Towards understanding $b\bar{b}$ production in $\gamma\gamma$ collisions

Jiří Chýla
Center for Particle Physics, Institute of Physics, Prague

Contents

- **$b\bar{b}$ production in $\gamma\gamma$ collisions: challenge for PQCD?**
- **Basic facts and formulae**
- **General form of $\sigma_{tot}(\gamma\gamma \to Q\overline{Q})$**
- **$b\bar{b}$ production at LEP2**
- **Can the NLO QCD corrections solve the puzzle?**
- **Conclusions**

Details in **hep-ph/0311074**
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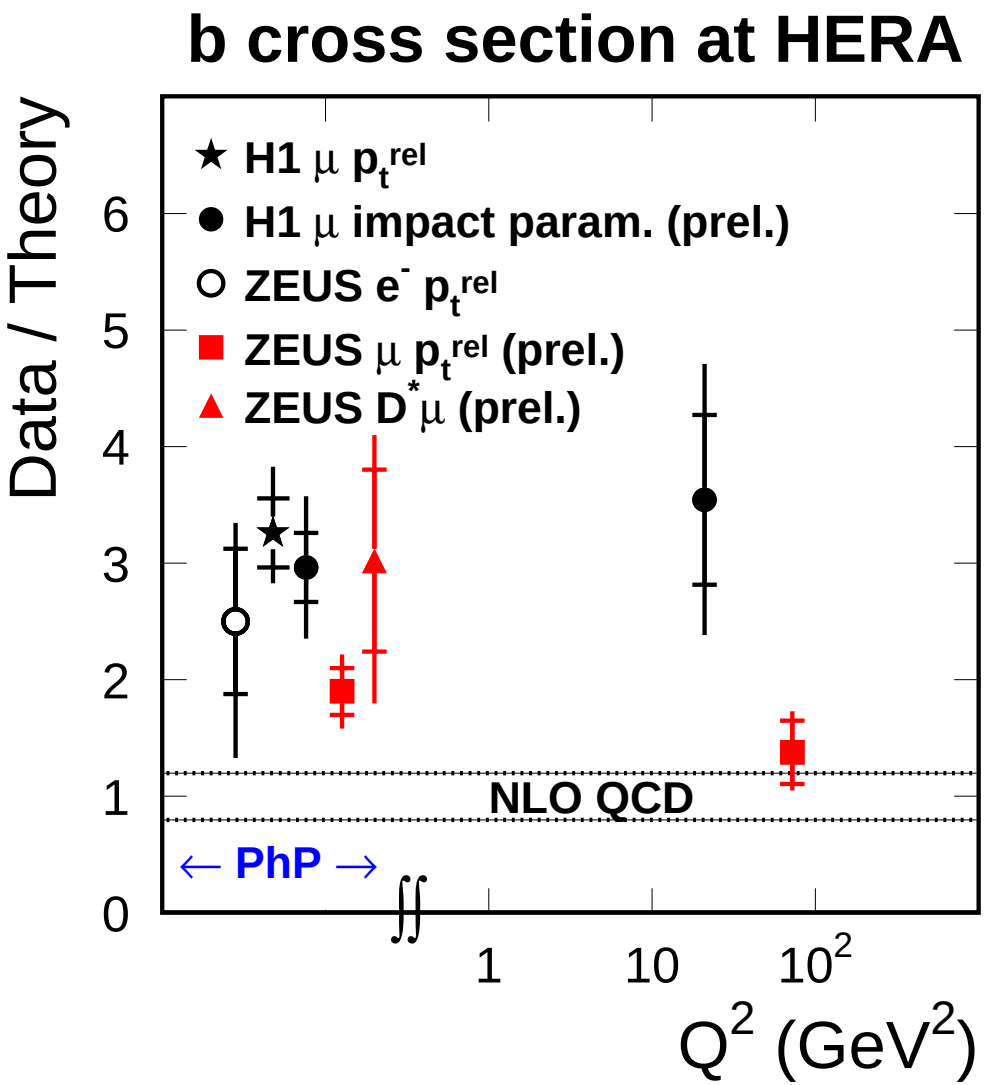
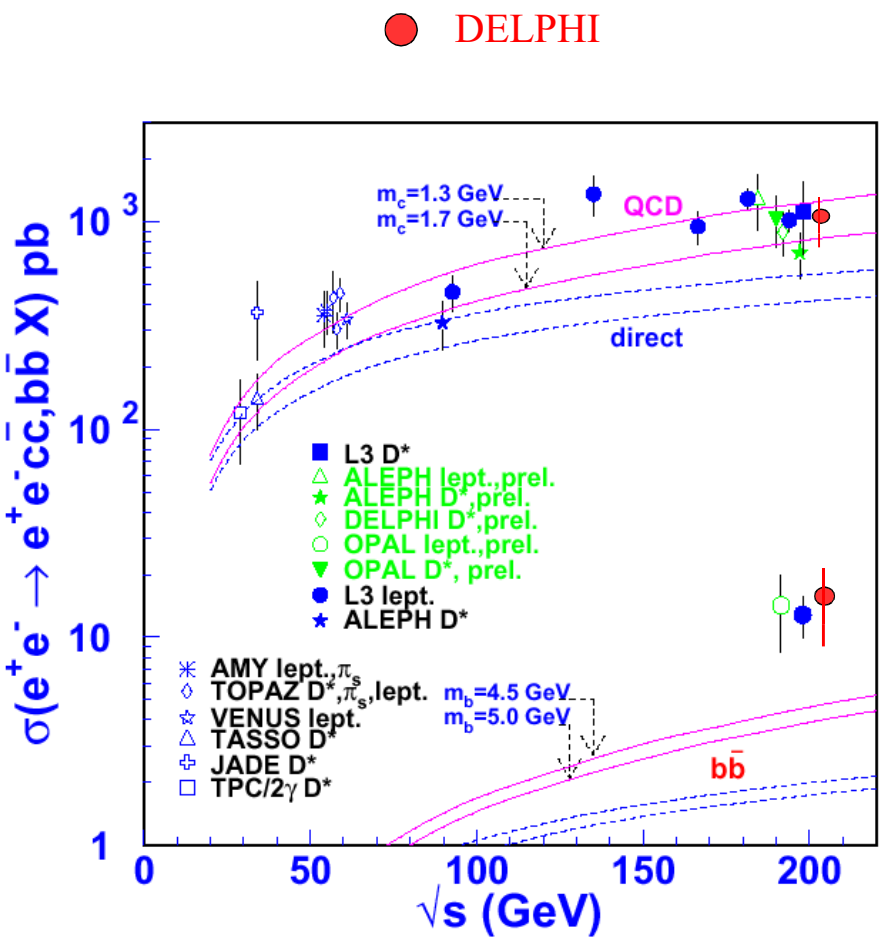
$\bar{b}b$ production in ep and $\gamma\gamma$ collisions



Comparison with L3 and OPAL



Basic facts and formulae

$\alpha_s(\mu)$ depends on the renormalization scale μ in a way governed by the equation

$$\frac{\mathrm{d}\alpha_s(\mu)}{\mathrm{d}\ln\mu^2} \equiv \beta(\alpha_s(\mu)) = -\frac{\beta_0}{4\pi}\alpha_s^2(\mu) - \frac{\beta_1}{16\pi^2}\alpha_s^3(\mu) + \cdots,$$

Its solutions depend also on the renormalization scheme (RS). At the NLO

$$\frac{\beta_0}{4\pi}\ln\left(\frac{\mu^2}{\Lambda_{\mathrm{RS}}^2}\right) = \frac{1}{\alpha_s(\mu)} + c\ln\frac{c\alpha_s(\mu)}{1+c\alpha_s(\mu)}, \quad c = \beta_1/(4\pi\beta_0).$$

At the NLO α_s is a function of the ratio $\mu/\Lambda_{\mathrm{RS}}$ and the variation of the RS for fixed scale μ is equivalent to the variation of μ for fixed RS. To vary both the renormalization scale and scheme **is legitimate but redundant**.

