Jets and inclusive hadronic processes in photon induced collisions: a theoretical overview

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Variety of related hard processes

- Jet production
- Inclusive particle production
- Jets + inclusive particles production
- Prompt photon + jet production
- Heavy quark production

just a remark

involving either (quasi)real or virtual photons in γp, γγ or hadron-hadron collisions.
I will restrict myself to the photoproduction-like region and will give an overview of the relevant theory with emphasize on

- current status of relevant QCD calculations
- recent progress in phenomenological applications
- potential problems in describing data.

I will concentrate on discussion of theoretical uncertainties of various QCD calculations because

these uncertainties are often bigger than experimental errors and thus prevent an unambiguous interpretation of the comparison of theory with data.
Basic concepts and notation

Evolution equations for PDF of the photon

\[
\frac{d\Sigma(x, M)}{d \ln M^2} = \delta \Sigma k_q + P_{qq} \otimes \Sigma + P_{qG} \otimes G,
\]

\[
\frac{dG(x, M)}{d \ln M^2} = k_G + P_{Gq} \otimes \Sigma + P_{GG} \otimes G,
\]

\[
\frac{dq_{NS}(x, M)}{d \ln M^2} = \delta_{NS} k_q + P_{NS} \otimes q_{NS},
\]

where

\[
k_q(x, M) = \frac{\alpha}{2\pi} k_q^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right) k_q^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \ldots
\]

\[
k_G(x, M) = \frac{\alpha}{2\pi} k_G^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right) k_G^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \ldots
\]

\[
P_{ij}(x, M) = \frac{\alpha_s(M)}{2\pi} P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \ldots
\]
General solution (in NS channel)

\[ D(x, M) = D_{\text{PL}}(x, M) + D_{\text{HAD}}(x, M) \]

Separation inherently ambiguous

Example of the pointlike part

\[ q_{\text{PL}}^{\text{NS}}(n, M_0, M) = \frac{4\pi}{\alpha_s(M)} \left[ 1 - \left( \frac{\alpha_s(M)}{\alpha_s(M_0)} \right)^{1-2P_{qq}^{(0)}(n)/\beta_0} \right] a_{\text{NS}}(n) \]

Which results from the resummation of the diagrams
the fact that for $M \to \infty$

$$q_{NS}^{PL}(x, M_0, M) \to \frac{4\pi}{\alpha_s(M)} a_{NS}(x) \equiv q_{NS}^{AP}(x, M) \propto \ln \frac{M^2}{\Lambda^2}$$

does not imply $q_{NS}^{PL} \propto \alpha / \alpha_s$

because when $\Lambda_{QCD} \to 0$

$$q_{NS}^{PL}(x, M, M_0) \to \frac{\alpha}{2\pi} k_{NS}^{(0)}(x) \ln \frac{M^2}{M_0^2}$$
Semantics (sometimes) matters

What do we mean under “LO” or “NLO”?

Recall:

\[ R_{e^+e^-}(Q) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \left(3 \sum_{i=1}^{n_f} e_i^2\right) (1 + r(Q)) \]

\[ r(Q) = \frac{\alpha_s(M)}{\pi} \left[ 1 + \frac{\alpha_s(M)}{2\pi} r_1(Q/M) + \cdots \right] \]

Always subtract pure QED contributions and keep the terms LO and NLO for QCD effects only!

Unfortunately, in some photon induced hard processes this convention is not observed, leading to unnecessary confusion.
Phenomenological input

colour coupling $\alpha_s(\mu, RS)$

renormalization scale

PDF of the proton
GRV94, GRV92, SaS96, GRS99, CJKL03, AFG05

PDF of the photon ($\gamma_T$)
AGF94, GRV92, SaS96, GRS99, CJKL03, AFG05

FF
BKK96, KKP2000, BFGW2001, Kretzer01

$D_p(M_p, FS_p)$

$D_\gamma(M_\gamma, FS_\gamma)$

$D_p^h(M_h, FS_h)$
Scales and schemes appear due to ambiguities in the treatment of singularities at

**short distances:** \[ \mu \] and renormalization scheme

**long distances:** \[ M_p, M_\gamma, M_F \] and factorization schemes

Freedom in the choice of renormalization and factorization schemes **almost unexploited** and phenomenological applications calculations done **mostly** in MS$_{\text{bar}}$ RS and MS FS.

