Motivation

Conventional formulation

Alternative:
  - Separating QED effects
  - $F_2^\gamma$ and evolution equations
  - Numerical results

Conclusions

Details in

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Motivation

Twofold:

- Clarify the meaning of the concepts “LO” and “NLO” in photon induced hard processes.
- Disentangle genuine QCD effects from those of pure QED.

My proposal builds in part on arguments advocated for a long time by J. Field and F. Kapusta and agrees with the approach to calculations of direct photon production at HERA pursued by M. Krawczyk.
Notation and basic formulae

\[
\frac{1}{x} F_2^\gamma(x, Q^2) = q_{NS}(M) \otimes C_q(Q/M) + \frac{\alpha}{2\pi} \delta_{NS} C_\gamma + \\
\langle e^2 \rangle \Sigma(M) \otimes C_q(Q/M) + \frac{\alpha}{2\pi} \langle e^2 \rangle \delta_{\Sigma} C_\gamma + \\
\langle e^2 \rangle G(M) \otimes C_G(Q/M)
\]

where PDF of the photon satisfy the evolution equations

\[
\frac{d\Sigma(x, M)}{d \ln M^2} = \delta_{\Sigma} k_q + P_{qq} \otimes \Sigma + P_{qG} \otimes G ,
\]
\[
\frac{dG(x, M)}{d \ln M^2} = k_G + P_{Gq} \otimes \Sigma + P_{GG} \otimes G ,
\]
\[
\frac{dq_{NS}(x, M)}{d \ln M^2} = \delta_{NS} k_q + P_{NS} \otimes q_{NS} ,
\]

with quark nonsinglet and singlets defined as

\[
\Sigma(x, M) \equiv \sum_{i=1}^{n_f} q_i^+(x, M) \equiv \sum_{i=1}^{n_f} [q_i(x, M) + \bar{q}_i(x, M)] ,
\]
\[
q_{NS}(x, M) \equiv \sum_{i=1}^{n_f} \left( e_i^2 - \langle e^2 \rangle \right) (q_i(x, M) + \bar{q}_i(x, M)) ,
\]

\[
\delta_{NS} = 6n_f \langle e^4 \rangle - \langle e^2 \rangle^2 , \quad \delta_{\Sigma} = 6n_f \langle e^2 \rangle .
\]

PDF separated into **hadronic** and **pointlike** parts

\[
D(x, M) = D^{PL}(x, M) + D^{HAD}(x, M).
\]

both of which contain **QCD effects**.
\[ k_q = \frac{\alpha}{2\pi} \left[ k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi} k_q^{(1)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 k_q^{(2)}(x) + \cdots \right], \]

\[ k_G = \frac{\alpha}{2\pi} \left[ \frac{\alpha_s(M)}{2\pi} k_G^{(1)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 k_G^{(2)}(x) + \cdots \right], \]

\[ P_{ij} = \frac{\alpha_s(M)}{2\pi} P_{ij}^{(0)}(x) + \left( \frac{\alpha_s(M)}{2\pi} \right)^2 P_{ij}^{(1)}(x) + \cdots, \]

where \( k_q^{(0)}(x) = (x^2 + (1-x)^2) \) and

\[ C_q(x, Q/M) = \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi} C_q^{(1)}(x, Q/M) + \cdots, \]

\[ C_G(x, Q/M) = \frac{\alpha_s(\mu)}{2\pi} C_G^{(1)}(x, Q/M) + \cdots, \]

\[ C_\gamma(x, Q/M) = C_\gamma^{(0)}(x, Q/M) + \frac{\alpha_s(\mu)}{2\pi} C_\gamma^{(1)}(x, Q/M) + \cdots, \]

\[ C_\gamma^{(0)}(x, Q/M) = (x^2 + (1-x)^2) \left[ \ln \frac{M^2}{Q^2} + \ln \frac{1-x}{x} \right] + 8x(1-x) - 1 \]

Basic question: where to truncate these expansions?
Conventional formulation: nonsinglet channel at the LO

The light quark contribution to the PL part of $F^\gamma_{NS}$

$$\frac{1}{x} F^\gamma_{NS}(x, Q^2) = \delta_{NS} \left[ q(M) \otimes C_q(Q/M) + \frac{\alpha}{2\pi} C_\gamma(Q/M) \right] =$$

$$\delta_{NS} \left[ q(M) + \frac{\alpha_s}{2\pi} q(M) \otimes C_q^{(1)}(Q/M) + \frac{\alpha}{2\pi} C_\gamma^{(0)}(Q/M) \right.$$

$$\left. + \frac{\alpha_s}{2\pi} \frac{\alpha}{2\pi} C_\gamma^{(1)}(Q/M) \cdots \right]$$

where $q \equiv u/3e_u^2 = d/3e_d^2 = s/3e_s^2$.