Quark and gluon distribution functions of the photon

$$\Sigma(x,M) \equiv \sum_{i=1}^{n_f}\left(q_i(x,M)+\overline{q}_i(x,M)\right), \; q_{\mathrm{NS}}(x,M) \equiv \sum_{i=1}^{n_f}\left(e_i^2 - \langle e^2\rangle\right)\left(q_i(x,M)+\overline{q}_i(x,M)\right)$$

satisfy the system of coupled **inhomogeneous** evolution equations

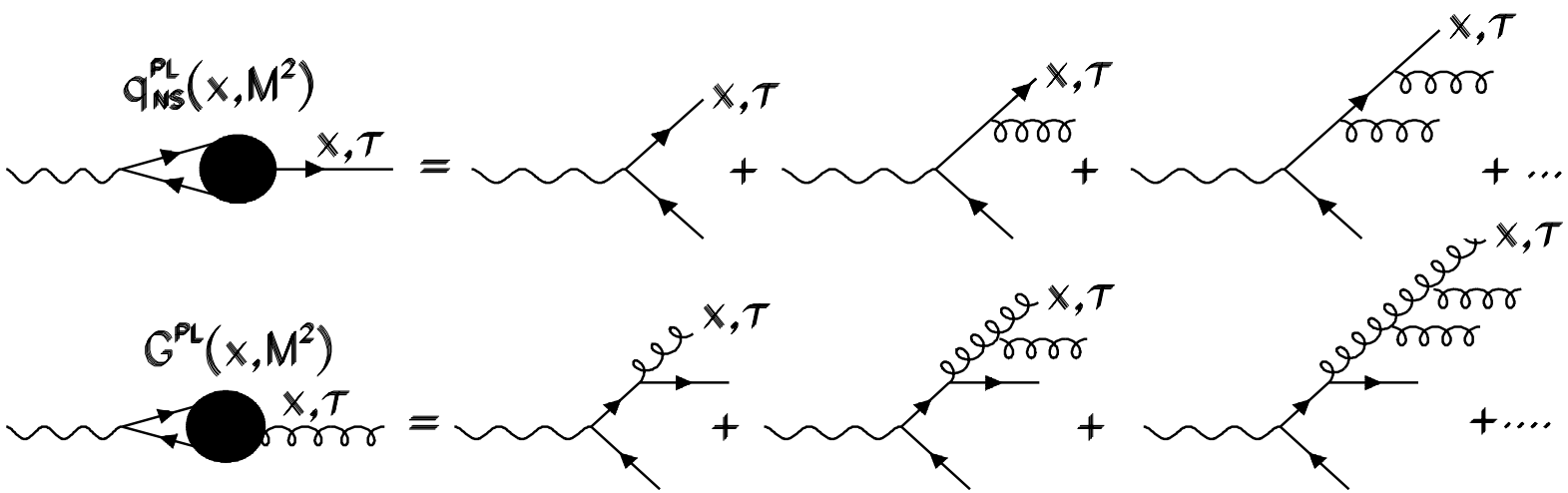
$$
\begin{aligned}
\frac{\mathrm{d}\Sigma(M)}{\mathrm{d}\ln M^2} &= \delta_\Sigma k_q(M) + P_{qq}(M)\otimes\Sigma(M) + P_{qG}(M)\otimes G(M),\\
\frac{\mathrm{d}G(M)}{\mathrm{d}\ln M^2} &= k_G(M) + P_{Gq}(M)\otimes\Sigma(M) + P_{GG}(M)\otimes G(M),\\
\frac{\mathrm{d}q_{\mathrm{NS}}(M)}{\mathrm{d}\ln M^2} &= \delta_{\mathrm{NS}} k_q(M) + P_{\mathrm{NS}}(M)\otimes q_{\mathrm{NS}}(M),
\end{aligned}
$$

where $\delta_{\mathrm{NS}} \equiv 6n_f\left(\langle e^4\rangle - \langle e^2\rangle^2\right)$ and $\delta_\Sigma = 6n_f\langle e^2\rangle$.

$$
\begin{aligned}
k_q(x,M) &= \frac{\alpha}{2\pi}\left[k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi}k_q^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \cdots\right],\\
k_G(x,M) &= \frac{\alpha}{2\pi}\left[\frac{\alpha_s(M)}{2\pi}k_G^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \cdots\right],\\
P_{ij}(x,M) &= \frac{\alpha_s(M)}{2\pi}P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \cdots.
\end{aligned}
$$

where $k_q^{(0)}(x) = (x^2 + (1-x)^2)$ as well as the homogeneous splitting functions $P_{ij}^{(0)}(x)$ **are unique**, whereas higher order splitting functions $k_q^{(j)}, k_G^{(j)}, P_{kl}^{(j)}, j \geq 1$ **depend on the choice of the factorization scheme** (FS).

General solutions of these eqs. can be written as a sum of a particular solution of the full inhomogeneous equations and a general solution, called **hadron-like**, of the corresponding homogeneous ones. A subset of the former resulting from the resummation of contributions of multiple parton emissions off the primary QED vertex $\gamma \to q\overline{q}$ and vanishing at $M = M_0$, are called **point-like**.



Due to the arbitrariness in the choice of M_0 the separation

$$D(x, M) = D^{\mathrm{PL}}(x, M, M_0) + D^{\mathrm{HAD}}(x, M, M_0).$$

into the hadronic and pointlike parts is, however, **ambiguous.**

The explicit form of the pointlike contribution to NS distribution function

$$q_{\rm NS}^{\rm PL}(n, M_0, M) = \frac{4\pi}{\alpha_s(M)} \left[1 - \left(\frac{\alpha_s(M)}{\alpha_s(M_0)} \right)^{1-2P_{qq}^{(0)}(n)/\beta_0} \right] a_{\rm NS}(n),$$

where

$$a_{\rm NS}(n) \equiv \frac{\alpha}{2\pi\beta_0} \frac{k_{\rm NS}^{(0)}(n)}{1 - 2P_{qq}^{(0)}(n)/\beta_0}$$

is often claimed to show that it behaves as $\mathcal{O}(\alpha/\alpha_s)$. However, the fact that $\alpha_s(M)$ appears in the denominator of $q_{\rm NS}^{\rm PL}$ **cannot be interpreted in this way** because switching QCD off by sending $\Lambda_{\rm RS} \to 0$ for fixed M, M_0 reduces, as expected, the expression (3) to **purely QED contribution**:

$$q_{\rm NS}^{\rm PL}(x, M, M_0) \to \frac{\alpha}{2\pi} k_{\rm NS}^{(0)}(x) \ln \frac{M^2}{M_0^2}.$$

The above explicit expression merely implies that **for asymptotically large** M

$$q_{\rm NS}^{\rm PL}(n, M_0, M) \propto \ln M^2.$$

As emphasized long time ago by Politzer, there is **no compelling reason for identifying the renormalization and factorization scales μ and M**.

$Q\overline{Q}$ production in $\gamma\gamma$ collisions

Conventional NLO QCD approximation, to $\sigma_{tot}(\gamma\gamma \to Q\overline{Q})$, performed with fixed **pole quark masses**, takes into account the first two terms in expansions

$$\begin{aligned}
\sigma_{\rm dir}(M) &= \sigma^{(0)}_{\rm dir} + \sigma^{(1)}_{\rm dir}\alpha_s(\mu) + \sigma^{(2)}_{\rm dir}(M,\mu)\alpha_s^2(\mu) + \sigma^{(3)}_{\rm dir}(M,\mu)\alpha_s^3(\mu) + \cdots, \\
\sigma_{\rm sr}(M) &= \sigma^{(1)}_{\rm sr}(M)\alpha_s(\mu) + \sigma^{(2)}_{\rm sr}(M,\mu)\alpha_s^2(\mu) + \sigma^{(3)}_{\rm sr}(M,\mu)\alpha_s^3(\mu) + \cdots, \\
\sigma_{\rm dr}(M) &= \sigma^{(2)}_{\rm dr}(M)\alpha_s^2(\mu) + \sigma^{(3)}_{\rm dr}(M,\mu)\alpha_s^3(\mu) + \cdots,
\end{aligned}$$

Starting at the order α_s^2 the direct photon contribution **depends on the factorization scale** and therefore **mixes with the single and double resolved ones**.

These approximations include **all currently known terms**, but we should be aware of its theoretical deficiency. The fact that the first two terms **start and end at different powers** of α_s is justified by claiming that PDF of the photon **behave as α/α_s**. Consequently, the first terms in all three expressions are claimed to be of order $(\alpha_s)^0 = 1$ and the second ones of order α_s. However, as shown above, the term $\ln M^2$ characterizing the large M behaviour of PDF of the photon comes from integration over the transverse degree of freedom of the purely QED vertex $\gamma \to q\overline{q}$ and **cannot be interpreted as $1/\alpha_s(M)$**.

