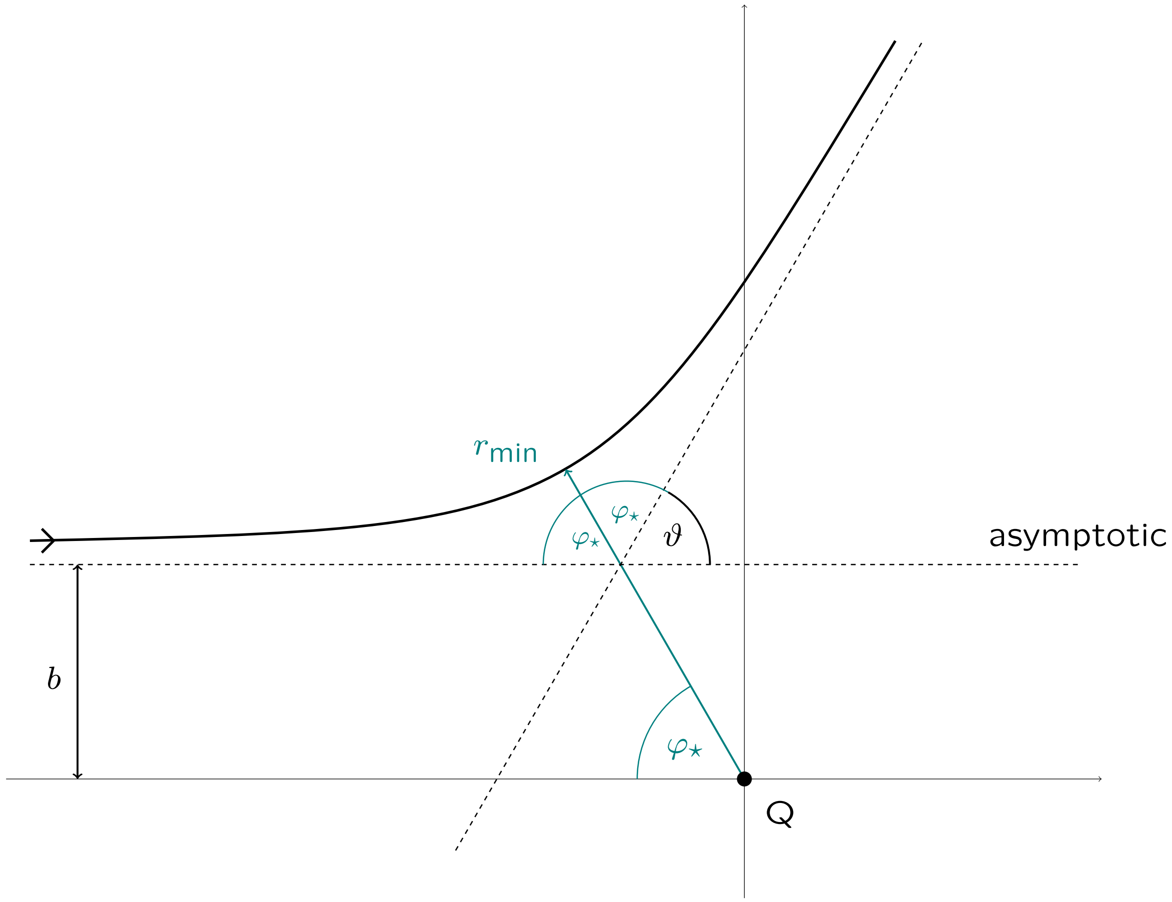
Periodic table of particle physics - Standard Model



- Same questions
  - What is behind this simple structure?
  - Can we see this new physics beyond SM experimentally?
  - What kind of theory is behind?
- Only hints
  - Dark matter and energy
  - Neutrino masses
- Many ideas
  - SUSY, strings, ...

- Jets (and QCD) represent an important tool for exploring the structure of matter
- With high p_T jets we directly probe the structure of matter and space-time at the smallest distances
- At hadron colliders, jets are everywhere - their understanding improves many measurements not related directly to them

Scattering in classical mechanics (CM)



- in CM, initial condition (positions + momenta) determine future (and past) trajectory
- scattering on potential

$$V(\vec{r}) = V(r), \quad \text{with force } \vec{F} = f(r)\vec{r}$$

  is happening in a plane (orbital momentum conservation)
- initial state (at $t \to -\infty$) can be specified by **impact parameter** b and **incident energy** E

- after scattering, at $t \to \infty$, particle direction is deflected by **scattering angle** ϑ
- solving the scattering problem in CM means finding $\vartheta(b, E)$
- for scattering on Coulomb potential $V(r) = \frac{\alpha}{r}$

$$\tan\frac{\vartheta}{2} = \frac{1}{\kappa b}, \quad \text{with} \quad \kappa = \frac{2E}{\alpha} \qquad \text{(Exercise 2.1)}$$

- enough to describe a scattering of one body (comet entering Solar system on hyperbolic trajectory)

Cross section

- Rutherford experiment - many particles hitting the target under random (unknown) impact parameter
- statistical nature of experiment - result described by **cross section** σ