The conventional approach is based on two assumptions

- $F^\gamma_{NS}$ expressed (dropping $\delta_{NS}$) in terms of $q$ as $F^p_{NS}$:

$$F^\gamma_{NS,LO}(x, Q^2) = q_{LO}(x, M)$$

- $q_{LO}$ satisfies the evolution equation with r.h.s. including $k_q^{(0)}$ and $P_{qq}^{(0)}$ only.

Note: the pure QED quantity $C_\gamma^{(0)}$ is assigned to NLO!

Consistency with evolution eqs. and factorization scale independence of $F^\gamma_{NS}$ requires that

$$q(x, M^2) = O(\alpha/\alpha_s)$$

because only then

$$\alpha_s(M) \left( q \otimes C_q^{(1)} \right) \approx \alpha C_\gamma^{(0)} = O(\alpha)$$

is of the “next-to-leading” order with respect to $q$!
Seemingly this is also suggested by the explicit form of the PL solutions:

\[
q_{\text{PL}}^{\text{NS}}(n, M, M_0) = \frac{4\pi}{\alpha_s(M)} \left[ 1 - \left( \frac{\alpha_s(M)}{\alpha_s(M_0)} \right)^{1-2P_{qq}^{(0)}(n)/\beta_0} \right] a_{\text{NS}}(n)
\]

where

\[
a_{\text{NS}}(n) \equiv \frac{\alpha}{2\pi \beta_0} \frac{k_{\text{NS}}^{(0)}(n)}{1 - 2P_{qq}^{(0)}(n)/\beta_0}
\]

All PL solutions share the same large \( M \) behavior

\[
q_{\text{PL}}^{\text{NS}}(x, M_0, M) \to \frac{4\pi}{\alpha_s(M)} a_{\text{NS}}(x) \equiv q_{\text{NS}}^{\text{AP}}(x, M) \propto \ln \frac{M^2}{\Lambda^2}
\]

defining the asymptotic pointlike solution \( q_{\text{NS}}^{\text{AP}} \).

**BUT:** the fact that \( \alpha_s(M) \) appears in the denominator of \( q_{\text{NS}}^{\text{AP}} \) cannot be interpreted as evidence that

\[
q(x, M) = \mathcal{O}(\alpha/\alpha_s)
\]

because provided \( M_0 \) is kept fixed when \( \alpha_s \to 0 \)

\[
q_{\text{NS}}^{\text{PL}}(x, M, M_0) \to \frac{\alpha}{2\pi} k_{\text{NS}}^{(0)}(x) \ln \frac{M^2}{M_0^2}
\]

corresponding to purely QED splitting \( \gamma \to q\bar{q} \).
Alternative formulation – the NS channel

Based on two related ingredients:

- **Separation of purely QED** effects, which actually **dominate** scaling violations of $F_{\text{NS}}^\gamma(x, Q^2)$, in particular its $\ln Q^2$ rise, from **genuine QCD** ones. To identify the latter one has to look for subtler effects, like the *$x$-dependence* of the slope

$$a(x) \equiv \frac{dF_{\text{NS}}^\gamma(x, Q^2)}{d \ln Q^2}$$

or low $x$ behaviour of $F_2^\gamma(x, Q^2)$.

- **Proper treatment of $\alpha_s$ dependence of PDF** in perturbation theory, i.e. as $\alpha_s \to 0$:

$$q(x, M), G(x, M) \propto (\alpha \ln M^2) = \mathcal{O}(\alpha)$$

rather than

$$q(x, M), G(x, M) \propto (\alpha/\alpha_s) = \mathcal{O}(\alpha/\alpha_s)$$

as in the conventional approach (recall my dispute with A. Vogt at PHOTON'99)

The point is simple:

$$\ln M^2 \neq \frac{1}{\alpha_s}$$

as the log comes from pure QED!
Vast difference in precision and scope of data on $F^\gamma_2$ and $F^p_2$

Similar plots for $F^p_2$ ⇒ (show Figs.)
Alternative approach in NS channel: definition of the LO

Define first the QED contribution to $F_{NS}^{\gamma}$

$$F_{NS,QED}^{\gamma}(Q^2) = q_{QED}(M) + \frac{\alpha}{2\pi} C_{x,\gamma}^{(0)}(Q/M)$$

$$q_{QED}(M) = \frac{\alpha}{2\pi} k_q^{(0)} \ln \frac{M^2}{M_0^2}$$

The pointlike part of quark distribution function

$$q_{PL}(M) = q_{QED}(M) + q_{QCD}(M)$$

satisfies evolution equation with $k_q^{(0)}$, $P_q^{(0)}$ and $k_q^{(1)}$.