Direct photon contribution

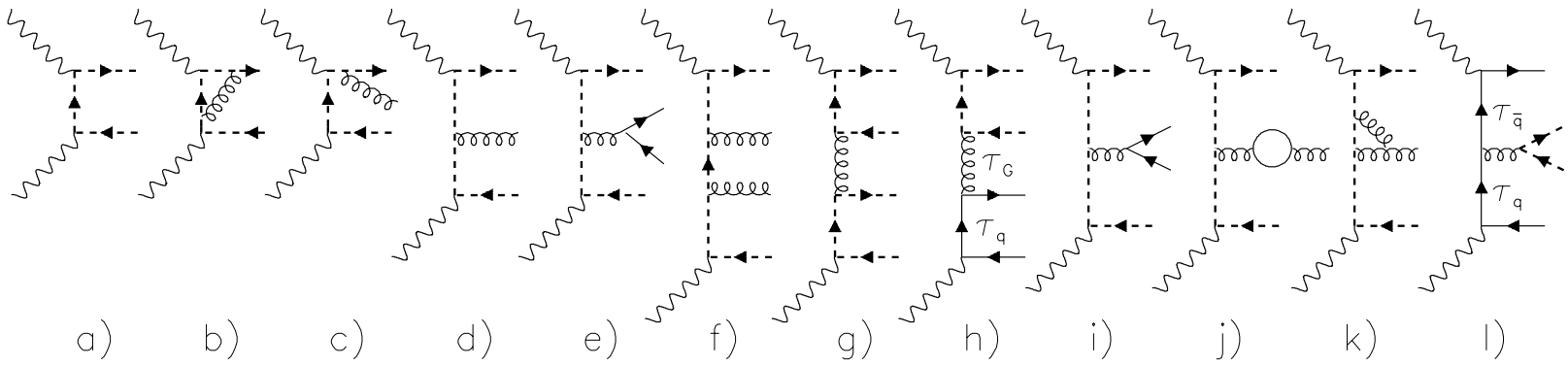
For proper treatment of the direct photon contribution to $\sigma_{tot}(\gamma\gamma \to Q\overline{Q})$, the total cross section of e^+e^- annihilations provides a suitable guidance.

$$\sigma_{\rm had}(\sqrt{S}) = \sigma^{(0)}_{\rm had}(\sqrt{S}) + \alpha_s(\mu)\sigma^{(1)}_{\rm had}(\sqrt{S}) + \alpha_s^2(\mu)\sigma^{(2)}_{\rm had}(\sqrt{S}/\mu) + \cdots = \sigma^{(0)}_{\rm had}(1 + r(\sqrt{S})),$$

where the lowest order term $\sigma^{(0)}_{\rm had}(\sqrt{S}) \equiv (4\pi\alpha^2/S)\sum_{f=1}^{n_f} e_f^2$ **comes from pure QED**, whereas genuine QCD effects are contained in the quantity

$$r(\sqrt{S}) = \frac{\alpha_s(\mu)}{\pi}\left[1 + \alpha_s(\mu) r_1(\sqrt{S}/\mu) + \cdots\right].$$

- For the purpose of QCD analysis of this quantity it is a generally accepted practice to **discard the lowest order term** $\sigma^{(0)}_{\rm had}(\sqrt{S})$, which is of **pure QED origin**, and denote as the "leading order" the second term $\sigma^{(0)}_{\rm had}\alpha_s/\pi$.
- The adjectives "LO" and "NLO" are reserved for **genuine QCD effects** described by $r(\sqrt{S})$.
- To work in a well-defined RS of α_s requires **including at least the first two consecutive powers of α_s**. The explicit μ-dependence of $r_1(\sqrt{S}/\mu)$ cancels to order α_s^2 the implicit μ-dependence of the LO term $\alpha_s(\mu)/\pi$.



Contrary to this practice, the conventional calculations of $\sigma_{tot}(\gamma\gamma \to Q\overline{Q})$ consider the purely QED contribution

$$\sigma^{(0)}_{\rm dir}(W) = \sigma_0 \left[\left(1 + \frac{4m_b^2}{W^2} - \frac{8m_b^4}{W^4} \right) \ln \frac{1+\beta}{1-\beta} - \beta \left(1 + \frac{4m_b^2}{W^2} \right) \right], \ \sigma_0 \equiv \frac{12\pi e_b^4 \alpha^2}{W^2},$$

where $\beta = \sqrt{1 - 4m_b^2/W^2}$, as the LO approximation. This **is legitimate** but implies that the NLO approximation includes only the lowest order term in α_s and **cannot therefore be associated to a well-defined RS** of α_s even if the NLO expression for α_s is used. For QCD analysis of $\sigma_{\rm dir}$ in a well-defined RS **the incorporation of the term proportional to $\alpha^2\alpha_s^2$ is indispensable.**

At the order $\alpha^2\alpha_s^2$ the diagrams with light quarks start appearing and **we can distinguish three classes** of direct photon contributions differing by the overall heavy quark charge factor CF:

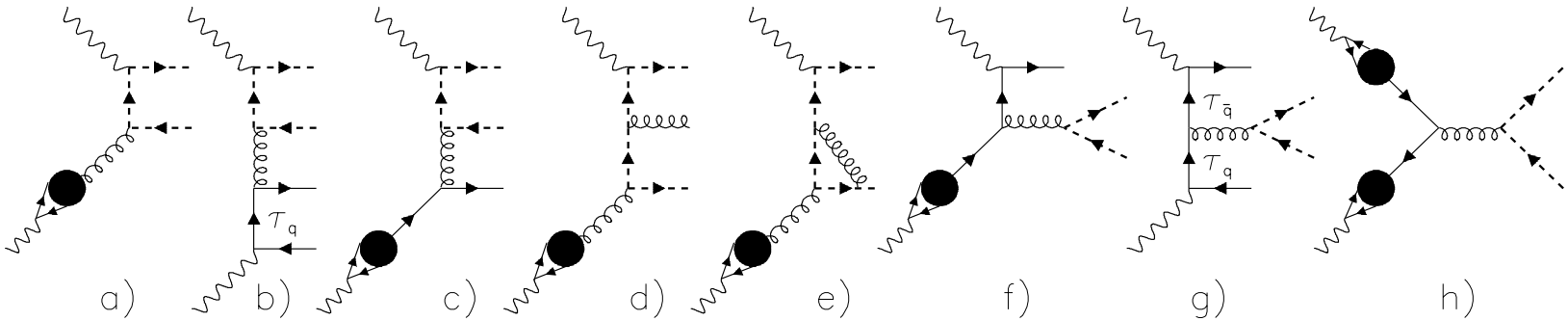
Class A: $CF = e_Q^4$. Comes from diagrams in which **both photons couple to heavy $Q\overline{Q}$** pairs. The KLN theorem implies that at each order of α_s **the sum of all contributions of this class to $\sigma_{\rm dir}$ is finite**. It is this class of direct photon contributions that is needed for the calculation of $\sigma_{\rm dir}$ to be performed in a well-defined RS.

Class B: $CF = e_Q^2$. Comes from diagrams in which **one of the photons couples to a heavy $Q\overline{Q}$** and the other to a light $q\overline{q}$ pair. For massless light quarks this diagram has initial state mass singularity, which is removed by **introducing the concept of the light quark (and gluon) distribution functions of the photon**. The resulting factorization scale dependence is related to that of the corresponding single resolved photon diagram.

Class C: $CF = 1$. Comes from diagrams in which **both photons couple to light $q\overline{q}$ pairs**. In this case the analogous subtraction procedure relates it to the corresponding single and double resolved photon contributions. Classes B and C are needed to guarantee the factorization scale (and scheme) invariance of single and double resolved photon contributions to order $\alpha^2\alpha_s^2$.

Because of different charge factors CF, the **classes A, B and C do not mix** under renormalization of α_s and factorization of mass singularities.

Resolved photon contribution



Let us write the sum of first two terms single and double resolved photon contributions explicitly in terms of PDF and parton level cross sections

$$
\begin{aligned}
\sigma^{(12)}_{\rm res}(M,\mu) \quad \equiv \quad & 2\alpha_s(\mu)\int {\rm d}x G(x,M)\left[\sigma^{(1)}_{\gamma G}(x)+\alpha_s(\mu)\sigma^{(2)}_{\gamma G}(x,M,\mu)\right]+ \\
& 4\alpha_s^2(\mu)\int {\rm d}x \sum_i q_i(x,M)\sigma^{(2)}_{\gamma q_i}(x,M)+ \\
& 2\alpha_s^2(\mu)\int\int {\rm d}x{\rm d}y \sum_i q_i(x,M)\overline{q}_i(y,M)\left[\sigma^{(2)}_{q\overline{q}}(xy)+\alpha_s(\mu)\sigma^{(3)}_{q\overline{q}}(xy,M,\mu)\right]+ \\
& \alpha_s^2(\mu)\int\int {\rm d}x{\rm d}y G(x,M)G(y,M)\left[\sigma^{(2)}_{GG}(xy)+\alpha_s(\mu)\sigma^{(3)}_{GG}(xy,M,\mu)\right]+ \\
& 2\alpha_s^3(\mu)\int\int {\rm d}x{\rm d}y \Sigma(x,M)G(y,M)\sigma^{(3)}_{qG}(xy,M)
\end{aligned}
$$