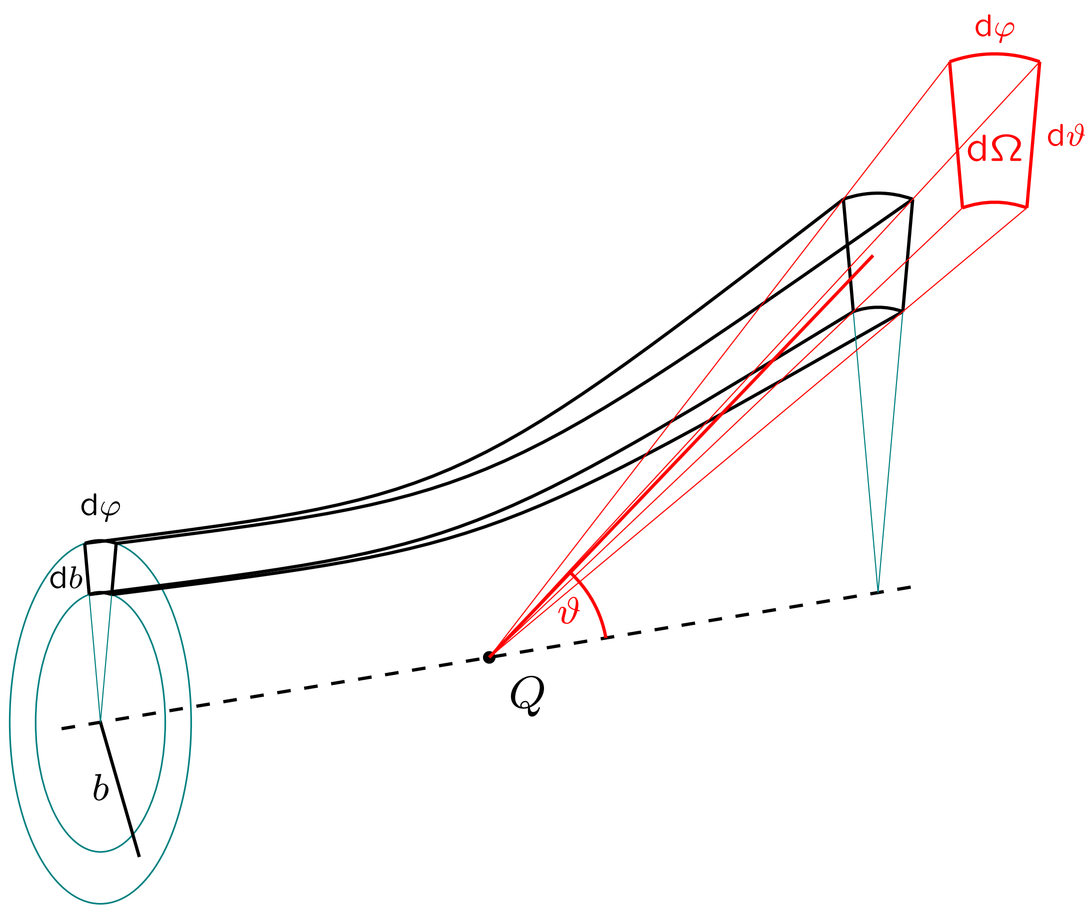
$$\mathrm{d}\sigma(\vartheta,\varphi) = \frac{N(\vartheta,\varphi)\mathrm{d}\Omega}{\mathcal{L}}$$

- $N(\vartheta,\varphi)$ - number of particles scattered in unit time to solid angle element $\mathrm{d}\Omega$ in direction (ϑ,φ)
- $\mathcal{L}$ - particle flux, **luminosity**

- if there is an inverse to $\vartheta(b,E)$, i.e. $b(\vartheta,E)$, then

$$\mathrm{d}\sigma = \frac{\mathcal{L}\, b\, \mathrm{d}b\, \mathrm{d}\varphi}{\mathcal{L}} = b\,\frac{\mathrm{d}b}{\mathrm{d}(\cos\vartheta)}\mathrm{d}(\cos\vartheta)\mathrm{d}\varphi$$

$$\Rightarrow \qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = b(\vartheta)\,\frac{\mathrm{d}b(\vartheta)}{\mathrm{d}(\cos\vartheta)}$$



- for scattering on Coulomb potential

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{4\kappa^2\sin^4\frac{\vartheta}{2}} \qquad \text{(Exercise 2.1)}$$

