Heavy quark production in $\gamma\gamma$ collisions

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Details in hep-ph/0010140
Related in part to JHEP04(2000)007
Rejected by referees in PLB as well as JHEP
b\bar{b} production in γp and γγ collisions: crisis of PQCD?

OPAL preliminary

σ^{\text{vis}} (ep \rightarrow b X)

\[ \sigma(e^+e^- \rightarrow e^+e^-b\bar{b}) [\text{pb}] \]

- OPAL (μ, \sqrt{s}=189-202 GeV, prel.)
- L3 (μ, e, \sqrt{s}=189-202 GeV, prel.)
- NLO (direct+resolved, GRV)
- NLO (direct only)

bands: \( m_b = 4.5 \text{ GeV}, \mu = m_b/2 \)
\( m_b = 5.2 \text{ GeV}, \mu = 2m_b \)

\( \sqrt{s_{ee}} [\text{GeV}] \)

Data / Theory

- H1 \( \mu \) p_{T,\text{rel}}
- H1 \( \mu \) impact param. (prel.)
- ZEUS e^− p_{T,\text{rel}}

NLO QCD

Q^2 < 1 \text{ GeV}^2

Q^2 (\text{GeV}^2)
QCD analysis of $\sigma(\gamma\gamma \rightarrow QQ)$: conventional approach

In the conventional approach the NLO QCD approximation is defined by taking into account the first two terms in expansions of direct, as well as single and double resolved photon contributions

$$
\sigma_{\text{dir}} = \sigma_{\text{dir}}^{(0)} + \sigma_{\text{dir}}^{(1)}(M, \mu)\alpha_s(M, \mu) + \sigma_{\text{dir}}^{(2)}(M, \mu)\alpha_s(M, \mu) + \cdots,$$

$$
\sigma_{\text{sr}} = \sigma_{\text{sr}}^{(1)}(M)\alpha_s(M) + \sigma_{\text{sr}}^{(2)}(M, \mu)\alpha_s(M, \mu) + \sigma_{\text{sr}}^{(3)}(M, \mu)\alpha_s(M, \mu) + \cdots,$$

$$
\sigma_{\text{dr}} = \sigma_{\text{dr}}^{(2)}(M)\alpha_s(M) + \sigma_{\text{dr}}^{(3)}(M, \mu)\alpha_s(M, \mu) + \cdots,$$

to the total cross section

$$
\sigma(\gamma\gamma \rightarrow QQ) = \sigma_{\text{dir}} + \sigma_{\text{sr}} + \sigma_{\text{dr}}.
$$

where $\sigma_{\text{dir}}^{(0)}$ comes from pure QED and equals

$$
\sigma_{\text{dir}}^{(0)} = \sigma_0 \left[ \left( 1 + \frac{4m_Q^2}{s} - \frac{8m_Q^4}{s^2} \right) \ln \frac{1 + \beta}{1 - \beta} - \beta \left( 1 + \frac{4m_Q^2}{s} \right) \right],
$$
PDF of the photon satisfy the system of inhomogeneous evolution equations

\[
\frac{d\Sigma(x, M)}{d \ln M^2} = \delta \Sigma k_q + P_{qq} \otimes \Sigma + P_{qG} \otimes G,
\]
\[
\frac{dG(x, M)}{d \ln M^2} = k_G + P_{Gq} \otimes \Sigma + P_{GG} \otimes G,
\]
\[
\frac{dq_{NS}(x, M)}{d \ln M^2} = \delta_{NS} k_q + P_{NS} \otimes q_{NS},
\]

where \(\delta_{NS} \equiv 6n_f \left( \langle e^4 \rangle - \langle e^2 \rangle^2 \right)\), \(\delta_\Sigma = 6n_f \langle e^2 \rangle\) and

\[
k_q(x, M) = \frac{\alpha}{2\pi} \left[ k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi} k_q^{(1)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 k_q^{(2)}(x) + \cdots \right],
\]
\[
k_G(x, M) = \frac{\alpha}{2\pi} \left[ \frac{\alpha_s(M)}{2\pi} k_G^{(1)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 k_G^{(2)}(x) + \cdots \right],
\]
\[
P_{ij}(x, M) = \frac{\alpha_s(M)}{2\pi} P_{ij}^{(0)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 P_{ij}^{(1)}(x) + \cdots.
\]

Due to the presence of \(k_q\) general solution of these equations can be split into the hadron-like (HAD) and point-like (PL) parts.

\[
D(x, M) = D^{\text{PL}}(x, M, M_0) + D^{\text{HAD}}(x, M, M_0).
\]
All complications of hard collisions of photons stem from $D^{PL}(x, M, M_0)$ which results from the resummations

\[
q_{NS}^{PL}(x, M^2) = \ldots + \ldots + \ldots + \ldots
\]

\[
g^{PL}(x, M^2) = \ldots + \ldots + \ldots + \ldots + \ldots
\]

Figure 1: Diagrams defining the point-like parts of nonsinglet quark and gluon distribution functions. The resummation involves integration over parton virtualities $\tau \leq M^2$ and is represented by the junction of the blob and the $\gamma \to q\bar{q}$ vertex. Partons going into the hard collision are denoted by $x, \tau$. 