Recalling the general form of the derivative $\mathrm{d}\sigma_{\mathrm{res}}/\mathrm{d}\ln M^2$

$$\frac{\mathrm{d}\sigma_{\mathrm{res}}}{\mathrm{d}\ln M^2} = \int \mathrm{d}x W_0(x,M) + \int \mathrm{d}x \left[\sum_i q_i(x,M) W_{q_i}(x,M) + G(x,M) W_G(x,M) \right] +$$

$$\iint \mathrm{d}x\mathrm{d}y \left[G(x,M)G(y,M)W_{GG}(xy,M) + \sum_i q_i(x,M)\overline{q}_i(y,M)W_{q\overline{q}}(xy,M) + \right.$$

$$\left. \Sigma(x,M)G(y,M)W_{qG}(xy,M) \right],$$

using the evolution equations and denoting $\alpha_s \equiv \alpha_s(\mu)$, $\dot{f} \equiv \mathrm{df}/\mathrm{d}\ln \mathrm{M}^2$ we find

$$\begin{aligned}
W_0(x,M) &= \frac{\alpha\alpha_s^2}{\pi} \left\{ \frac{1}{2\pi} k_G^{(1)}(x)\sigma_{\gamma G}^{(1)}(x) + 6k_q^{(0)}(x) \sum_i e_i^2 \sigma_{\gamma q_i}^{(2)}(x,M) \right\} + \cdots \\
W_{q_i}(x,M) &= \frac{\alpha_s^2}{\pi} \left\{ 4\pi\dot{\sigma}_{\gamma q}^{(2)}(x) + \int \mathrm{d}z \left[P_{Gq}^{(0)}(z)\sigma_{\gamma G}^{(1)}(xz) + 3e_i^2\alpha k_q^{(0)}(z)\sigma_{q\overline{q}}^{(2)}(xz) \right] \right\} + \cdots \\
W_G(x,M) &= \frac{\alpha_s^2}{\pi} \left\{ 2\pi\dot{\sigma}_{\gamma G}^{(2)}(x) + \int \mathrm{d}z P_{GG}^{(0)}(z)\sigma_{\gamma G}^{(1)}(xz) \right\} + \cdots \\
W_{GG}(x,M) &= \frac{\alpha_s^3}{\pi} \left\{ \pi\dot{\sigma}_{GG}^{(3)}(x) + \int \mathrm{d}z P_{GG}^{(0)}(z)\sigma_{GG}^{(2)}(xz) \right\} + \cdots
\end{aligned}$$

$$W_{q\overline{q}}(x,M) = \frac{\alpha_s^3}{\pi}\left\{2\pi\dot{\sigma}^{(3)}_{q\overline{q}}(x) + 2\int \mathrm{d}z P^{(0)}_{qq}(z)\sigma^{(2)}_{q\overline{q}}(xz)\right\} + \cdots$$

$$W_{qG}(x,M) = \frac{\alpha_s^3}{\pi}\left\{2\pi\dot{\sigma}^{(3)}_{qG}(x) + \int \mathrm{d}z\left[P^{(0)}_{qG}(z)\sigma^{(2)}_{q\overline{q}}(xz) + P^{(0)}_{Gq}(z)\sigma^{(2)}_{GG}(xz)\right]\right\} + \cdots$$

The factorization scale invariance of $\sigma^{(12)}_{\rm res}(M,\mu)$ requires that its variation with respect to $\ln M^2$ is of higher order in α_s than the approximation itself. There is no dispute that direct photon contributions of classes B and C are needed to guarantee this property. The question is **which terms W_{ij} must vanish** if the "NLO approximation" is defined by the expression $\sigma^{(12)}_{\rm res}(M,\mu)$.

Conventional approach: $q(M), G(M) \propto \alpha/\alpha_s$ and $\sigma^{(12)}_{\rm res}(M,\mu)$ is thus **of order $\alpha^2\alpha_s$**. Consequently, W_q and W_G must vanish to order α_s^2 and $W_{GG}, W_{q\overline{q}}$ and W_{qG} to order α_s^3 respectively, which, indeed, they do. The fact that **the first term in W_0 does not vanish is of no concern** as it is manifestly of the order $\alpha\alpha_s^2$ and thus of higher order.

My view: taking into account that, actually, $q(M), G(M) \propto \alpha$, we see that W_0 is of the same order $\alpha^2\alpha_s^2$ as the products $q_iW_{q_i}, GW_G$ etc. and **must therefore also vanish** for theoretical consistency of $\sigma^{(12)}_{\rm res}(M,\mu)$. This necessitates the inclusion of class B direct photon contributions of the order $\alpha\alpha_s^2$.

$\bar{b}b$ production at LEP2

At LEP the incoming leptons act as sources of **transverse and longitudinal** virtual photons:

$$f_T^\gamma(y,Q^2) = \frac{\alpha}{2\pi}\left(\frac{1+(1-y)^2)}{y}\frac{1}{Q^2} - \frac{2m_e^2 y}{Q^4}\right),$$
$$f_L^\gamma(y,Q^2) = \frac{\alpha}{2\pi}\frac{2(1-y)}{y}\frac{1}{Q^2},$$

The LEP data includes photon virtualities up to moderate Q^2, but are dominated $b\bar{b}$ production by two quasireal photons with $\langle Q^2\rangle \simeq 0.01$ GeV^2 and longitudinal virtual photons can be neglected.

Only cross sections integrated over the whole phase space are available, but we shall analyze also the following differential distributions

$$\mathrm{d}\sigma_k(\mathrm{e}^+\mathrm{e}^- \to b\bar{b})/\mathrm{d}W$$

$$F_k(W) \equiv \int_{2m_b}^{W} \mathrm{d}w \frac{\mathrm{d}\sigma_k(\mathrm{e}^+\mathrm{e}^- \to b\bar{b})}{\mathrm{d}w}, \quad G_k(W) \equiv \int_{W}^{\sqrt{S}} \mathrm{d}w \frac{\mathrm{d}\sigma_k(\mathrm{e}^+\mathrm{e}^- \to b\bar{b})}{\mathrm{d}w}$$

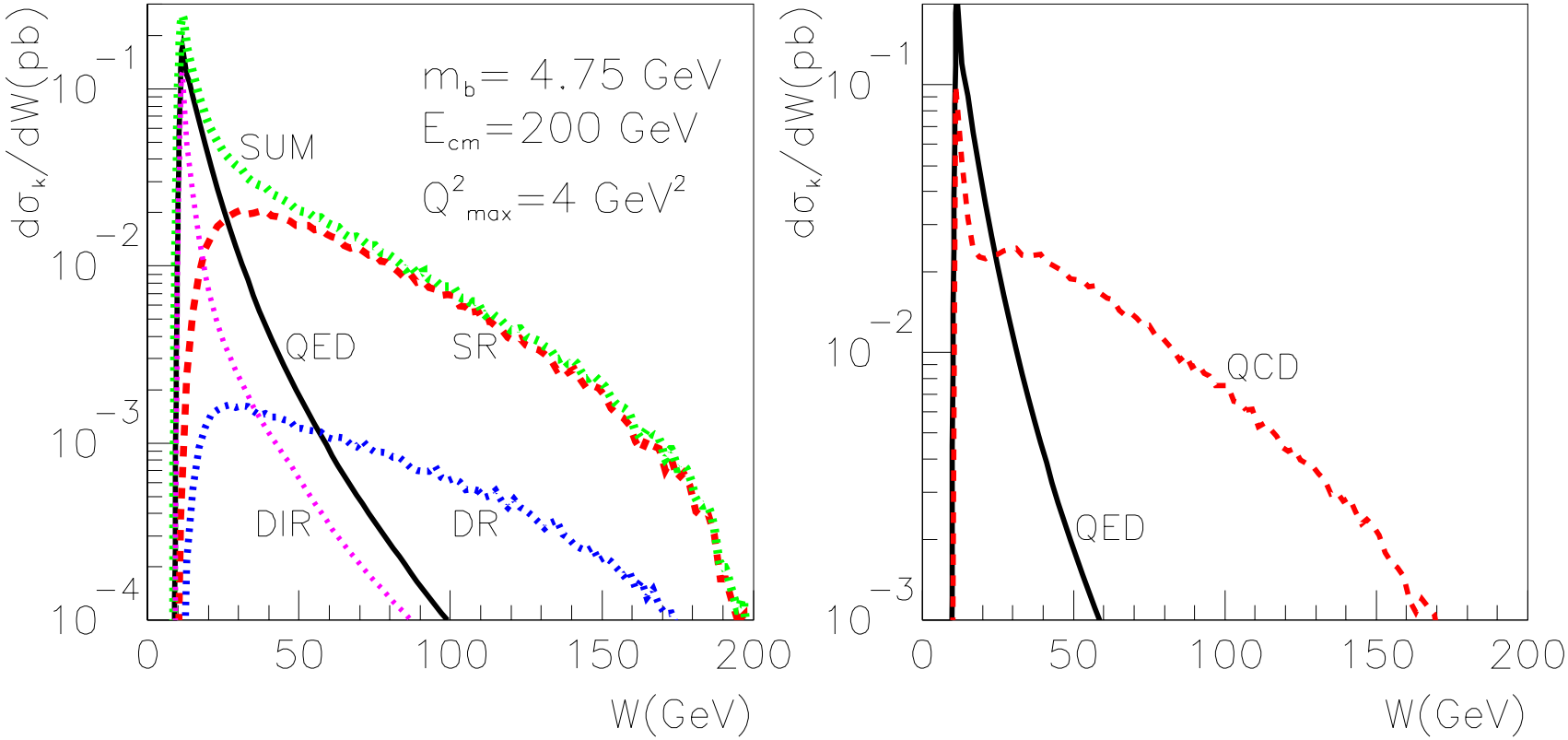
$$r_k(W) \equiv \frac{\mathrm{d}\sigma_k(\mathrm{e}^+\mathrm{e}^- \to b\bar{b})}{\mathrm{d}W} \Big/ \frac{\mathrm{d}\sigma_{tot}(\mathrm{e}^+\mathrm{e}^- \to b\bar{b})}{\mathrm{d}W}.$$