Rewrite $F(Q^2)$ as the sum of its QED and QCD parts

$$F_{NS}^{\gamma}(Q^2) = q_{QED} + \frac{\alpha}{2\pi} C_{\gamma}^{(0)} +$$

$$\underbrace{\text{A}_0; \text{pure QED}}_{q_{QCD} + \frac{\alpha_s}{2\pi} C_q^{(1)} q_{QED} + \frac{\alpha}{2\pi} \frac{\alpha_s}{2\pi} C_{\gamma}^{(1)} +}$$

$$\equiv A_1, \text{starting as } O(\alpha \alpha_s)$$

$$\underbrace{\frac{\alpha_s}{2\pi} C_q^{(1)} q_{QCD} + \frac{\alpha}{2\pi} \left( \frac{\alpha_s}{2\pi} \right)^2 C_{\gamma}^{(2)} + \left( \frac{\alpha_s}{2\pi} \right)^2 C_{q}^{(2)} q_{QED}}_{\equiv A_2, \text{starting as } O(\alpha^2 \alpha_s^2)}$$

The LO QCD correction to $F_{NS}^{\gamma}(Q^2)$ is identified with

$$F_{NS,LO}^{\gamma}(Q^2) = q_{QCD} + \frac{\alpha_s}{2\pi} C_q^{(1)} q_{QED} + \frac{\alpha}{2\pi} \frac{\alpha_s}{2\pi} C_{\gamma}^{(1)}$$

whereas in the conventional approach

$$F_{NS,LO}^{\gamma}(Q^2) = q_{LO}$$
The difference concerns *semantics* (the conventional approach includes also the QED part) as well as *substance*. To see the latter construct the sum

\[
F_{\gamma,\text{QED}}^{\gamma} + F_{\gamma,\text{LO}}^{\gamma} = q_{\text{QED}} + q_{\text{QCD}} + \frac{\alpha}{2\pi} C_{\gamma}^{(0)} + \frac{\alpha_s}{2\pi} C_{q}^{(1)} q_{\text{QED}} + \frac{\alpha}{2\pi} \frac{\alpha_s}{2\pi} C_{\gamma}^{(1)}
\]

which differs from that of the conventional approach

- by the absence of photonic c. f. \( C_{\gamma}^{(0)} \) and \( C_{\gamma}^{(1)} \),
- by the absence of the convolution \( q_{\text{QED}} \otimes C_{q}^{(1)} \)
- by the fact \( k_{q}^{(1)} \) is included in the evolution equation for \( q(M) \).

These differences are important, but as all quantities are known, there is no obstacle to performing LO QCD analysis in the alternative approach. On the other hand, the NLO QCD analysis requires so far uncalculated quantities.

Note: within the conventional approach \( C_{\gamma}^{(0)} \) and \( C_{q}^{(1)} \) enter the NLO expression

\[
\frac{1}{x} F_{\gamma,\text{NLO}}^{\gamma} = q + \frac{\alpha_s}{2\pi} C_{q}^{(1)} q + \frac{\alpha}{2\pi} C_{\gamma}^{(0)} \tag{1}
\]

but \( C_{\gamma}^{(1)} \), though known, is not used even at NLO!
Numerical results

Calculations proceed in four stages:

- solve analytically evolution equations in momentum space taking into account $k_q^{(0)}$, $k_1^{(1)}$, $P_{qq}^{(0)}$
- Convert the results into the $x$-space using numerical inverse Mellin transformation
- perform in $x$-space the convolution $q \otimes C_1^{(1)}$
- add in $x$-space the contributions of $C_\gamma^{(0)}$ and $C_\gamma^{(1)}$
Comparison of individual contributions as well as of the full expressions for $F_{NS}^\gamma$ in the two approaches reveals \textbf{phenomenological importance} of terms proportional to $C_{\gamma}^{(1)}$ and $k_q^{(1)}$, both of them absent in the conventional one.
Conclusions and outlook

1. The proposed approach to QCD analysis of $F_2^\gamma$ differs substantially from the conventional one. It satisfies factorization scale invariance in a way that does not rely on physically untenable assumption $q = O(\alpha/\alpha_s)$.

2. To be useful for phenomenological applications the proposed approach needs to be further elaborated by
   - extending it to the singlet sector
   - merging it with the hadronic contributions
   Work on this is in progress.

3. The NLO QCD analysis requires several so far unknown quantities and is thus currently impossible to perform. In view of the quality and number of experimental data on $F_2^\gamma$, this is at the moment no serious drawback.