Rutherford experiment

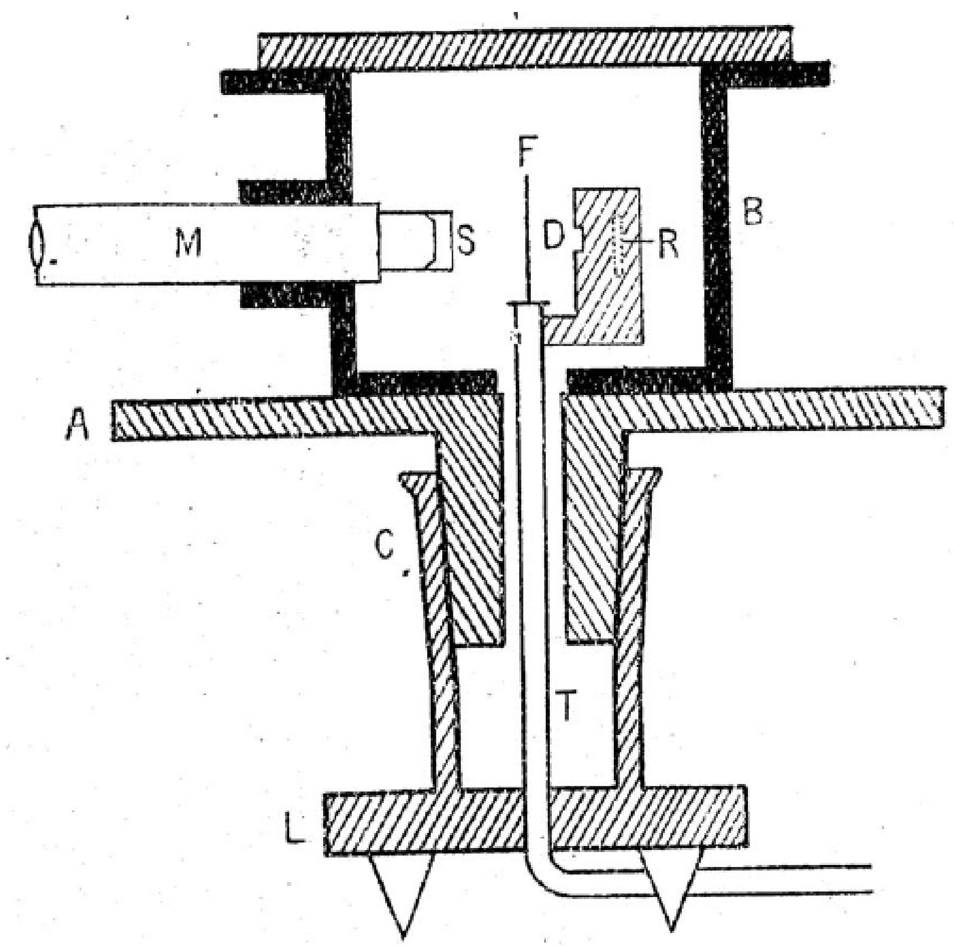


Table II.
Variation of Scattering with Angle. (Collected results.)

I. Angle of deflexion, ϕ.	II. $\frac{1}{\sin^4 \phi/2}$.	III. Silver. Number of scintillations, N.	IV. Silver. $\frac{N}{\sin^4 \phi/2}$.	V. Gold. Number of scintillations, N.	VI. Gold. $\frac{N}{\sin^4 \phi/2}$.
150°	1·15	22·2	19·3	33·1	28·8
135	1·38	27·4	19·8	43·0	31·2
120	1·79	33·0	18·4	51·9	29·0
105	2·53	47·3	18·7	69·5	27·5
75	7·25	136	18·8	211	29·1
60	16·0	320	20·0	477	29·8
45	46·6	989	21·2	1435	30·8
37·5	93·7	1760	18·8	3300	35·3
30	223	5260	23·6	7800	35·0
22·5	690	20300	29·4	27300	39·6
15	3445	105400	30·6	132000	38·4
30	223	5·3	0·024	3·1	0·014
22·5	690	16·6	0·024	8·4	0·012
15	3445	93·0	0·027	48·2	0·014
10	17330	508	0·029	200	0·0115
7·5	54650	1710	0·031	607	0·011
5	276300	...	...	3320	0·012

Figure 1.3: Schematic layout of the apparatus used by Geiger and Marsden (left) and the Table containing the results of their measurements.

- observed yields in Rutherford experiment followed $\sin^{-4}(\vartheta/2)$ formula
- incident energy of α particles was $E = 7.86$ MeV $\Rightarrow r_{\text{min}} \sim 30$ fm (Exercise 2.2)

  $\Rightarrow$ positive charge must be localized in sphere with radius smaller then 30 fm

  for $\varrho(r)$, the potential energy is given only by charge inside sphere of radius r (Gauss law)

Scattering in quantum mechanics

- cross section is a necessity as the notion of trajectory has no meaning in QM
- QM cookbook (Exercise 2.4)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\vec{p},\vec{p}\,')|^2$$

$$f(\vec{p},\vec{p}\,') = -4\pi^2 m\hbar \,\langle \vec{p}\,'|\hat{T}(\vec{p})|\vec{p}\rangle$$

- from Lipmann-Schwinger eq.

$$\hat{T}(\vec{p}) = \hat{H}_I + \hat{H}_I \hat{G}^{(+)}(E)\hat{H}_I$$

- retarded Green operator $\hat{G}^{(+)}(E)$

$$\hat{G}^{(+)}(E) = \frac{1}{E - \hat{H} + i\varepsilon}$$

- ◦ $\vec{p}$ and $\vec{p}\,'$ - incoming and outgoing momenta
- ◦ $f(\vec{p},\vec{p}\,')$ - **scattering amplitude**
- ◦ $\hat{T}(\vec{p})$ - **transition operator**
- ◦ Hamiltonian $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_I$

  free Hamiltonian $\hat{\mathcal{H}}_0$ and interaction Hamiltonian $\hat{\mathcal{H}}_I$
- ◦ retarded Green operator of free particle

$$\hat{G}_0^{(+)}(E) = \frac{1}{E - \hat{H}_0 + i\varepsilon}$$

- usually solved by perturbative expansion in $\hat{H}_I$

$$\hat{T}(\vec{p}) = \hat{H}_I \;+\; \hat{H}_I\hat{G}_0^{(+)}(E)\hat{H}_I \;+\; \hat{H}_I\hat{G}_0^{(+)}(E)\hat{H}_I\hat{G}_0^{(+)}(E)\hat{H}_I \;+\; \ldots$$

Scattering in quantum mechanics

- cross section is a necessity as the notion of trajectory has no meaning in QM
- QM cookbook (Exercise 2.4)

$$\frac{d\sigma}{d\Omega} = |f(\vec{p},\vec{p}\,')|^2$$

$$f(\vec{p},\vec{p}\,') = -4\pi^2 m\hbar \, \langle \vec{p}\,' | \hat{T}(\vec{p}) | \vec{p} \rangle$$

