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**RELATIVISTIC TRANSFORMATION OF TEMPERATURE AND
MOSENGEIL–OTT’S ANTINOMY**

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SHADOWS OVER THE SPEED OF LIGHT

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PHENOMENOLOGICAL MEANING OF TEMPERATURE

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AN ATTEMPT AT QUANTUM THERMAL PHYSICS

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This paper outlines an alternative exposition of the structure of quantum thermodynamics which is essentially based on Carnot's theory where fluxes of caloric are identified with negative information fluxes. It is further assumed that the thermal energy evolved by thermal processes is identical with the electromagnetic zero-point background energy evolved by the destruction of information inscribed in a structural unit (qubit). Theoretical arguments on an elementary level are accompanied by illustrative examples.

Keywords: caloric, entropy, information, quantum thermodynamics, zero-point radiation

Introduction

Motivation for this work were mainly informal discussions at two conferences [1] concerning the validity of the Second law of thermodynamics in case where the quantum nature of the system must be taken into account. As we were in the past engaged in research into practical problems requiring application of thermodynamics to quantum systems [2–6], we feel ourselves in the position to express our meaning also to the fundamental questions involved. Therefore, the main purpose of this work is to express our opinion on these questions and to present a sketch of an alternative conceptual structure of quantum thermal physics rather than to investigate a particular problem in detail. Our subject should be, of course, distinguished from that of quantum thermodynamics based on the consequent implementation of quantum-mechanical concepts into classical thermodynamics as is known from standard literature (e.g. [1, 7]).

Besides analytical mechanics and theory of electromagnetic field, it is thermodynamics that is considered to be a well-established, logically closed theory. There are even various axiomatic forms of the thermodynamics, which seem to guarantee absolute clearness of concepts involved. In spite of that we have serious difficulty in finding any book where the subject is treated in a way really clear to an ordinary student. As we are convinced, the very origin of the difficult understanding of thermodynamics is connected just with an inconvenient choice of conceptual basis more than 150 years ago. Traditionally the most obscure is an artificial concept of entropy and rather exceptional form of the 'Second law' of thermodynamics. Whereas the universal laws have mostly the form of

conservation laws, the logical structure of the Second law is quite different. Ultimately formulated, it is a law of irreparable waste of 'something' in every real physical process. This imperative negativistic and pessimistic nature of the Second law is very likely, for philosophers but also for many active researches in the field, the permanent source of dissatisfaction. That is why the criticism aimed at the Second law has the history as long as the Second law itself. Moreover, in recent decade an unprecedented number of challenges have been raised against the Second law from the position of quantum mechanics [1]. These arguments, however, are as a rule, enormously complicated with numerous approximations and neglects and consequently rather questionable.

It is a very old empirical fact that the thermal processes in the nature are submitted to certain restrictions strongly limiting the class of possible processes. The exact and sufficiently general formulation of these restrictions is extremely difficult and sometimes incorrect (cf. e.g. the principle of antiperistasis [8], Braun-le Chatelier's principle [9] and Second law) but in spite of it very useful. That is why the authors of this paper believe that the Second law (or another law which puts analogous limitations on thermal processes) does reflect experimental facts with an appreciable accuracy and thus it should be incorporated into the formalism of thermodynamics. On the other side, being aware of the fact that the contemporary structure of thermodynamics with its rigid conceptual basis may have intrinsic flaws, we claim that the absolute status of the Second law should not be criticized or denied from the point of view of another physical theory (e.g. quantum mechanics) prior the correction of these imperfections has been made.

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Reformulation of fundamental laws of thermodynamics