Dependence on scales has a **clear intuitive interpretation**, but their choice is **not sufficient** to define unambiguously perturbative QCD calculations.

**Common practice:** \[ \mu = M_p = M_\gamma = M_F \] = “natural scale”

But there is no good reason for such assumption (Politzer 87)
Factorization: XS of physical processes can be written as convolutions of PDF, FF and partonic hard cross sections

\[
\sum_{a,b,c} D_{a/A}(M_a) \otimes D_{b/B}(M_b) \otimes \sigma^\text{hard}_{ab \rightarrow c+-+-} (M_a, M_b) \otimes D_c^h (M_h)
\]

The renormalization scale enters only when partonic hard scattering cross section is expanded in PQCD

\[
\sigma^\text{hard}_{ab \rightarrow c+-+-} = \alpha_s^k(\mu)\sigma^{LO} + \alpha_s^{k+1}(\mu)\sigma^{NLO}(M_a, M_b, \mu)
\]

- **k=0**: DIS, Drell-Yan, jets in direct \( \gamma\gamma \)
- **k=1**: jets in direct ep
- **k=2**: jets in hadron-hadron

In defining perturbative calculation the order of all PDF, FF and hard scattering cross sections must be specified.

Note: NLO in \( \sigma^\text{hard}_{a+b \rightarrow c} \) requires \( k \geq 1 \).
The **same** choice of the renormalization scale gives different results in different RS! In fact the schemes are as important as scales, but there no “natural” RS or FS!

The conventional procedure which implicitly assumes working in MS\textsubscript{bar} is thus based on entirely **ad hoc** choice of RS.

I think the choice of scales and schemes should be done more in a **more sophisticated and educated** way.

This means keeping \( \mu \) and \( M_p, M_\gamma, M_F \) independent and investigating the stability of perturbative results on these free parameters looking for regions of local stability - Principle of Minimal Sensitivity (PMS)

fastest apparent convergence - Effective Charges (EC)

**Such investigation makes sense** even if one does not subscribe to PMS or EC!!
Theoretical “uncertainty” of perturbative calculations conventionally comprises several components:

- dependence on PDF and FF
- hadronization effects
- scale and scheme variation

Conventional way of estimating theoretical uncertainty due to scale choice, i.e. identifying all scales with some “natural scale” $Q$ and plotting the band of results corresponding to $Q/2 \leq \mu \leq 2Q$ makes little sense because it depends on selected scheme and is actually misleading.
Take the LO where

\[
\frac{\alpha_s(\mu/2) - \alpha_s(2\mu)}{\alpha_s(\mu)} \equiv \beta_0 \alpha_s(\mu) \ln 4 \to 0 \text{ as } \mu \to \infty
\]

so the LO QCD predictions would appear to have very small “uncertainty” at short distances, which is nonsense. Because of the way the “uncertainty” due to scale/scheme choice is conventionally defined it should not be mixed with the other sources of genuine theoretical uncertainty.

We should make a choice of scales and schemes, based on some general idea, and look whether it leads to meaningful phenomenology for wide range of processes.

To make a judicious choice we should spend computer time on investigation of scale dependence as mentioned above.
Ambiguities do not decrease at higher orders

The usual claim that at higher orders the ambiguities of finite order perturbative approximations decrease is not true, because at each order of perturbation theory additional new free parameters, like higher order $\beta$-function coefficients and splitting functions do appear. We can ignore them and continue to work in the conventional (renormalization and factorization) schemes, which reflect our preference for dimensional regularization, but this does not make the mentioned claim true.

Note, that technically, it makes no sense to compare functions of different number of variables.
Hadronization corrections and related effects

What we want to correct?
Standard way of estimating hadronization corrections to jet observables, i.e.

jet algo on hadrons ($\eta, p_T$)
jet algo on partons ($\eta, p_T$)

has **little relevance** for correcting NLO parton level calculations.