- from Lipmann-Schwinger eq.

$$\hat{T}(\vec{p}) = \hat{H}_I + \hat{H}_I \hat{G}^{(+)}(E) \hat{H}_I$$

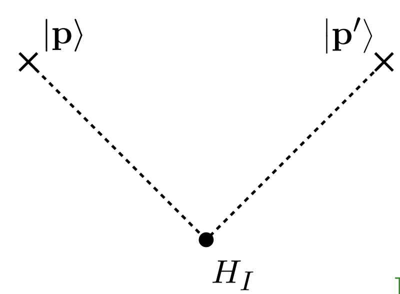
- retarded Green operator $\hat{G}^{(+)}(E)$

$$\hat{G}^{(+)}(E) = \frac{1}{E - \hat{H} + i\varepsilon}$$

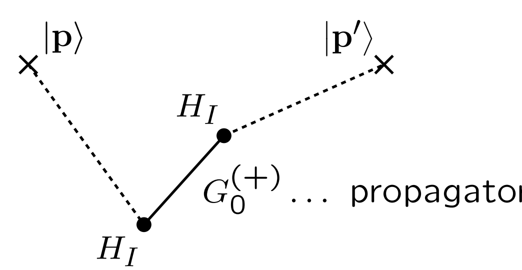
- usually solved by perturbative expansion in $\hat{H}_I$

$$\hat{T}(\vec{p}) = \hat{H}_I \ + \ \hat{H}_I \hat{G}_0^{(+)}(E) \hat{H}_I \ + \ \hat{H}_I \hat{G}_0^{(+)}(E) \hat{H}_I \hat{G}_0^{(+)}(E) \hat{H}_I \ + \ \ldots$$

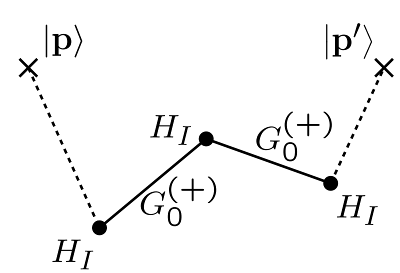
- connection with interpretation of QM based on Feynman path integrals and Feynman diagrams



$\mathbf{H_I}$



$\mathbf{+H_I G_0^{(+)} H_I}$



$\mathbf{+H_I G_0^{(+)} H_I G_0^{(+)} H_I}$

Born approximation

- leading term in perturbative expansion: $\hat{T}(\vec{p}) = \hat{H}_I(\vec{p})$
- in space representation

$$\langle \vec{x}|\vec{p}\rangle = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}} \exp\left(\frac{i}{\hbar}\vec{x}\cdot\vec{p}\right), \qquad \hat{H}_I = V(\vec{x})$$

we find

$$f(\vec{p},\vec{p}\,') = -4\pi^2 m\hbar \frac{1}{(2\pi\hbar)^3} \int \mathrm{d}^3\vec{x}\, V(\vec{x}) \exp\left(\frac{i}{\hbar}\vec{x}\cdot(\vec{p}-\vec{p}\,')\right)$$

- we see that scattering amplitude is just a function of momentum transfer $\vec{q} = \vec{p} - \vec{p}\,'$

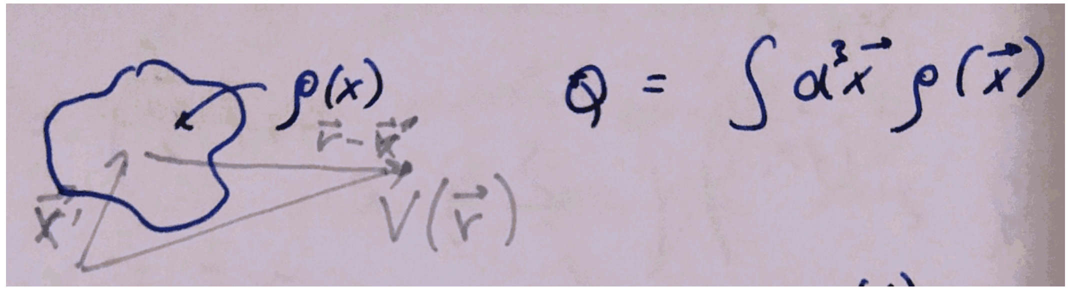
$$f(\vec{q}) = -\frac{m}{2\pi}\int \mathrm{d}^3\vec{x}\, V(\vec{x})\, e^{i\vec{x}\cdot\vec{q}} \qquad \text{(switched to } \hbar = c = 1\text{)}$$

apart the normalization, it is nothing but a Fourier transformation of potential $V(\vec{x})$

- Born approximation for scattering on Coulomb potential gives the same result as CM
  - it is a little bit tricky as we need to regularize the potential first

$$V(r) = \frac{\alpha}{r} \quad \to \quad V(r) = \frac{\alpha}{r}\exp(-\mu r), \text{ with } \mu > 0,\ \mu \to 0 \qquad \text{(Exercise 2.4)}$$

Scattering on target with internal structure



- target described by charge density $\varrho(\vec{x})$
- potential is due to the superposition principle

$$V(\vec{x}) = \int \mathrm{d}^3\vec{x}'\varrho(\vec{x}\,')V^{Q=1}_{\text{point}}(\vec{x}-\vec{x}').$$

$V^{Q=1}_{\text{point}}(\vec{y})$ is potential of point-like particle with unit charge $Q=1$ placed at $\vec{y}=0$