A serious flaw in the conceptual basis of classical thermodynamics concerns even the so-called First law of thermodynamics. The first step toward this law was made by Benjamin Count of Rumford by the generalisation of his observations made at an arsenal in Munich (1789) [10]. Accordingly, practically unlimited quantity of heat was possible to produce only by mechanical action i.e. by boring of cannon barrels by a blunt tool and this experimental fact was by Rumford analysed as follows: 'It is hardly necessary to add, that any thing which any insulated body, or system of bodies, can continue to furnish without limitations, cannot possibly be a material substance: and it appears to me extremely difficult, if not quite impossible, to form any distinct idea of anything, capable of being excited and communicated, in the manner the heat was excited and communicated in these experiments, except it be motion'. The same idea that heat absorbed by a body, which is particularly responsible e. g. for the increase of its temperature, is identical with the kinetic energy of its invisible components was further apparently supported by arguments due to J. P. Joule [11]. Results of his ingenious and marvelously accurate experiments have been summarized into two points: The quantity of heat produced by the friction of bodies, whether solid or liquid is always proportional to the quantity of force expended. The quantity of heat capable of increasing the temperature of a pound of water by 1° Fahrenheit requires for its evolution expenditure of a mechanical force represented by the fall of 772 lbs. through the space of one foot (here the term 'force' has evidently meaning of energy). In spite of clearness of these correct statements, Joule did not stressed out explicitly the fact that in his experiment we have to do only with one-way transformation of work into the heat. Instead he tacitly treated throughout the paper the heat as it were a physical entity fully equivalent or identical with mechanical energy. It was probably due either to influence of Rumford or to the reasoning that in the experiment heat appears just when mechanical work disappears and ipso facto these two entities must be identical. Such an extremely suggestive but incorrect idea was later canonized by Clausius [12] who proclaims an object of thermodynamics to be 'die Art der Bewegung, die wir Wärme nennen' i.e. the kind of motion we call heat.

In the history of thermodynamics objections appeared against such an energetic interpretation of the heat. Unfortunately, these objections were only rare and with no adequate response. One of them is due e.g. to Mach [13]. Accordingly, it is quite easy to realize device of Joule's type where a given amount of energy W is

completely dissipated and simultaneously the heat in amount $Q=JW$ is evolved, where J is universal Joule's proportionality factor. On the other side, as far as it is known, there is no single real case where the same amount of heat Q is transformed back into mechanical work $W=Q/J$ only by reversion the original process. Taking into account this circumstance together with the very generic property of the energy, which can be principally converted into another form of energy without any limitation, we must exclude the logical possibility that the heat is energy at all. Of course, postulating the equivalence of energy and heat a meaningful mathematical theory of thermal processes can be and actually has been established. The price paid for the equivalence principle is, however, rather high. In order to make the theory consistent it was necessary to create somewhat artificial and highly abstract quantities like entropy, enthalpy, free energy, and various thermodynamic potentials the meaning of which is more formal than physical. The mathematical manipulations with their ~ 720 derivatives and differentials (which are sometimes total) [14] actually do provide results the interpretation of which is, however, rather matter of art than of science.

Heat as an Entropy-Caloric

Astonishingly an elegant way leading out from these problems was very likely for the first time suggested by Callendar [15] and later in more sophisticated form worked out by Job in his impressive book [16]. The main idea is that the heat in common sense (e.g. as a cause of elevation of temperature of bodies exposed to the heating) should not be identified with a kind of energy but with the entropy, which is known from classical thermodynamics. It was shown by Larmor [17] and especially by Lynn [18] in a very pregnant way that the heat could be measured in energy and entropy units as well. In the latter case the heat-entropy concept attains the content identical with the concept of Carnot's 'caloric' ζ [19], whereas the empirical temperature θ , i.e. 'hotness' [13], automatically starts to play the role of its potential. (We are using for caloric Greek final letter ζ as this letter involves both, usual S for entropy and C for caloric.) For the increase of potential energy $d\varepsilon$ of the amount of caloric ζ due to the increase of temperature by $d\theta$ we may, namely, write:

$$d\varepsilon = \zeta F'(\theta) d\theta \quad (1)$$

where $F'(\theta)$ is so-called Carnot's function. It is an experimental fact that this function can be reduced to the universal constant = 1 using instead of arbitrary empirical temperature scale θ the ideal gas temperature scale T equivalent to the absolute Kelvin scale [20, 21]. (Notice that Carnot's function in (1) corresponds to the situation

where the heat is measured in entropy and not in energy units [22]). In this case for the potential energy ε corresponding to the given amount of caloric ζ kept at the temperature T we can write:

$$\varepsilon = \zeta T \quad (2)$$

The perfect analogy with other potentials known from physics, such as gravitational and electrostatic potentials, is then evident. After the terminological substitution of heat-energy by heat-entropy it is only a technical problem to reformulate two fundamental laws in a manner which is common in classical axiomatic thermodynamics [23], namely:

I.) Energy is conserved in any real thermal process

II.) Caloric (heat) cannot be annihilated in any real thermal process

Notice that the first and second law, formulated in such a way are conceptually disjunctive because caloric has nothing to do with energy. The possible link between these laws and quantities, however, provides formula (2). We shall not discuss here application of theorem II) to particular cases known from empirical observations of real processes (it is already done e.g. in [16]) but, instead, we proceed further making use of an important relation existing between entropy and information. It was recognized by Szilárd in his pioneering work [24] that in a certain thermal process the exchange of information must play an essential role. Afterwards, the establishment of fundamentals of information theory [25] enabled Brillouin to reformulate this idea with an appreciable mathematical rigor [26]. (For more recent review on the information-entropy relation, see e.g. [27].) Accordingly, the information Y has a character of negative entropy (i.e. we are allowed to write $Y = -\zeta$) and therefore, in our old-new provisional terminology, we can identify the production of caloric with the destruction of information and the flux of caloric with the information flux in an opposite direction. Theorem II) can thus be reformulated in terms of information as:

II*) Information (Y) is destroyed in any real thermal process

Veracity of this theorem seems to be very obvious at first glance. Indeed, almost everybody has experience that by combustion of newspapers in a stove or petrol in a car engine these materials are lost forever, together with the information involved. On the other hand, it is little convincing that such a 'tiny thing' as the information is, can really be able to control natural thermal processes. Isn't it more likely that statement II*) concerns only side effects taking place in certain cases? We do not think so and we assume that the validity of postulate II*) is quite general and apt for substitution of the Second law of thermodynamics. Moreover, besides the properties of the caloric al-

ready discussed, just the existence of the direct link between caloric (i.e. heat) and information is the very reason for which we prefer to use for the quantum description of thermal processes rather the conceptual basis of caloric theory than that of classical thermodynamics. In conclusion of this paragraph and as illustration of such an approach, let us paraphrase Rumford's original analysis of his experiments cited above by simply writing there instead of the word 'motion' the phrase 'perished information'.

Quantum nature of information bound to caloric

In order to involve the information into the physical reasoning it is convenient to convert information coded, as usual, in binary units Y_2 (bits) into the information Y_p expressed in physical units. This relation obviously reads:

$$Y_p = (k \ln 2) Y_2 \quad (3)$$

where k is Boltzmann's constant ($k = 1.38 \cdot 10^{-23} \text{ J K}^{-1}$). It should be stressed here that by choosing Boltzmann's constant as a conversion factor simultaneously the absolute Kelvin scale was chosen for temperature measurements.

We assume now that there is no information 'an sich' or in other words information needs in all cases a material carrier. From the point of view of macroscopic thermal physics there is, however, fundamental difference between e.g. genetic information inscribed in the DNA and information provided by a gravestone inscribed with personal data. Whereas in the former case for coding of information structural units on molecular level are used, which should be described by microscopic many-body formalism, to the later case rather a macroscopic description in terms of boundary-value problem is adequate. To distinguish without ambiguity between these two extreme cases we need, however, a criterion which, having a sign of universality specifies what the 'molecular level is'. As far as we know, a good candidate for such a criterion is modified Sommerfeld's condition distinguishing between classical and quantum effects [28, 29]. It reads:

$$\Omega \leq 2\pi\hbar \quad (4)$$

where Ω is phase space occupied by a structural unit ('qubit') where minimally 1 bit information is stored and \hbar is the Planck's universal constant ($\hbar = 1.05 \cdot 10^{-34} \text{ Js}$). Direct computation of the action Ω corresponding to one atom built in an ordinary crystal, liquid or gas confirms the validity of condition (4) in these cases. It proves the fact that every atom together with its nearest neighbourhood should be treated as a quantum structural unit responsible for in-

formation storage on a ‘molecular level’. Generalizing this result, we can conclude that the very nature of Carnot’s caloric is the destructed information originally coded in occupied quantum states of structural units of which the macroscopic system under investigation consists.