For that purpose **all effects**, absent in hard partonic calculations, **should be included.**
PYTHIA and PHOJET use the same QED XS, which dominates at large $E_T$, so what makes such a dramatic difference?
Some of the features of full NLO QCD effects are mimicked within MC event generators, which use only LO partonic cross sections, by means of parton showers and noncollinear kinematics of initial state partons. But LO MC also use different input (LO PDF and $\alpha_s$), which were extracted in global analysis from data, and so have a chance to describe also other data as well. To identify genuine NLO effects is thus not simple.
Jet production in $\gamma p$ collisions

Complete framework for quasireal photons laid out in
Frixione, Ridolfi (97)
Klasen, Kramer, Kleinwort (98)
Aurenche, Bourhis, Fontannaz, Guillet (00)

and recently reviewed in Klasen: RMP 74 (2002), 1221

Differences in regularization method: slicing vs subtraction

Suitable cuts on dijets developed

- Single jets
- Dijets
- Multijets
- Jets in diffraction

Closely related to dynamics of jet + prompt photon
Extensive application to HERA data lead in general to reasonable agreement, except for some IR sensitive quantities but suffers from non-negligible scale dependence of many QCD calculations.

Concept of the resolved photon indispensable.

For full consistency to order also direct photon contribution to that order needed!

Structure of calculations

Direct contribution: \( \alpha \alpha_s + \alpha \alpha_s^2 \)

Resolved contribution: \( \alpha \alpha_s^2 + \alpha \alpha_s^3 \)

Not necessary to order \( \alpha \alpha_s^2 \)

For full consistency to order also direct photon contribution to that order needed!

Extensive application to HERA data lead in general

- to reasonable agreement, except for some IR sensitive quantities
- but suffers from non-negligible scale dependence of many QCD calculations.
- Concept of the resolved photon indispensable.
Klasen, Kramer, Kleinwort (99): single jets for $E_T = 20$ GeV, $\eta = 1$
All scales identified and set equal to M.

What do these plots indicate? (In)stability?
LO MC clearly better than NLO parton level calculations!
Jet production in $\gamma^* p$ collisions

Many people contributed to theory and phenomenology of hard collisions of virtual photons
Kramer, Klasen, Pötter, GRV, Sjöstrand, Friberg, J.Ch., Taševský, Aurenche, Fontannaz, Guillet, Heinrich, De Florian, Sassot, ….sorry for any omission

Of particular interest the region of transition between photo-production (where $E_T$ sets the scales) and genuine DIS, where $Q^2$ is unquestionably the hard scale.

In this region the concept in resolved (virtual) photon does not have to be introduced, but it turns out to be very useful phenomenologically. But not everybody agrees.

But, in this region playing with scales does wonders and fakes (almost) everything else.
NLO parton level Monte Carlos: (black: no longer attended)

MEPJET (Mirkes, Zeppenfeld)
DISENT (Catani, Seymour)
DISASTER (Graudenz)
JETVIP (Pötter), may be resuscited by Klasen
NLOJET (Nagy, Troczsanyi)
plus possibly other.

In principle JETVIP most general, but
- has problem with stability of NLO RES
- disagrees with other MC in NLO DIR

MC event generators: resolved virtual photons available in

PYTHIA: $\gamma_T$ as well as $\gamma_L$
HERWIG: $\gamma_T$ and in H1 version also $\gamma_L$
RAPGAP: $\gamma_T$ only

Only JETVIP includes resolved $\gamma^*$
The only NLO generator for genuine 3-jet region
Processes:

- Single jets
- Dijets
- Multijets
- Forward jets

Of great theoretical interest because of potential implications for identifying BFKL dynamics.