- The scattering amplitude is then

$$f(\vec{q}) = -\frac{m}{2\pi}\int \mathrm{d}^3\vec{x}\,e^{i\vec{x}\cdot\vec{q}}\int \mathrm{d}^3\vec{x}\,'\varrho(\vec{x}\,')V^{Q=1}_{\text{point}}(\vec{x}-\vec{x}')$$

- Reorganizing the integral, and using substitution $\vec{y}=\vec{x}-\vec{x}\,'$, we get

$$f(\vec{q}) = -\frac{m}{2\pi}\int \mathrm{d}^3\vec{x}\,'\varrho(\vec{x}\,')e^{i\vec{x}\,'\cdot\vec{q}}\int \mathrm{d}^3\vec{x}e^{i\vec{x}\cdot\vec{q}-i\vec{x}\,'\cdot\vec{q}}V^{Q=1}_{\text{point}}(\vec{x}-\vec{x}\,') = -\frac{m}{2\pi}\int \mathrm{d}^3\vec{y}\,V^{Q=1}_{\text{point}}(\vec{y})e^{i\vec{y}\cdot\vec{q}}\int \mathrm{d}^3\vec{x}'\varrho(\vec{x}\,')e^{i\vec{x}\,'\cdot\vec{q}}$$

- we factored out the point-like amplitude, putting back the charge Q into it

$$f(\vec{q}) = f^{Q}_{\text{point}}(\vec{q})F(\vec{q}), \qquad \text{where} \quad F(\vec{q}) = \frac{1}{Q}\int \mathrm{d}^3\vec{x}'\varrho(\vec{x}')e^{i\vec{x}\,'\cdot\vec{q}} \quad \ldots \textbf{ form factor}$$

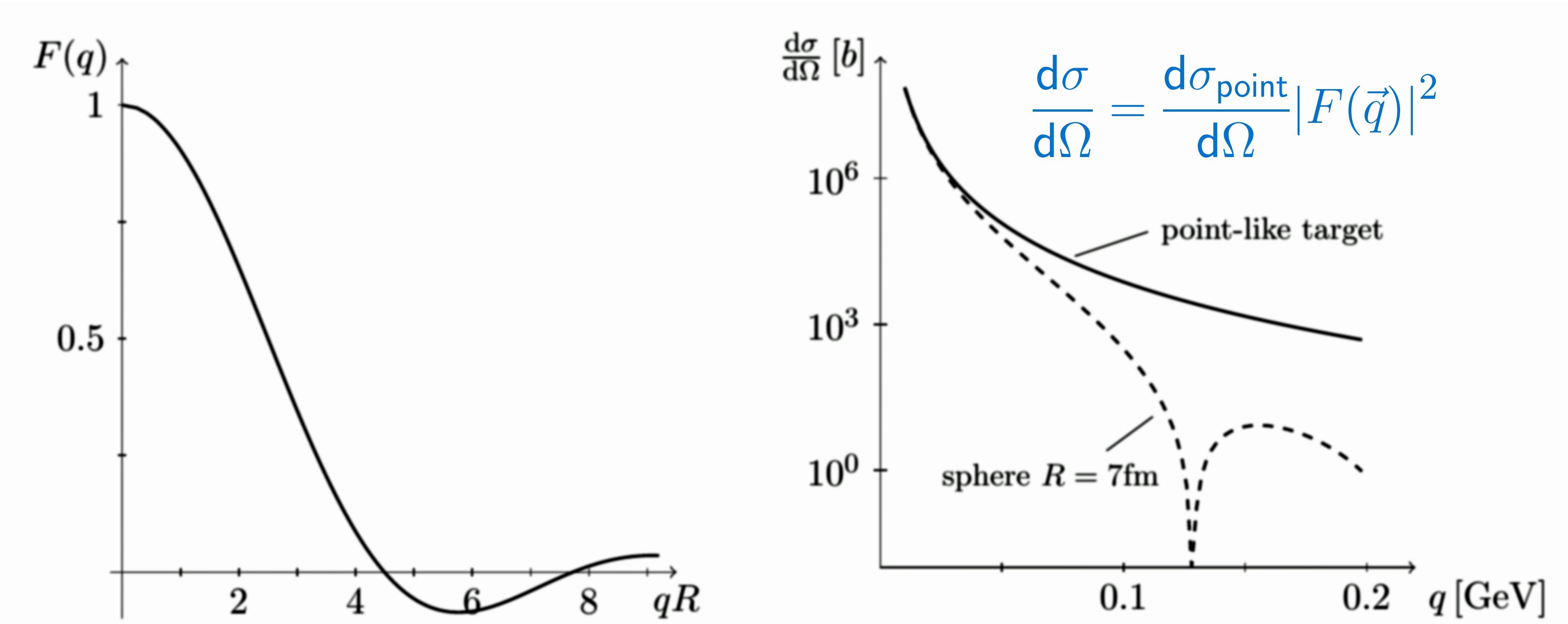
Fourier transformation of charge density