QED contribution

$$\frac{\mathrm{d}\sigma_{\mathrm{QED}}(\mathrm{e}^+\mathrm{e}^- \to b\bar{b})}{\mathrm{d}W} = \frac{6\alpha^4 e_b^4}{\pi S}\frac{A(W)}{W}\left[\left(1+\frac{4m_b^2}{W^2}-\frac{8m_b^4}{W^4}\right)\ln\frac{1+\beta}{1-\beta}-\beta\left(1+\frac{4m_b^2}{W^2}\right)\right],$$

where $\beta = \sqrt{1-4m_b^2/W^2}$ and

$$A(W) = \iint \mathrm{d}y\mathrm{d}z\delta\left(\frac{W^2}{S}-yz\right)\left[\frac{1+(1-y)^2}{y}\right]\left[\frac{1+(1-z)^2}{z}\right]\ln\frac{Q^2_{max}(1-y)}{m_e^2y^2}\ln\frac{Q^2_{max}(1-z)}{m_e^2z^2},$$



Lowest order QCD contribution

Three types: **direct, single resolved and double resolved photon**.
Given as convolutions of f_T^γ with appropriate partonic cross sections.
u, d, s and c considered as intrinsic and massless $\Rightarrow$ $n_f = 4$.

Direct photon contribution

$$\frac{\mathrm{d}\sigma_{\mathrm{dir}}^{\mathrm{LO}}(W)}{\mathrm{d}W} = \frac{6\alpha^4 e_b^4}{\pi S}\frac{A(W)}{W}\alpha_s(\mu)\sigma_{\mathrm{dir}}^{(1)}(W/m_b)$$

coming form real or virtual emission of one gluon is exclusively of class A.
The value μ **is completely arbitrary**. Peaks more sharply at small W than pure QED, because $\sigma_{\mathrm{dir}}^{(1)}(W/m_b)$ **does not vanish at the threshold**.

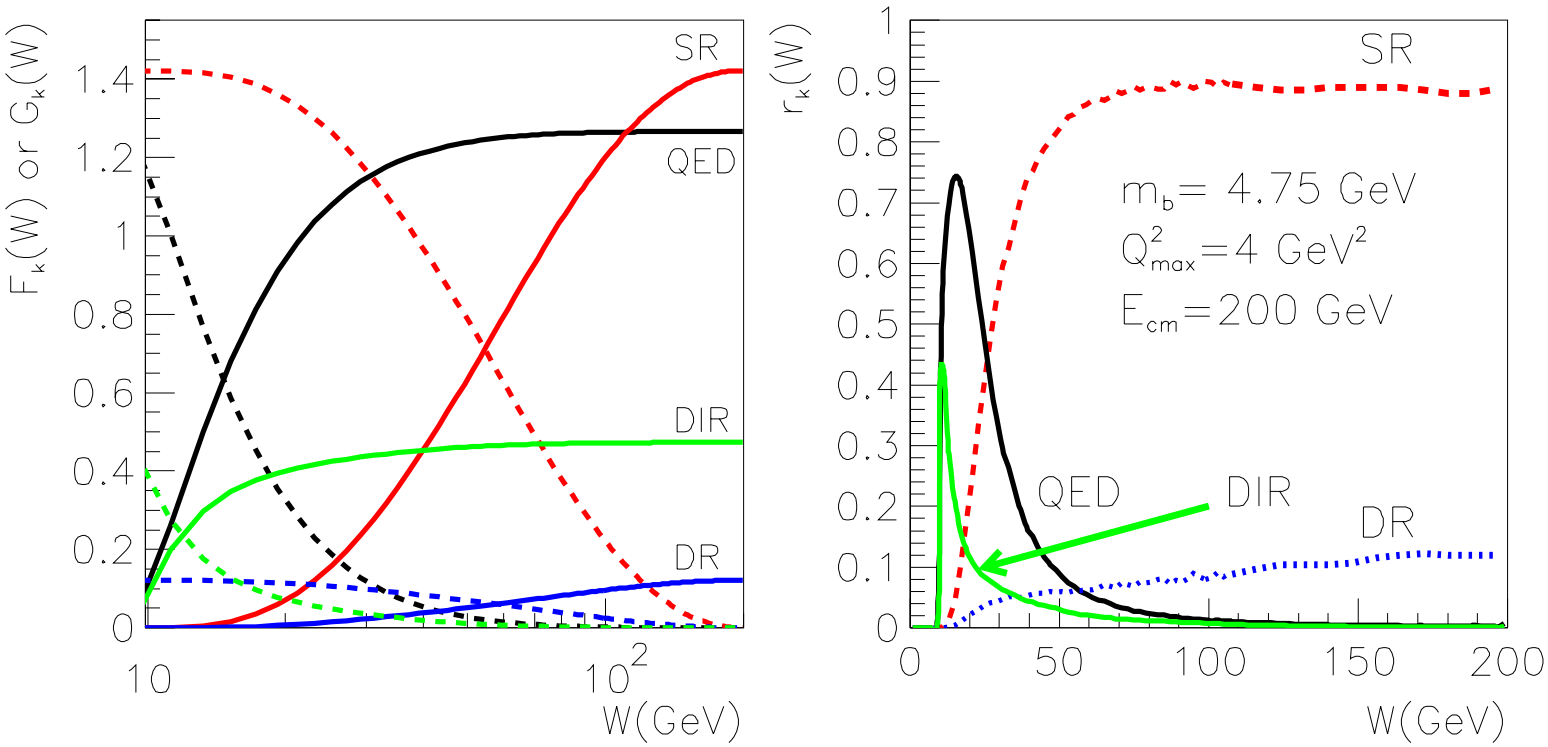
Resolved photon contribution

The LO single and double resolved photon contributions were computed with HERWIG, which calculates LO cross sections of the processes

$$\begin{aligned} \gamma + G &\rightarrow b + \overline{b}, \\ G + G &\rightarrow b + \overline{b}, \quad q + \overline{q} \rightarrow b + \overline{b} \end{aligned}$$

and convolutes them with photon fluxes and PDF of quasireal photons setting $\mu = M \doteq M_T \equiv \sqrt{E_T^2 + M^2}$. $\langle M_T \rangle$ depends weakly on W with $\langle M_T \rangle \simeq 7$ GeV.

Parameters			QED	LO QCD			Total
m_b	$\Lambda^{(4)}$	PDF		DIR	SR	DR	
4.75	0.27	GRV LO	1.27	0.473	1.415	0.121	3.28
4.5	0.27	GRV LO	1.40	0.478	1.746	0.146	3.77
4.75	0.35	GRV LO	1.27	0.520	1.542	0.141	3.47
4.75	0.27	SAS1D	1.27	0.473	0.904	0.077	2.73



Comparison of individual contributions

- Pure **QED** as well as the **LO direct photon** one **peak at very small** W and are negligible above $W \simeq 50$ GeV: 95% of the QED contribution comes from the region $W \lesssim 30$ GeV.
- The onset of **single and double resolved photon** contributions is **much slower**, but **both distributions are broader.**
- The **double resolved photon** contribution is **negligible** practically everywhere.
- The pure **QED and single resolved photon** contributions are of **comparable size** and together make up about 85% of the total integrated cross section,
- For $W \lesssim 30$ GeV $\mathrm{d}\sigma_{tot}/\mathrm{d}W$ is dominated by **pure QED** contribution, whereas for $W \gtrsim 30$ GeV, **QCD contributions** take over.