Inexhaustible source of energy for thermal processes

The mechanism of information transfer through the macroscopic system is assumed to be due to erasing information in one particular structural unit which is influenced by the neighbouring one in the field of long range forces defined by macroscopic system as a whole. There are, however, limitations of such a process. First, as the information storage in both neighbouring structural units is submitted to the same condition (4) it is impossible to exchange more information from one unit to another than ~ 1 bit per $2\pi\hbar$ of the occupied phase space. Second, the exchange of information must be in agreement with boundary conditions put on the macroscopic system as a whole, which are locally realized e.g. by long range forces. It may thus happen that the transfer of some information from one unit to the neighbouring unit is incompatible with these external conditions and information is lost. The loss of information physically means that some characteristic pattern of structural unit has disappeared and a wider class of quantum states becomes accessible. In the frame of the presented model any loss of information should be accompanied with the development of energy. How to explain where the energy comes from?

We are inclined to interpret the stability of quantum objects as a result of existence of zero-point electromagnetic vacuum fluctuations exactly compensating energy loses due to the recoil radiation from this object. Such an approach well known from stochastic and quantum electrodynamics [30, 31], confines our considerations to the systems controlled only by electromagnetic interactions, namely, low temperature plasma, gases, condensed matter and chemical reactions in these systems. Accordingly, the cohesion energy of any such a system is nothing but the energy of electromagnetic modes of the background zero-point radiation accommodated in such a way that they fit the geometry of the system. Characterizing the dimensions of the quantum electromagnetic system (for which the universal constants \hbar and c must be taken into consideration) by a single length parameter a , we immediately obtain for cohesion energy a formula of Casimir’s type by applying dimensional analysis [32]:

$$\varepsilon \approx \gamma(\hbar c / a) \quad (5)$$

where the dimensionless parameter γ should be determined from a particular geometry of the system (usually γ ranges from 0.1–0.001 [30]). The change of dimension a or complete destruction of a structural unit with energy (5) during thermal process has a consequence that just this amount of energy is developed at the place. As this energy is in fact a modified energy of all-pervasive universal zero-point background, we have to do with an energy supply from practically inexhaustible non-local source of energy. Therefore, within the frame of stochastic electrodynamics every thermodynamic quantum system should be interpreted as an open system even in the case where it is finite.

Examples

In order to make the presented system of quantum thermodynamics more intelligible we have given three examples illustrating how should be some common observations within the frame of this system interpreted.

1) How does the heat engine work? Heat engine in the sense of original Carnot’s theory is nothing but a kind of mill driven by caloric ζ falling from a higher potential T_1 (boiler) to a lower potential T_2 (cooler). Information thus flows from the cooler with condensed water (better ordered than steam) to the cylinder of engine where the information is destroyed (by weakening of correlations among molecules during the expansion) giving rise to useful work originating in zero-point background. Then the residual information continues to flow to the heater where it is dissolved during ordering of configuration of the steam. Notice that the flow of information and the flow of water are just opposite in this case and that the question how the boiler is heated is put aside. In a typical combustion engine at low temperature the fuel with high information content flows into the cylinder of engine. During the combustion of fuel the information which is coded in its structure is destroyed and the useful work from the zero-point quantum electromagnetic energy is produced there. The information, however, flows inside the combustion space for this type of engine also through the exhaust-pipe so that special attention must be paid to this part.

2) There is an interesting device called Bunsen’s ice calorimeter. As this apparatus works at a well defined temperature T_M (i.e. melting temperature of ice =273 K) it, in fact, according to equation (2), measures directly inputted caloric and may thus serve as ‘entropy-meter’. Indeed, the information destroyed and the latent energy of melting is connected here in an especially obvious way. An estimate of the latent energy ε_M per one mol of ice can be obtained as follows. We make a use of an important fact that the H₂O

molecules retain their integrity in both water and ice, simultaneously neglecting the effect of clustering of molecules at temperatures well above the melting point which is responsible for non-trivial macroscopic behaviour of water [33, 34]. Within the frame of such a simplified model it would be then necessary for the melting of ice to break down 4 well-oriented bonds per every water molecule and substitute them by quasi-continuum of states. Such a transformation corresponds approximately to the destruction of $\Upsilon_2=4$ bits of information per molecule [27]. Taking into account equations (2) and (3) we can thus write:

$$\varepsilon_M = (Nk \ln 2)T_M \Upsilon_2 \quad (6)$$

where N is Avogadro's constant ($N=6.02 \cdot 10^{23} \text{ mol}^{-1}$). The estimate of ε_M then reads $\approx 6288 \text{ J mol}^{-1}$ in an excellent agreement with the experimental value $=6007 \text{ J mol}^{-1}$.

