Too much beauty all around

Jiří Chýla and Jiří Srbek
Center for Particle Physics, Institute of Physics, Prague

Contents

• Data on $b\bar{b}$ production in $\bar{p}p$, $\gamma p$ and $\gamma\gamma$ collisions
• It there a common explanation?
• Choosing the scales
• General structure of $\sigma(\bar{p}p \to Q\bar{Q})$
• Numerical results
• Conclusions

Details in JHEP03(2003)042 and hep-ph/0111469
Clear excess over PQCD at $Q^2 = 0$ but data not quite consistent at high $Q^2$
\( \bar{b}b \) production in \( \gamma\gamma \) collisions

Comparison with L3 and OPAL

- New DELPHI data suggest striking agreement between the three LEP experiments
- and dramatic disagreement of their data with PQCD
- despite the fact that this process is expected to be the cleanest test of PQCD
- my view: current calculations not truly NLO QCD
- but no relation to low x physics
Clear excess of both data over PQCD that comes from the transition region to low $x$ as $\langle x \rangle \approx 0.01$. 
(In)consistency of $\bar{p}p$ results at TEVATRON and SPSC?

Ratio of $\sigma(b)$ at 630 GeV to 1800 GeV

<table>
<thead>
<tr>
<th>0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ratio of Cross Sections

CDF

UA1

NLO QCD

$\sigma_\ell(p_T > p_{T,\text{min}}/2, |y|<1.5)$ (nb)
No problems at low energy?

Message:
- Good agreement with PQCD at low energies!
- but large experimental errors
- and theoretical uncertainties to draw strong conclusions and
- and not low $x$ physics.
New physics?

low $x$ (BFKL/CCFM) or Supersymmetry?

$p\bar{p} \rightarrow bX$, $\sqrt{s}=1.8$ TeV, $|y^b|<1$

DØ Data
(Errors have correlations)

- Dimuons
- Muons+Jets
  (This Analysis)
- Inclusive Muons

$\sigma^b(p_T>\min p_T) (\text{nb})$

$\sqrt{s} = 1.8$ TeV

- $m_g = 14$ GeV
- $m_b = 3.5$ GeV
- $m_b = 4.75$ GeV
Or subtleties of conventional calculations?

There are several aspects of QCD calculations that must be taken properly into account in the comparison to data as they may significantly enhance the conventional results:

- correctly extracted \textbf{b-quark fragmentation functions}
- threshold effects
- small $x$ effects
- resummation of large logs of the type $\ln(p_T/m_b)$
- choice of \textbf{renormalization} and \textbf{factorization} scales
Effect of proper parameterization of $D_b^D(z)$ (Cacciari, Nason)

\[ i f \frac{d\hat{\sigma}}{d\hat{\rho}_T} = A\hat{\rho}_T^{-n} \Rightarrow \frac{d\sigma}{dp_T} = \frac{A}{\hat{\rho}_T^n} D(n), \quad D(N) \equiv \int dz D_b^B(z) \]
Among them those damned scales

General form of perturbative expansion involving \(a(s) \equiv \alpha_s(\mu)/\pi\) in a given RS:

\[
r(Q) = a_s^k(\mu, \text{RS}) \left( r_0(Q) + r_1(Q/\mu, \text{RS})a_s(\mu, \text{RS}) + r_2(Q/\mu, \text{RS})a_s^2(\mu, \text{RS}) \cdots \right)
\]

\[
\frac{\beta_0}{4\pi} \ln \left( \frac{\mu^2}{\Lambda_{\text{RS}}^2} \right) = \frac{1}{\alpha_s(\mu)} + c \ln \frac{c\alpha_s(\mu)}{1 + c\alpha_s(\mu)},
\]

Example:

\[
R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu)} = 3 \left( 1 + r_\tau \right)
\]

\[
r_1(Q/\mu, \text{RS}) = kb \ln \frac{\mu}{\Lambda_{\text{RS}}} - \rho(Q),
\]

\[
\rho(Q) \equiv kb \ln (Q/\Lambda_{\text{RS}}) - r_1(1, \text{RS})
\]

is a renormalization scale and scheme invariant.

which RS scale to choose? Only two points truly exceptional:
maximum which defines the Principle of Minimal Sensitivity and
intersection LO=NLO=NNLO defining the Effective Charges approach
A common origin of the discrepancies? Not quite.

\( \bar{p}p: \text{complete NLO} \)
- \( \langle x_1x_2 \rangle \doteq 6.5 \times 10^{-2} \) for \( \sqrt{S} = 50 \text{ GeV} \)
- \( \langle x_1x_2 \rangle \doteq 8 \times 10^{-4} \) for \( \sqrt{S} = 600 \text{ GeV} \)
- \( \langle x_1x_2 \rangle \doteq 1.3 \times 10^{-4} \) for \( \sqrt{S} = 1.8 \text{ TeV} \Rightarrow \text{low } x? \)
- \( \langle x_1x_2 \rangle \doteq 6.5 \times 10^{-6} \) for \( \sqrt{S} = 14 \text{ TeV} \Rightarrow \text{low } x! \)

\( \gamma p: \text{incomplete NLO, } \langle x \rangle \doteq 0.03 \text{ at HERA} \)

\( \gamma\gamma: \text{incomplete NLO, } \langle x_1x_2 \rangle \gtrsim 0.01 \text{ at LEP} \)