Dependence on input parameters

Results depend on the **numerical values** of $m_b, \Lambda^{(4)}_{\rm QCD}, Q^2_{max}$, choice of PDF of the photon and **renormalization and factorization scales μ and M.**
In all calculations shown so far I set $\mu = M = m_b$. The "central" calculation was performed for $\sqrt{S} = 200$ GeV, $Q^2_{max} = 4$ GeV^2, $m_b = 4.75$ GeV, $\Lambda^{(4)} = 0.27$ GeV using the GRV LO PDF of the photon. To see the sensitivity of the LO results to these assumptions we varied some of these parameters:

- m_b was lowered to $m_b = 4.5$ GeV,
- $\Lambda^{(4)}$ was increased to 0.35 GeV,
- GRV set of PDF of the photon was replaced with that of Schuler-Sjöstrand set SAS1D.

Lowering m_b increases all four contributions, as does, except for the pure QED one, increasing $\Lambda^{(4)}$. SAS1D PDF, on the other hand, yield markedly lower results for single and double resolved photon contributions. It is clear that **varying the input parameters within reasonable bounds does not bring the sum of lowest order QED and QCD calculations significantly closer to the data.**

Explanations?

- P. Ferreira (hep-ph/0309156) explains the excess as evidence for Hahn-Nambu **integer quark charges** arguing that instead of the usual $3e_b^4 = 1/27$, the charge factor equals

$$\frac{1}{3}\left(\sum_{i=1}^{3} e_b^{(i)}\right)^4 = \frac{1}{3} = 9\frac{1}{27},$$

  If true the whole discrepancy would have to come **from the region of small W**, where QED contribution dominates.

- The **light gluino production** via gluon-gluon fusion (suggested by Berger for explanation of similar excess in $p\overline{p}$ collisions) would have to come **from the large W-region** dominated by the double resolved photon contribution.

The above observations suggest that in order to pin down the origins of the mentioned discrepancy of the total integrated cross sections, **it would be very helpful if the data could be separated in two subsamples according to their associated hadronic energy** W, say $W \lesssim 30$ GeV and $W \gtrsim 30$ GeV.

Next-to-leading order QCD corrections

With the sum of LO QED and QCD contributions way below the data one might ask whether the NLO contributions could bridge this gap.

Direct photon contribution

The genuine next-to-leading order QCD approximation can be written as

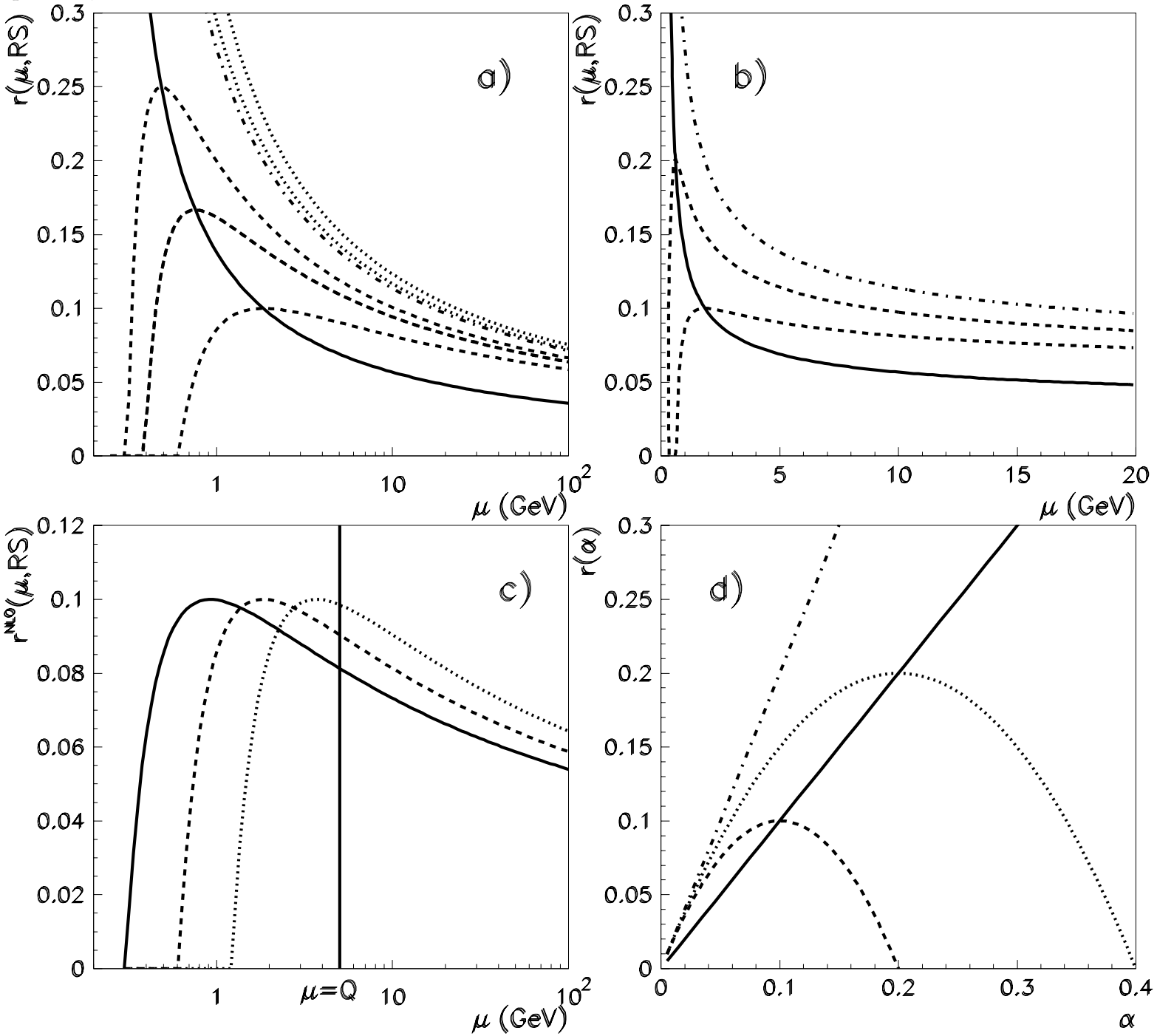
$$\sigma_{\rm dir}^{\rm NLO}(\mu) = \sigma_{\rm dir}^{(1)}\alpha_s(\mu)\left[1 + \frac{\sigma_{\rm dir}^{(2)}(\mu/m_b)}{\sigma_{\rm dir}^{(1)}}\alpha_s(\mu)\right] = \sigma_{\rm dir}^{(1)}\alpha_s(\mu)\left[1 + r_1(\mu/m_b)\alpha_s(\mu)\right],$$

As the leading term is of class A, **only the class A** of order $\alpha^2\alpha_s^2$ direct photon contributions is needed. The renormalization scale invariance of $\sigma_{\rm dir}^{\rm NLO}(\mu)$ implies:

$$r_1(W/m_b, \mu/m_b, {\rm RS}) = \frac{\beta_0}{4\pi}\ln\frac{\mu^2}{\Lambda_{\rm RS}^2} - \rho(W/m_b),$$

where $\rho(W/m_b)$ **is renormalization scale and scheme invariant**, which can be evaluated using the results of a calculation in any given RS:

$$\rho(W/m_b, m_b/\Lambda^{(4)}) = \frac{\beta_0}{4\pi}\ln\frac{m_b^2}{\Lambda_{\rm RS}^{(4)}} - r_1(W/m_b, 1, {\rm RS}).$$


a)
b)
c)
d)
r(μ,RS)
r^NLO(μ,RS)
r(α)
μ (GeV)
α
μ=Q

$\rho > 0$: $\sigma_{\rm dir}^{\rm NLO}(\mu)$ exhibits a **local maximum,** where the prediction is stable. This point is also very **close to the point where $r_1 = 0$.** The value of $\sigma_{\rm dir}^{\rm NLO}$ at this point is proportional to $1/\rho$ implying very large NLO corrections for small ρ. For $n_f = 4$, $m_b = 4.75$ GeV and $\Lambda_{\overline{\rm MS}}^{(4)} = 0.27$ GeV, we get

$$\rho(W/m_b) = 3.88 - r_1(W/m_b, 1, \overline{\rm MS}).$$

$r_1(W/m_b, 1, \overline{\rm MS})$ does not have to be outrageously large to get small ρ!

$\rho \leq 0$: $\sigma_{\rm dir}^{\rm NLO}(\mu)$ is a monotonous function **steeper** than $\sigma_{\rm dir}^{\rm LO}$! NLO thus **does not improve** the stability of the calculation!