The region of interest: \( Q^2 \leq E_T^2 \) or even better \( Q^2 \ll E_T^2 \)

where:

- the concept of resolved virtual photon is relevant
- jet \( E_T \) provides the only natural scale and
- photon virtuality \( Q^2 \) enters only through PDF of \( \gamma_{T,L} \)

However, in most analyses of HERA data only the weaker condition \( 1/\kappa \leq Q^2 / E_T^2 \leq \kappa \) typically with \( \kappa = 2 \) demanded.
JETVIP direct unsubtracted for asymmetric $E_T$ cuts:

$$E_T^{(1)} \geq 7, \ E_T^{(2)} \geq 5 \text{ GeV}$$

Resolved in direct

NLO direct is well approximated by

LO DIR+ LO RES

with QED PDF of the virtual photon

provided also the contribution of $\gamma_L$ is included.
Relevance of resolved $\gamma_L$

HERWIG MC simulations:

$1.4 < P^2 < 2.4 \text{ GeV}^2$, $0.05 < y < 0.95$, $E_T > 5 \text{ GeV}$, $-5 < \eta < 5$

Similar calculations with PYTHIA.
PDF of virtual $\gamma_T$ and $\gamma_L$

$\gamma_T$: GRV, GRS, SaS, AFG  $\quad\gamma_L$: J.Ch.

Virtuallity dependence of PDF very different for the point-like and hadron-like components

In most of practical circumstances modest QCD evolution

Kramer, Pötter

Crucial improvement over DIRECT unresolved: the inclusion of NLO resolved contribution
Resolved virtual photon as approximate method for including higher order direct photon terms.

NLOJET in 3-jet mode allows us to investigate this idea quantitatively.

Graphs contributing to the definition of the pointlike part of quark and gluon distribution functions of the photon
Example:

**QCD analysis of dijet production at low $Q^2$ at HERA**

(J. Ch., J. Cvach, K. Sedlák, M. Taševský 05)

Note the disagreement between JETVIP DIR a DISENT/NLOJET

Still higher order direct terms, **effectively included in resolved $\gamma^*$, will hopefully fill the remaining gap to data.**
Note: the data are described very well by HERWIG with both $\gamma_T$ and $\gamma_L$!
Forward jets at HERA

Mueller 92: best place to look for BFKL effects.

But difficult to distinguish from effects of resolved virtual photon!
Forward jet production at small $x$ in NLO QCD
Kramer, Pötter 99

DIR alone below data

Inclusion of NLO RES leads to nice agreement!

However: scale choice crucial!

Figure 1. Dijet cross section in the forward region compared to HERA data: (a) and (b) ZEUS; (c) and (d) H1. (a) NLO DIS, $E_T > 5$ GeV; (b) NLO DIRS + RES, $E_T > 5$ GeV; (c) NLO DIS, $E_T > 3.5$ GeV; (d) NLO DIRS + RES, $E_T > 3.5$ GeV.
But are NLO resolved terms really important?

Jung, Jönsson, Küster 99:

NLO calculations including contributions from direct as well as resolved photons, are similar to using LO matrix elements for both the photon and the proton with the addition of parton showers, as implemented in the RAPGAP generator. This indicates that

higher order contributions are well simulated by the inclusion of parton showers in LO Monte Carlo generators.
Inclusive particle production in $\gamma^* p$ collisions

Aurenche, Fontannaz, Basu, Godbole, Daleo, De Florian, Sassot, Kramer, Kniehl, Maniatis, Gordon,....

Recently most attention to forward pions to search for BFKL

Situation even more ambiguous and puzzling than for jets:

| BFKL | or merely direct $\gamma$ | or resolved $\gamma$ needed? |

Systematic investigation of scale dependence crucial for proper choice of four scales! (Aurenche et al.)

Conventional choice

$$\mu^2 = M_p^2 = M_\gamma^2 = M_h^2 = \kappa^2 (Q^2 + E_T^2)$$

even less justified than for jets!

It results in resolved contribution even for $Q^2 \gg E_T^2$ !!
Daleo, de Florian, Sassot 04: only direct photon contribution

KKP as well as Kretzer FF, setting \( M^2 = \frac{Q^2 + E_T^2}{2} \)

Sizable dependence on both FF and common scale!
The central (solid) line corresponds to setting the factorization and renormalization scales to $(Q^2 + p_T^2)/2$ and the upper and lower (dashed) ones to $(Q^2 + p_T^2)/4$ and $Q^2 + p_T^2$, respectively.
Their conclusion:

In spite of the reasonable agreement between data and the NLO DGLAP estimate, the rather large K-factors and the significant factorization scale dependence, ..., suggest the presence of non-negligible NNLO effects....