Form factors

- thanks to the normalization, for a target of finite size R

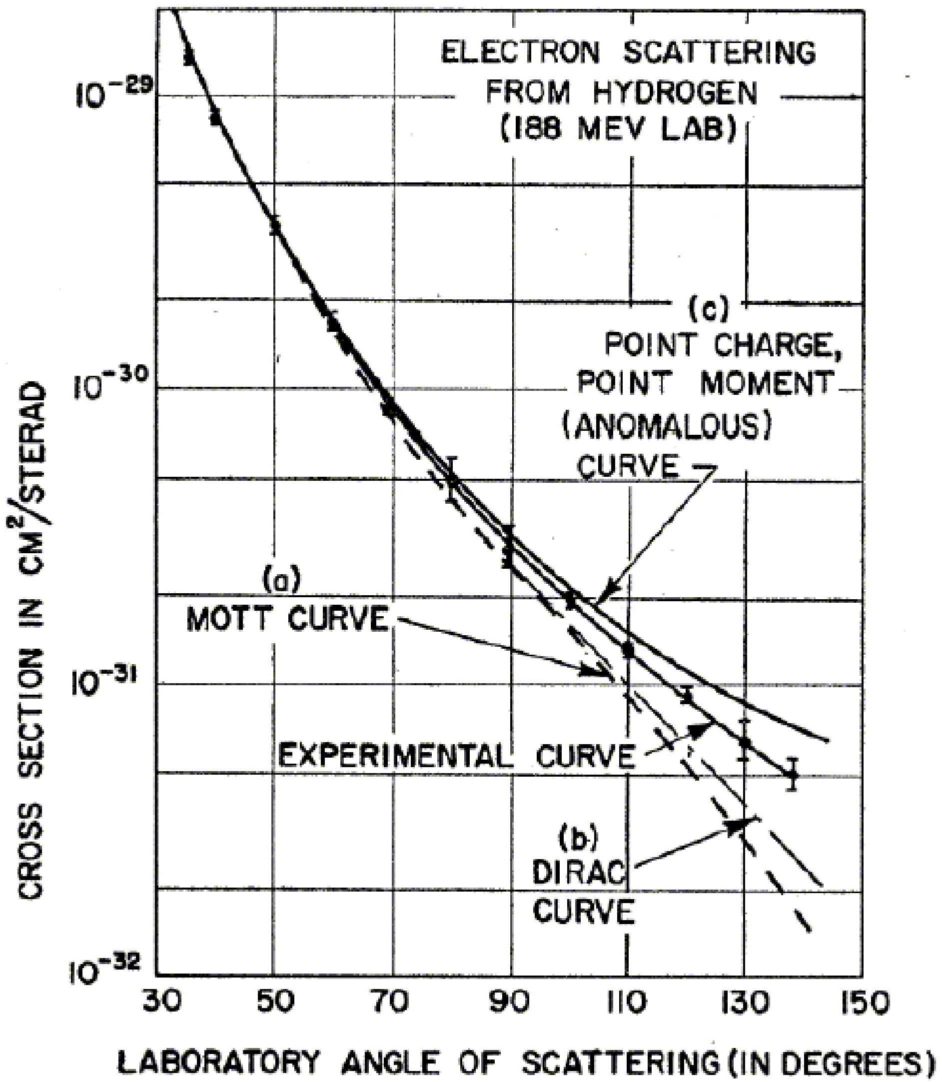
$$F(\vec{q}) \to 1 \qquad \text{for small enough momentum transfers, } qR \ll 1$$

- scattering with low enough energy beams is like a scattering on a point-like particle
- scattering on homogeneous sphere (Exercise 2.5)



$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\text{point}}}{d\Omega}|F(\vec{q})|^2$$

Figure 1.4: *Left.* Formfactor for scattering on homogeneously charged sphere with radius R. *Right.* Corresponding cross section for scattering of α particles on gold nuclei.



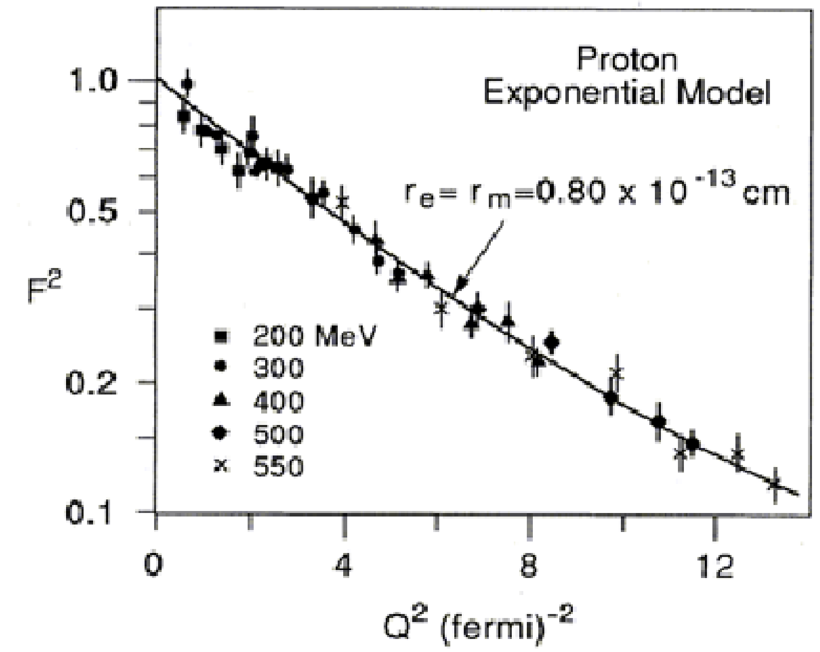


FIG. 8. The proton form factor for various energies and momentum transfers as measured in early experiments using the 36-inch spectrometer facility at HEPL. The value of F^2 was calculated from the original Rosenbluth formula which defined form factors $F_1(Q^2)$ and $F_2(Q^2)$. F_1 corresponds to the form factor for a Dirac (spin-$\frac{1}{2}$) proton, and F_2 to the form factor for the anomalous magnetic moment. In the analysis of the data it was assumed that $F_1 = F_2$. At higher values of Q^2 it became evident that $F_1 \neq F_2$, but rather that $G_E = G_M/\mu_p$ for the proton, and the use of the G's then became universal.