These features of $\sigma_{\rm dir}^{\rm NLO}$ are obvious for $\beta_1 = 0$, when

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\rm RS}^2)} \quad \Rightarrow \quad \sigma_{\rm dir}^{\rm NLO} = \sigma_{\rm dir}^{(1)} \alpha_s(\mu)\,[2 - \rho\alpha_s(\mu)]$$

- $\sigma_{\rm dir}^{\rm NLO}(\mu)$ exhibits a **local maximum at** $\alpha_s^{max} = 1/\rho$ where $\sigma_{\rm dir}^{\rm NLO} = \sigma_{\rm dir}^{(1)}/\rho$
- For negative ρ there is **no region of local stability** for $\alpha_s > 0$, but weak logarithmic dependence plotted in a linear scale, **fakes it.**
- The curve representing $\sigma_{\rm dir}^{\rm NLO}(\mu)$ depends on the RS. Identifying μ with a "physical" scale Q **does not resolve** the renormalization scale ambiguity.

- Although the position of the local maximum also depends on the choice of the RS, the value of the $\sigma_{\rm dir}^{\rm NLO}(\mu)$ **at this point does not!**
- Instead of varying both the renormalization scale and scheme, which is redundant, **we may use the couplant α_s itself.**

As there is **no "natural" renormalization scheme,** there are only two general lines of arguments for choosing the renormalization scheme:

- either one looks for the **maximum local stability** (Stevenson's PMS) or
- **smallest α_s^2 corrections** (Grunberg's Effective charges).

Summary:
a) As $\sigma_{\rm dir}^{(2)}$ in has not yet been calculated, **we cannot associate** the class A direct photon contribution to a well-defined RS.
b) As the magnitude of $\sigma_{\rm dir}^{\rm NLO}$ is determined by the ratio $\sigma_{\rm dir}^{(2)}/\sigma_{\rm dir}^{(1)}$, the coefficient r_1 **may be very large** even when both the numerator and denominator are on average of comparable and small magnitude.
c) Without the knowledge of $\sigma_{\rm dir}^{(2)}$ **we cannot make a meaningful estimate** of the importance of higher order corrections.

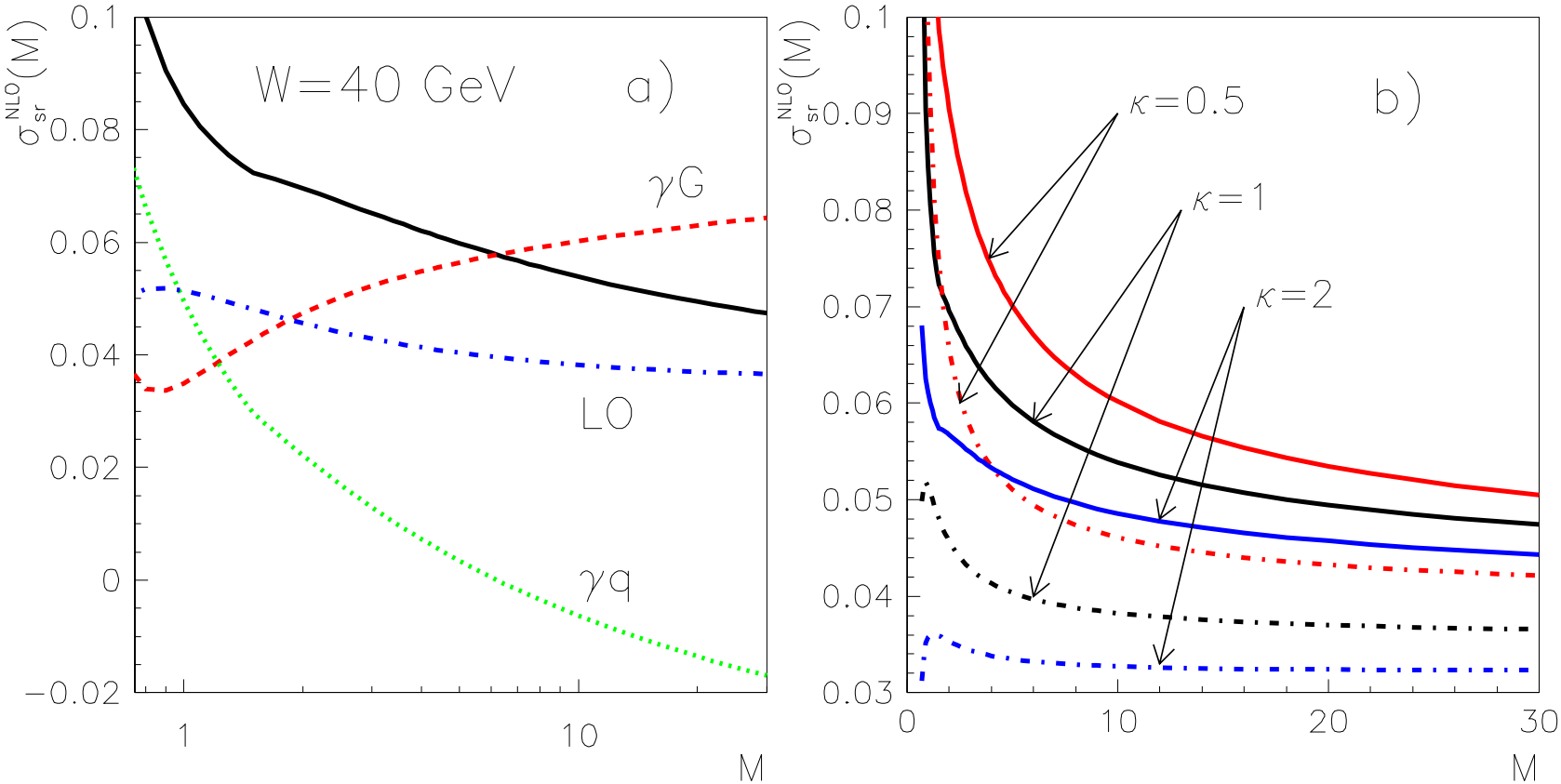
Single resolved photon contribution

Spectrum of $\mathrm{d}\sigma_{\mathrm{sr}}/\mathrm{d}W$ peaks at about $W = 30$ GeV with the mean value $\langle W \rangle \doteq 65$ GeV $\Rightarrow$ **energy range $30 \lesssim W \lesssim 65$ GeV decisive.**

$$\sigma_{\mathrm{sr}}^{\mathrm{NLO}}(W, M, \mu) = 2\alpha_s(\mu) \int \mathrm{d}x G(x, M) \left[\sigma_{\gamma G}^{(1)}(x) + \alpha_s(\mu)\sigma_{\gamma G}^{(2)}(x, M, \mu)\right] + 4\alpha_s^2(\mu) \int \mathrm{d}x \sum_i q_i(x, M)\sigma_{\gamma q_i}^{(2)}(x, M)$$