In all cases the renormalization and factorization scales play different role and should therefore be kept as independent parameters of the QCD calculations.
General form of $\sigma_{\text{tot}}(Q\overline{Q})$

$$\sigma_{\text{tot}}(pp \rightarrow Q\overline{Q}, S) = \int \int dx dy \sum_{ij} D^p_i(x, M) D^p_j(y, M) \sigma_{ij}(s = xyS, M)$$

$$\sigma_{ij}(s, M) = \alpha_s^2(\mu) \sigma^{(2)}_{ij}(s) + \alpha_s^3(\mu) \sigma^{(3)}_{ij}(s, M, \mu) + \cdots,$$

at the NLO

$$\sigma_{\text{tot}}^{\text{NLO}}(M, \mu) = \alpha_s^2(\mu) \left\{ \int \int dx dy \sum_{i=1}^{2n_f} q_i(x, M) q_i(y, M) \left[ \sigma^{(2)}_{qq}(xy) + \alpha_s(\mu) \sigma^{(3)}_{qq}(xy, M, \mu) \right] + \right.$$

$$2 \int \int dx dy \Sigma(x, M) G(y, M) \alpha_s(\mu) \sigma^{(3)}_{GG}(xy, M) +$$

$$\left. \int \int dx dy G(x, M) G(y, M) \left[ \sigma^{(2)}_{GG}(xy) + \alpha_s(\mu) \sigma^{(3)}_{GG}(xy, M, \mu) \right] \right}\}$$

**Crucial point:** keep the factorization and renormalization scales **independent**!

Similar expression for differential cross sections as well.
Factorization scale dependence of the NLO approximation:

\[
\frac{d\sigma_{tot}^{NLO}(M,\mu)}{d\ln M^2} = \int \int dx dy G(x,M)G(y,M)W_{GG}(xy,M,\mu) + \\
\int \int dx dy \left[ \sum_{i=1}^{2n_f} q_i(x,M)q_i(y,M)W_{qq}(xy,M,\mu) + \Sigma(x,M)G(y,M)W_{qG}(xy,M,\mu) \right].
\]  

Denoting \( \hat{f} \equiv df/d\ln M^2 \), the functions \( W_{ij} \) are given as

\[
W_{GG}(x,M,\mu) = \frac{\alpha_s^3(\mu)}{\pi} \left\{ 2\pi \hat{\sigma}_{GG}^{(3)}(x) + \int dz P_{GG}^{(0)}(z) \sigma_{GG}^{(2)}(xz) \right\} + \cdots 
\]

\[
W_{q\bar{q}}(x,M,\mu) = \frac{\alpha_s^3(\mu)}{\pi} \left\{ 2\pi \hat{\sigma}_{q\bar{q}}^{(3)}(x) + 2\int dz P_{qq}^{(0)}(z) \sigma_{q\bar{q}}^{(2)}(xz) \right\} + \cdots 
\]

\[
W_{qG}(x,M,\mu) = \frac{\alpha_s^3(\mu)}{\pi} \left\{ 2\pi \hat{\sigma}_{qG}^{(3)}(x) + \int dz \left[ P_{qG}^{(0)}(z) \sigma_{q\bar{q}}^{(2)}(xz) + P_{Gq}^{(0)}(z) \sigma_{GG}^{(2)}(xz) \right] \right\} + \cdots
\]

Theoretical consistency requires that the expressions standing in the above expressions by \( \alpha_s^3 \) vanish which, indeed, they do.
What is wrong with the conventional assumption $M = \mu$?

Fakes the stability where there is none
Leads away from genuine stability region
Numerical results at the NLO

\[ \bar{b}b, \sqrt{S} = 1800 \text{ GeV} \]
\[ p_T = 1 \text{ GeV} \]

\[ r^{(N)} \equiv \sum_{j=k}^{N} r_{j-k} a_s^j \]

\[ \frac{d r^{(N)}}{d \ln M} = \frac{d r^{(N)}}{d \ln \mu} = O(a_s^{N+1}) \]

Saddle point defines the most stable prediction
\[ \sigma_{\text{tot}}^{\text{NLO}}(M, \mu) \text{ at } \sqrt{S} = 62 \text{ GeV} \]

**Quark dominated processes**

**Saddle close to the diagonal** \( M = \mu \)
\[ \sigma_{\text{tot}}^{\text{NLO}}(M, \mu) \text{ at } \sqrt{S} = 630 \text{ GeV} \]
Non-diagonal energy dependence of the saddle
Different energy dependence of $\sigma_{NLO}^{\text{tot}}(saddle)$
particularly in the TEVATRON energy range
The effect decreases with increasing $m_b$
$p_T$ dependence at $\sqrt{S} = 630$ GeV
$p_T$ dependence at $\sqrt{S} = 1800$ GeV
Top is safe at the TEVATRON

$\sqrt{S}=1800$ GeV
$\Lambda_q=200$ MeV, $m=175$ GeV
GRV 94
as well as LHC
Conclusions

• The proper choice of scales is crucial for application of PQCD

• Renormalization and factorization scales should not be identified

• PMS optimized results that are
  – significantly above the conventional ones in the TEVATRON energy range
  – but close to them at SPSC and LHC energies

• Predictions for the top quark are safe

• Not the sole explanation of the discrepancy

• Analyses of $\gamma p$ and $\gamma\gamma$ will follow