These rather large uncertainties in their (and my) view hide any potential disagreement between the data and the DGLAP prediction, restraining, for the moment, any empirical suggestion of dynamics different to plain DGLAP evolution.

Note: basically the same perturbative partonic cross sections as for jets, so the explanation should be applicable to both.
Kniehl, Kramer and Maniatis 05

Same scale choice and FF as in previous analysis

$\pi^0$ and charged hadrons spectra compared to H1(99) data.

Again a factor of 2 difference

$M^2 = \xi \left( Q^2 + E_T^2 \right) / 2$

Agreement for

$\xi = 1$

range

$1/2 \leq \xi \leq 2$
Compared to Daleo et al., they are more optimistic in the interpretation of the H1 data:

**NLO correction is found to produce a sizeable enhancement in cross section, of up to one order of magnitude, bringing the theoretical prediction to good agreement with recent measurements for neutral pions and charged hadrons at HERA.**

But they obviously put (too?) much faith in their choice of scales:

**Our default predictions, endowed with theoretical uncertainties estimated by moderate unphysical scale variations led to a satisfactory description of the HERA data in the preponderant part of the accessed phase space.**
The most recent analysis of the same H1 data on forward $\pi^0$ done by Aurenche, Basu, Fontannaz and Godbole taking into account the resolved virtual photon as well.

Includes up to now the most detailed analysis of the separate dependence on different scales finding very different dependence on the renormalization factorization, fragmentation and factorization scales.

Default calculations done for the common scale set to

$$M^2 = (Q^2 + E_T^2)$$

Compared to Daleo et al. and Kniehl et al. the common scale is twice bigger and thus the NLO direct substantially smaller!

Consequently, also the conclusions are very different.
very different dependences
Variations under the proton and photon factorization scales are under control. However a large instability is observed when varying independently the renormalization and fragmentation scales. This prevents a really quantitative prediction for the single pion inclusive distribution in the forward region.
Inclusive particle production in $\gamma\gamma$ collisions

Problems with inclusive particle production are not new recall

Gordon 94:

FIG. 16. Comparison of the LO and NLO predictions using the GRV photon distributions and the new charged hadron fragmentation functions of Ref. [40] with the Mark II data.
FIG. 1. Cross section $d\sigma/dp_T^2$ of inclusive charged-hadron production in single-tagged $\gamma\gamma$ collisions. The TASSO data [15] ($\sqrt{s}=33.1$ GeV on average) are compared with our corresponding NLO calculations for scale choices $\xi=1/2$, 1, and 2.

FIG. 2. Cross section $d\sigma/dp_T$ of inclusive charged-hadron production in single-tagged $\gamma\gamma$ collisions. The MARK II data [16] ($\sqrt{s}=29$ GeV) are compared with our corresponding NLO calculations for scale choices $\xi=1/2$, 1, and 2.
FIG. 4. NLO cross section $d\sigma/dp_T$ of inclusive charged-hadron production in double-tagged $\gamma\gamma$ collisions at LEP2 ($\sqrt{s} = 175$ GeV). The DD, DR, and RR components are also shown.

FIG. 5. Same as in Fig. 4 for the $y$ dependence of $d^2\sigma/dy\,dp_T$ at $p_T = 10$ GeV.
Fig. 4. (a) Inclusive $\pi^\pm$ differential cross section $d\sigma/dp_t$ compared to NLO QCD calculations [19] for $W_{\gamma\gamma} > 5$ GeV. The dashed-dotted line corresponds to the direct subprocess. The dashed lines represent the scale uncertainty of the calculations. (b) Inclusive $\pi^\pm$ differential cross section $d\sigma/dp_t$ with different $W_{\gamma\gamma}$ cuts. The average $p_t$ value of each bin, $\langle p_t \rangle$, is used.
Difficult to understand as for large $p_T$ hadrons the pure QED contribution should overwhelmingly dominate!