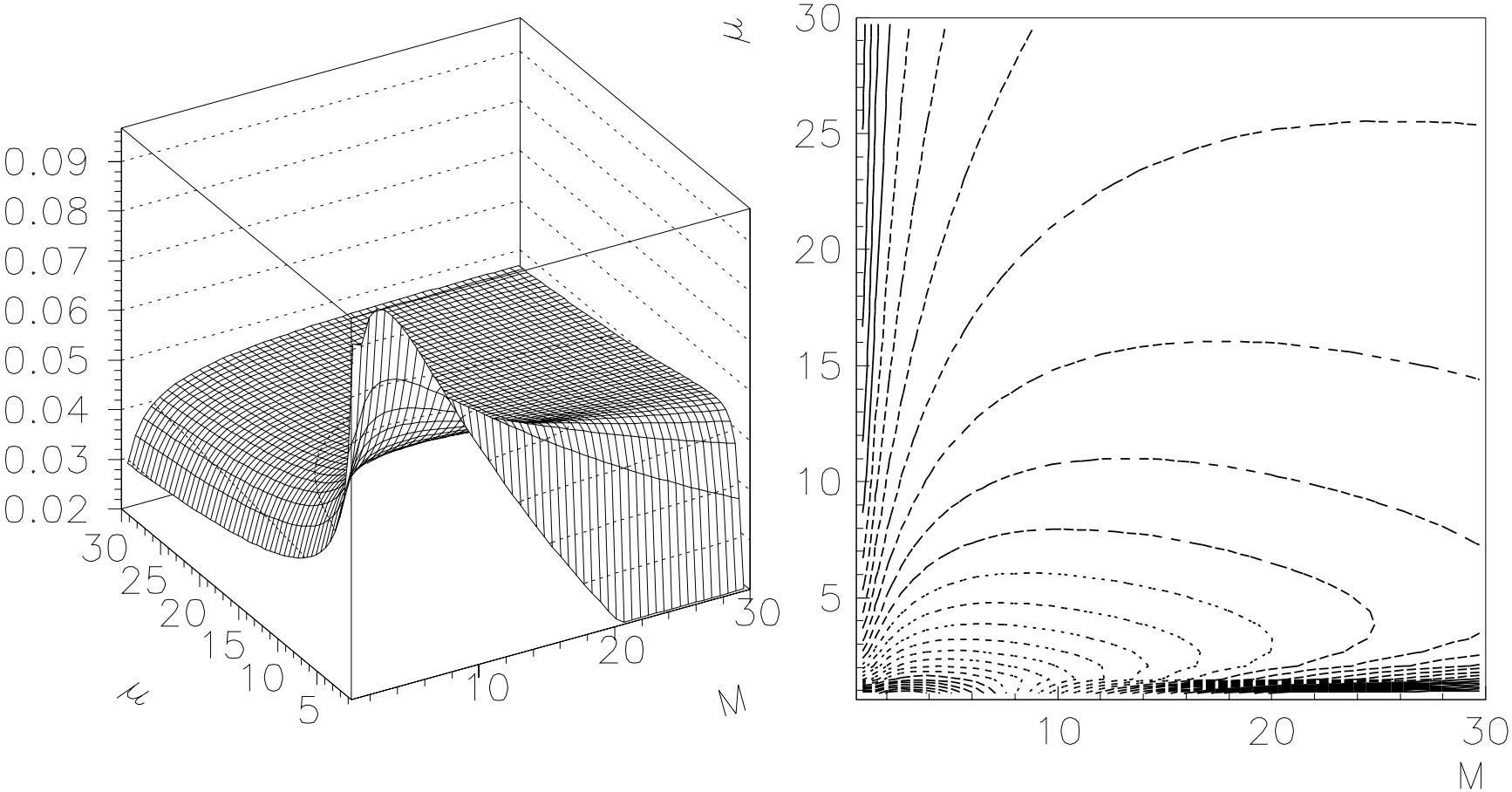
- Partonic cross sections $\sigma_{ij}^{(k)}$ as given by **Ellis and Nason.**
- Even if one does not agree with my claim that the approximations used by **Zerwas, Krämer and Laenen** do not represent complete NLO approximation, one might wish to **know their renormalization and factorization scale dependence.**
- **Separate dependence on μ and M implemented** by adding to $\sigma_{\gamma\mathrm{G}}^{(2)}(x, M, M)$ the term $(\beta_0/4\pi)\sigma_{\gamma G}^{(1)} \ln(\mu^2/M^2)$
- For fixed M, $\sigma_{\mathrm{sr}}^{\mathrm{NLO}}(W, M, \mu)$ has the form of the $\sigma_{\mathrm{dir}}^{\mathrm{NLO}}$
- GRV HO set of PDF of the photon and $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 0.274$ GeV used.

$$\sigma_{\rm sr}^{\rm NLO}(W, M, \mu = \kappa M)$$



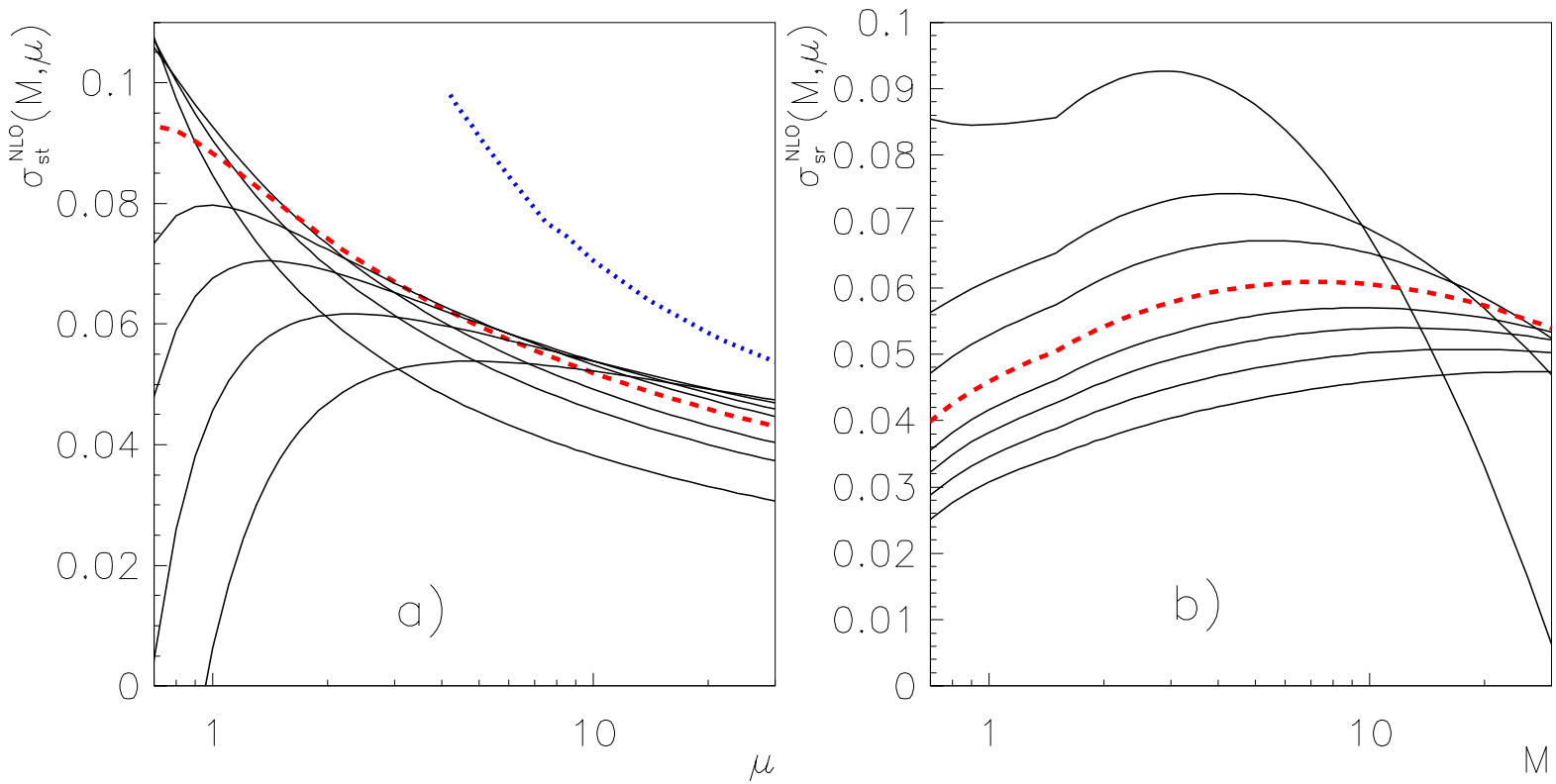
- Significantly **different scale dependence** of the γG and γq channels, the latter turning negative for $M \gtrsim 6$ GeV,
- conventional NLO approximation is **a monotonously decreasing function** of the common scale, which falls off **even faster** than the LO expression!
- **Going to the NLO does not improve the stability!**

$\sigma_{\rm sr}^{\rm NLO}(W, M, \mu)$



Contrary to analogous process in $\overline{p}p$ collisions $\sigma_{\rm sr}^{\rm NLO}(W, M, \mu)$ **does not exhibit a true saddle point** but there seems to be a sort of quasistability region at large scales, say for $M \gtrsim 10$ GeV, $\mu \gtrsim 20$ GeV, **but** $\cdots$

this is misleading as is evident if we slice the surface plot along both axes



- For $M \lesssim 4.2$ GeV $\sigma^{\rm NLO}(M,\mu)$ corresponds to negative ρ in and exhibits thus **no local stability point.**
- For higher M the local maximum in the μ-dependence of $\sigma_{\rm sr}^{\rm NLO}(M,\mu)$ exists at the associated $\mu_{max}(M)$. The M-dependence of $\sigma_{\rm sr}^{\rm NLO}(M,\mu_{max}(M))$, shown by the dotted curve, is, however, **even steeper** that those at fixed M.

Conclusions

In order to explain the excess of LEP2 data on $b\bar{b}$ production in $\gamma\gamma$ collisions over the theoretical calculations, two things should be performed.

1. On the **experimental side**, the separation of data into at least **two bins of the hadronic energy** W say $W \lesssim 30$ GeV and $W \gtrsim 30$ GeV, could be instrumental in narrowing the possible mechanisms or phenomena responsible for the observed excess.

2. On the **theoretical side** the evaluation of **direct photon contribution at the order** $\alpha^2\alpha_s 2$ is needed to make the existing theoretical expressions of genuine next-to-leading order in α_s. In their absence, the existing (in my view incomplete) NLO calculations are **highly sensitive to the variation of renormalization and factorization scale** with no region of local stability. This prevents us from making any reasonable estimate of the the associated theoretical uncertainty.