(Exercise 2.6)

Figure 5.3: Left: Comparison of data obtained in [87] with theoretical calculations described in the text. The best fit corresponds to exponential charge distribution with rms radius equal to 0.74 fm. Right: Q^2 dependence of proton formfactors as measured [88]. and plotted in units of inverse square fm, which means that the point 12.5 fm^{-2} corresponds to $Q^2 = 0.5$ GeV^2.

Elastic proton-proton scattering at LHC

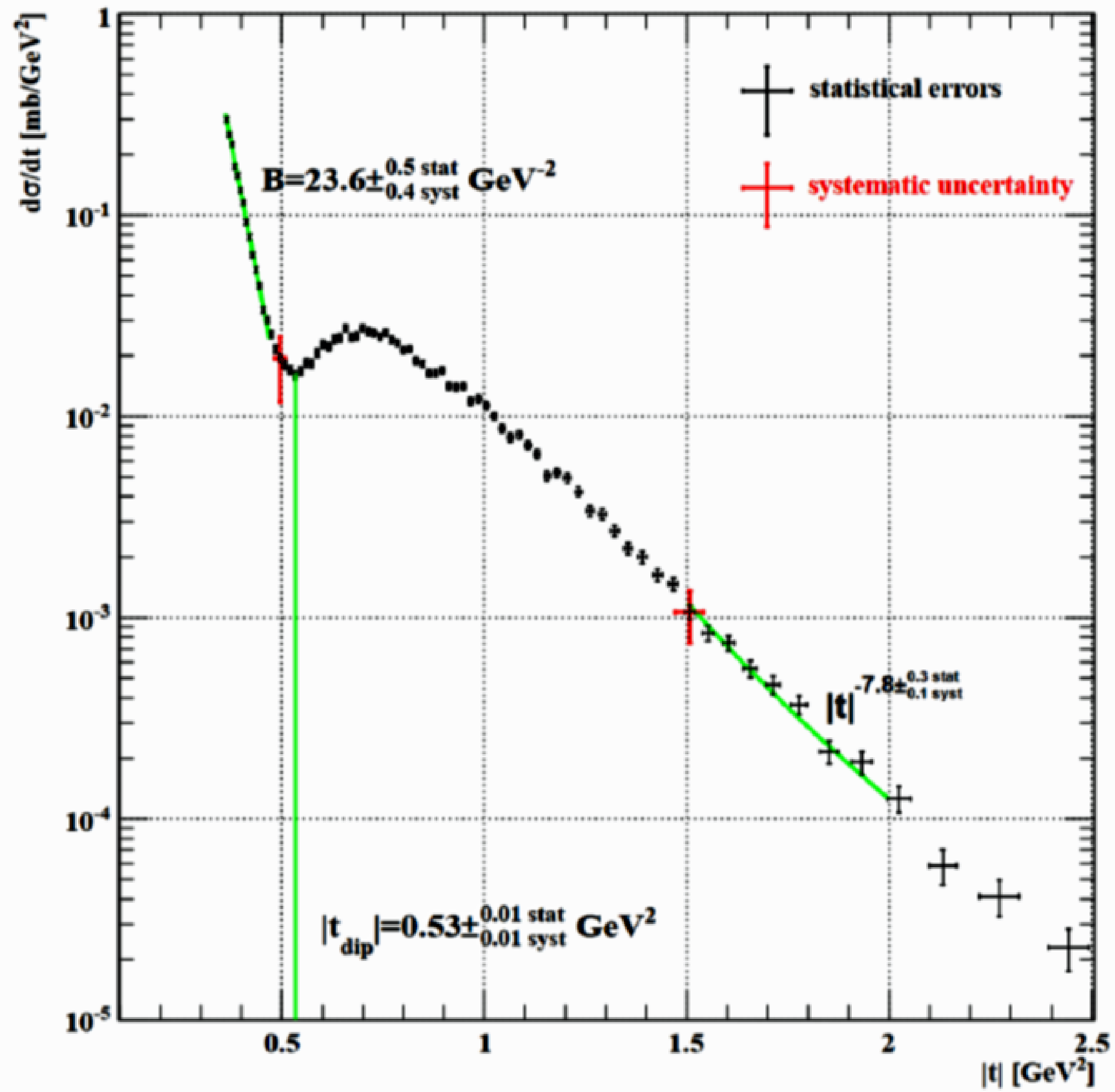


Figure 1.5: Elastic cross section of proton-proton scattering at center of mass energy 7 TeV as measured by experiment TOTEM, from [6]. Mandlestam variable $t = (p - p')^2$, where p and p' are four-momenta of incoming and outgoing protons.