Defining the NLO approximation

Recall the definition of the term **next-to-leading** for $\sigma(e^+e^-\to\text{hadrons})$

$$\sigma_{\text{had}}(Q) = \sigma_{\text{had}}^{(0)}(Q) + \alpha_s(\mu)\sigma_{\text{had}}^{(1)}(Q) + \alpha_s^2(\mu)\sigma_{\text{had}}^{(2)}(Q/\mu) + \cdots = \sigma_{\text{had}}^{(0)}(Q)(1 + r(Q)),$$

where $\sigma_{\text{had}}^{(0)}(Q) \equiv (4\pi\alpha^2/Q^2)\sum_f e_f^2$ comes from pure QED whereas genuine QCD effects are contained in

$$r(Q) = \frac{\alpha_s(\mu)}{\pi} \left[ 1 + \frac{\alpha_s(\mu)}{\pi}r_1(Q/\mu) + \cdots \right]. \quad (1)$$

For this quantity the terms **LO** and **NLO** are applied to genuine QCD effects!
NOTE: inclusion of first two term in (??) is needed in order to work in a well-defined renormalization scheme of $\alpha_s$!

For purely perturbative quantities like $\sigma_{\text{had}}(Q)$ the association of the term NLO QCD approximation with a well-defined RS is a generally accepted convention, worth retaining for physical quantities in any hard scattering process, like the point-like part of $F_2^\gamma(x, Q^2)$, measured in DIS on the photon:

and discussed in detail in JHEP04(2000)007.
Direct photon contribution to $\sigma(\gamma\gamma \rightarrow Q\bar{Q})$

At the order $\alpha^2\alpha_s^2$ diagrams with light quarks appear and we can distinguish (to all orders) three classes of contributions, differing by the charge factor $CF$:

**Class A:** $CF = e_Q^4$. Comes from diagrams in which both primary photons couple to heavy quarks or antiquarks.

**Class B:** $CF = e_Q^2e_q^2$. Comes from diagrams in which one of the primary photons couples to a heavy and the other to a light quark-antiquark pair.

**Class C:** $CF = e_q^4$. Comes from diagrams in which both photons couple to light $q\bar{q}$ pairs.
• For classes B and C the corresponding diagrams involve (and massless light quarks) initial state mass singularities that must be subtracted and put into the corresponding PDF of the photons.

• Because of different charge factors $C_F$ classes A, B and C do not mix under renormalization of $\alpha_s$ and factorization of mass singularities.

• The order $\alpha^2\alpha_s^2$ direct photon contributions of all three classes are needed for theoretical consistency:
  A: for the calculation of class A direct photon contribution (which does not mix with any other) to be performed in a well-defined RS
  B: For factorization scale invariance of the sum of direct and resolved photon contributions.
  C: dtto.

• Classes B,C can be defined also for the PL parts of single and class C for PL parts of double resolved photon contributions.

• Classes A,B,C can be treated separately as they do not mix.
Class A direct photon contributions

In the conventional approach the "leading" and "next-to-leading" order approximations of $\sigma_{\text{dir}}$ are defined as follows

**LO:** $\sigma_{\text{dir}}^{(0)}$, which is of purely QED origin

**NLO:** $\sigma_{\text{dir}}^{(01)} \equiv \sigma_{\text{dir}}^{(0)} + \alpha_s(\mu)\sigma_{\text{dir}}^{(1)}$

But the latter expression has the same form as the first two terms in $\sigma_{\text{had}}(Q)$

$\sigma_{\text{dir}}^{(01)}$ cannot be associated to a well-defined RS

$\sigma_{\text{dir}}^{(01)}$ does not deserve the label NLO!

For QCD analysis of $\sigma_{\text{dir}}$ in a well-defined RS the the third term in $\sigma_{\text{dir}}$, proportional to $\alpha_s^2(\mu)$ is indispensable. We need in particular the diagrams involving loops which contribute to the renormalization of $\alpha_s(\mu)$. The regular, $\mu$-dependent parts of their contributions provide the $\ln \mu^2$ term canceling the $\mu$ dependence of $\alpha_s(\mu)$ in the second term of $\sigma_{\text{dir}}^{(01)}$. 
The lesson from $Q\bar{Q}$ production in pp collisions