But maybe, we just do not understand hadronization?

Must have the same reason as excess in jet production observed by L3 as well.

LP05 Uppsala: Trivial explanation by DELPHI?
Do we really know QCD this well?

- 2003: L3 $\gamma\gamma$ results show weird discrepancy at high transverse momentum
- Similar effect in both charged hadrons (shown) and in jets
- Scale dependence (theory error) certainly doesn't account for it!
- If true, a real surprise & a big problem for QCD
Single Particle Inclusive Cross Section in $\gamma\gamma$ events

- New DELPHI result (this conference)
- Shows good agreement with QCD.
Single Particle Inclusive Cross Section in $\gamma\gamma$ events

- Use L3 cuts, let in annihilation background.

Jon: Seems to be resolved. QCD wins again!

Really?
Puzzling $\gamma\gamma$ collisions: and now also jets

![Graphs showing data and predictions for $d\sigma/dp_t$ in $\gamma\gamma$ collisions with different models.]
L. Bertora at
PHOTON03, DIS04

Data from Di-Jet Production in photon-photon collisions at sqrt(s) from 189 to 209 GeV,
OPAL coll., hep-ex/0301013

Leonardo Bertora, 2003 April 8
Results: Theo vs. L3

Good shape, but not really good normalization

Wrong shape and normalization:
Preliminary data seem to confirm L3-hadron production discrepancy with NLO prediction

L. Bertora at
PHOTON03, DIS04

Leonardo Bertora, 2003 April 8
Remarks on theory:

At large $p_T$:  
- **direct $\gamma\gamma$ contribution dominant** (as for hadrons)
- QED part dominates direct $\gamma\gamma$ contribution
- QCD contribution does not represent full NLO but cannot be expected to close the gap
- threshold resummation may be important
- Hadronization effects should be looked into

Direct contribution: \[ \alpha^2 + \alpha^2\alpha_s + \alpha^2\alpha_s^2 \]
Single resolved: \[ \alpha^2\alpha_s + \alpha^2\alpha_s^2 + \alpha^2\alpha_s^3 \]
Double resolved: \[ \alpha^2\alpha_s^2 \]

Needed for full consistency to order \[ \alpha^2\alpha_s^2 \]
Moreover:

the same problem in $b\bar{b}$ production in $\gamma\gamma$ collisions!

New L3 analysis confirms their older result which is in excellent agreement with those of OPAL and DELPHI.

So, I think we still have a problem.
Prompt photon + jet production

Bawa, Stirling, Gordon, Wogelsang, Krawczyk, Zembruski, Fontannaz, Heinrich,...

Two approaches differing by counting of orders of $\alpha_s$

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<th>Approach</th>
<th>A: Direct contribution</th>
<th>$\alpha^2$</th>
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Neither included

Direct contribution: $\alpha^2 + \alpha^2 \alpha_s + \alpha^2 \alpha_s^2 + \alpha^2 \alpha_s^3$

Single resolved: $\alpha^2 \alpha_s + \alpha^2 \alpha_s^2 + \alpha^2 \alpha_s^3$

Double resolved: $\alpha^2 \alpha_s^2 + \alpha^2 \alpha_s^3$

Direct contribution: $\alpha^2 + \alpha^2 \alpha_s + \alpha^2 \alpha_s^2$ (box)

Single Resolved: $\alpha^2 \alpha_s$

Double Resolved: $\alpha^2 \alpha_s^2$
Conclusions

In the interesting transition region between photoproduction and DIS the scale and other uncertainties prevent us from calculating with \textit{(in)accuracy} better than a factor of 2, potentially masking signals of “new physics” like BFKL.

Calls for \textbf{NNLO calculations} should be accompanied with systematic investigation of scale and scheme ambiguities of existing LO/NLO calculations.

We should also \textbf{understand} better the relation between LO \textbf{MC event generators} and NLO partonic calculations. Way forward: \textbf{MC&NLO}.

In theoretically seemingly clean case of \textit{jet}, inclusive hadrons and heavy quark production in photon-photon collisions we face serious disagreement with (some?) data which we do not understand.