Scattering on target with point-like constituents

- target is made out of N point-like constituents with charges Q_i positioned at x_i
- let's keep for now the charge in the definition of form factor

$$\varrho(\vec{x}) = \sum_i^N Q_i \delta(\vec{x} - \vec{x}_i) \qquad \Rightarrow \qquad F(\vec{q}) = \sum_i^N Q_i \, e^{i\vec{x}_i \cdot \vec{q}}$$

- in cross section we have

$$|F(\vec{q})|^2 = \Big(\sum_i Q_i \, e^{i\vec{x}_i \cdot \vec{q}}\Big) \cdot \Big(\sum_j Q_j \, e^{-i\vec{x}_j \cdot \vec{q}}\Big) = \sum_i Q_i^2 + 2\mathrm{Re} \sum_{i<j} Q_i Q_j e^{i\vec{q}\cdot(\vec{x}_i - \vec{x}_j)}$$

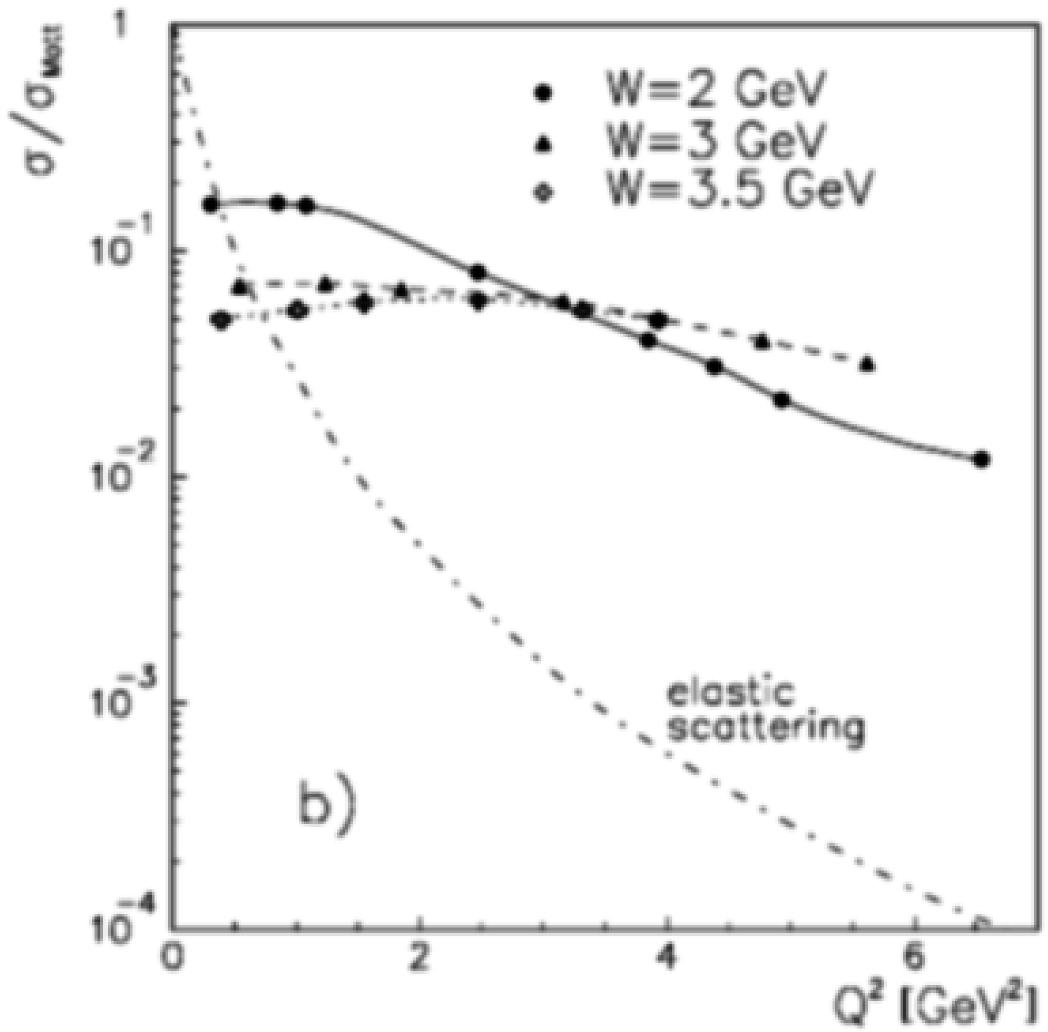
- for high enough momentum transfer, $q\Delta_{\mathrm{min}} \gg 1$, where Δ_{min} is minimal distance between the constituents, the phase becomes erratic

$$|F(\vec{q})|^2 \to \sum_i Q_i^2 \quad \Rightarrow \quad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \to \frac{\mathrm{d}\sigma_{\mathrm{point}}^{Q=1}}{\mathrm{d}\Omega} \sum_i Q_i^2 = \sum_i \frac{\mathrm{d}\sigma_{\mathrm{point}}^{Q_i}}{\mathrm{d}\Omega}$$

- in this limit, the cross section is a **sum of cross sections** on individual constituents
  - example - target is made out of N identical constituents each carrying charge $Q_i = Q/N$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_{\mathrm{point}}^{Q=1}}{\mathrm{d}\Omega} \sum_i \frac{Q^2}{N^2} = \frac{1}{N} \frac{\mathrm{d}\sigma_{\mathrm{point}}^{Q}}{\mathrm{d}\Omega}$$

Parton model



- DIS data from SLAC:
  same Q^2 dependence as for a point-like source (σ_{Mott})
- Feynman 1968: proton behaves in DIS as a bag of quasi-free point-like particles
- already first data were showing that there must be many constituents, not just three quarks like in hadron model

$$\frac{\sigma}{\sigma_{\text{Mott}}} \sim \frac{1}{20}$$

- this was probably the reason why Feynman called them partons and not quarks (quark model of hadrons was already known)

For reading and for exercises

- For reading (J. Chyla textbook)
  - chapters 2.1-2.4
- Exercises: 2.1-2.6