Recall how factorization operates for heavy quark production in $pp$ collisions, where at the NLO

$$\sigma^{\text{NLO}}(\bar{p}p \rightarrow Q\bar{Q}) = D_1(M) \otimes \left[ \alpha_s^2(\mu)\sigma^{(2)} + \alpha_s^3(\mu)\sigma^{(3)}(M, \mu) \right] \otimes D_2(M),$$

and $D_1(M)$ and $D_2(M)$ satisfy the **homogeneous** evolution equations. Factorization scale invariance of this expression requires that

$$\frac{d\sigma^{\text{NLO}}(\bar{p}p \rightarrow Q\bar{Q})}{d \ln M} = \alpha_s^2(\mu) \left[ \frac{dD_1(M)}{d \ln M} \otimes \sigma^{(2)} \otimes D_2(M) + D_1(M) \otimes \sigma^{(2)} \otimes \frac{dD_2(M)}{d \ln M} + \alpha_s^3(\mu)D_1(M) \otimes \frac{d\sigma^{(3)}(M)}{d \ln M} \otimes D_2(M) \right] + \alpha_s^3(\mu) \left( \frac{dD_1(M)}{d \ln M} \otimes \sigma^{(3)} \otimes D_2(M) + D_1(M) \otimes \sigma^{(3)} \otimes \frac{dD_2(M)}{d \ln M} \right) \propto \alpha_s^4D_1(M) \otimes D_2(M)$$

which is guaranteed by the fact that

$$\left[ \ldots \right] \propto \alpha_s^2, \quad \left( \ldots \right) \propto \alpha_s \quad \text{due to} \quad \frac{dD_i(M)}{d \ln M} \propto \alpha_s(M)D_i(M)$$
Resolved photon contribution to $\sigma(\gamma\gamma \rightarrow Q\bar{Q})$

There are 5 classes of resolved photon contributions:

- **Single resolved** photon using
  - hadron-like parts of PDF ($\sigma_{srh}$),
  - point-like parts of PDF ($\sigma_{srp}$).

- **Double resolved** photon using
  - hadron-like parts of PDF on both sides ($\sigma_{drhh}$),
  - hadron-like parts of PDF on one side and point-like on the other ($\sigma_{drhp}$),
  - point-like parts of PDF on both sides ($\sigma_{drpp}$).

With this subdivision of $\sigma_{sr}$ and $\sigma_{dr}$ in mind we can write

$$\sigma(\gamma\gamma \rightarrow Q\bar{Q}) = \sigma_{dir} + \sigma_{srh} + \sigma_{srp} + \sigma_{drhh} + \sigma_{drhp} + \sigma_{drpp}.$$ 

Only cross sections involving **point-like parts** of PDF will be considered further
**Q̅Q** production in single resolved photon contribution

\[
\sigma_{\text{srp}}^{(12)} \equiv \alpha_s(\mu)\sigma_{\text{srp}}^{(1)}(M) + \alpha_s^2(\mu)\sigma_{\text{srp}}^{(2)}(M, \mu) = \alpha_s(\mu) \left[ \sigma^{(1)} \otimes G(M) + \alpha_s(\mu)\sigma^{(2)}(M, \mu) \otimes D(M) \right]
\]

**Class B** single resolved photon terms mix with direct photon ones

\[
\frac{d\sigma_{\text{srp}}^{(12)}}{d \ln M} = \alpha_s(\mu) \left[ \sigma^{(1)} \otimes \frac{dG(M)}{d \ln M} + \alpha_s(\mu) \frac{d\sigma^{(2)}(M)}{d \ln M} \otimes D(M) + \alpha_s(\mu)\sigma^{(2)}(M) \frac{dD(M)}{d \ln M} \right]
\]

\[
\frac{dG(M)}{d \ln M} \propto \alpha \alpha_s, \quad \frac{d\sigma^{(2)}(M)}{d \ln M} \propto \alpha, \quad \text{but} \quad \left[ \ldots \right] \propto \alpha \alpha_s
\]

only by including class B direct photon contribution which have the form

\[
\alpha_s^2(\mu)\sigma^{(2)}_{\text{dir}}(B) \propto \alpha \alpha_s^2(\mu) \ln M \Rightarrow \frac{d\alpha_s^2(\mu)\sigma^{(2)}_{\text{dir}}(B)}{d \ln M} \propto \alpha \alpha_s^2
\]

can the factorization scale invariance be guaranteed!
Phenomenological consequences

Conventional expression for the NLO approximation

$$\sigma^{\text{conv}} \equiv \sigma_{\text{dir}}^{(01)}(\mu) + \left[ \sigma_{\text{sr}}^{(12)}(M, \mu) + \sigma_{\text{dr}}^{(23)}(M, \mu) \right]$$

where first term does not mix with the sum in [..], is incomplete because

- the direct photon contribution $\sigma_{\text{dir}}^{(01)}$ is of the LO only,
- the resolved photon contribution is not factorization scale invariant.

Both shortcomings stem from the absence of direct photon contributions of the order $\alpha^2 \alpha_s^2$. These come in three classes, each of them needed for different reasons to make the expression

$$\sigma^{\text{NLO}} \equiv \sigma_{\text{dir}}^{(02)}(A; M, \mu) + \left[ \sigma_{\text{sr}}^{(12)}(M, \mu) + \sigma_{\text{dr}}^{(23)}(M, \mu) + \sigma_{\text{dir}}^{(02)}(B, M, \mu) + \sigma_{\text{dir}}^{(02)}(C, M, \mu) \right]$$

genuine NLO QCD approximation. What to do when $\sigma_{\text{dir}}^{(2)}$ is not available?

Check the stability of $\sigma^{\text{conv}}$ by

investigating its factorization scale dependence!