3) There are different microscopic parameters characterizing the configuration of a structural unit where the information is stored which can be in principle constructed from quantum numbers describing this system. The relation connecting these microscopic parameters and macroscopic boundary conditions is evidently very complicated. If we, however, as above, confine ourselves only to a single parameter a – characteristic dimension of the structural unit, this relation can be found in an explicit form and compared directly with experimental data. Combining formulae (2) and (3) the temperature change of the potential energy of a structural unit which is due to the erasing of information Υ_2 from it is given by:

$$d\varepsilon / dT \approx -(k \ln 2)\Upsilon_2 \quad (7)$$

Substituting for ε the Casimir's quantum cohesion energy (5) we immediately obtain an estimate for the corresponding relative expansion of the unit:

$$d \ln a / dT \approx a(k \ln 2 / \gamma \hbar c)\Upsilon_2 \quad (8)$$

Assuming that the thermal process is homogeneous and isotropic, this coefficient must be within the order of magnitude identical with the expansion coefficient macroscopically observed. For typical condensed matter where bond length $a \approx 4 \cdot 10^{-10} \text{ m}$ and $\Upsilon_2=1$ we obtain from (8) for coefficient of relative thermal expansion a value of $1.2 \cdot 10^{-7} / \gamma$ which is near to the values experimentally observed (typically $\approx 10^{-5}$), provided that $\gamma \approx 0.01$.

Conclusions

In conclusion, using elementary arguments without mathematical rigor, changes in the conceptual basis and in the structure of quantum thermodynamics have

been suggested. The resulting theory is based essentially on the following points:

- Modified form of Carnot's theory where caloric is identified with the entropy.
- Equivalence of information and negative entropy.
- Interpretation of stability of quantum objects as the consequence of the existence of electromagnetic zero-point vacuum radiation.

The authors are aware that the structure of quantum thermodynamics as sketched out in this paper is far from to be mature, however, they are simultaneously convinced that it is potentially apt to reflect the empirical facts in a more intelligible way than the present theories.

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Relativistic transformation of temperature and Mosengeil–Ott’s antinomy

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ABSTRACT

A not satisfactorily solved problem of relativistic transformation of temperature playing the decisive role in relativistic thermal physics and cosmology is reopened. It is shown that the origin of the so-called Mosengeil–Ott’s antinomy and other aligned paradoxes are related to the wrong understanding of the physical meaning of temperature and application of Planck’s Ansatz of Lorentz’s invariance of entropy. In the contribution, we have thus reintroduced and anew analyzed fundamental concepts of hotness manifold, fixed thermometric points and temperature. Finally, on the basis of phenomenological arguments the Lorentz invariance of temperature and relativistic transformations of entropy are established.

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1. Introduction

There is a non-trivial problem connected with the relativistic transformation of temperature which can be, however, reduced to a simple question: is the body moving with the velocity v relatively to the rest system of coordinates in its own coordinate system colder or hotter than if it were measured in the rest system of coordinates? Very early after the appearance of the special theory of relativity in 1905 the problem was solved by a pupil of von Laue, von Mosengeil, who provided the following result [1]:

$$T = T_0 \sqrt{1 - v^2/c^2}, \quad (\text{Mosengeil, 1907}) \quad (1)$$

where T_0 is the Kelvin temperature as measured in the rest system of coordinates and T the corresponding temperature detected in the moving system. The satisfaction with this formula which is up to now serving as a standard in textbooks on special theory of relativity was put in doubts by a challenging paper due to Ott [2] where just an inverse formula for the relativistic transformation of temperature is derived, namely

$$T = T_0 / \sqrt{1 - v^2/c^2}. \quad (\text{Ott, 1963}) \quad (2)$$

After the appearance of Ott’s paper a vivid discussion in “Nature” [3–8] broke out which, however, stopped after some time without bringing any clear decision which of these two formulae is true [9,10]. At the end of the 20th century, during the last wave of interest in the problem [11,12], however, another

opinion appeared, namely that the temperature must be Lorentz invariant [13].

As we believe for the solution of this fundamental problem of relativistic thermal physics, it is inevitable to first make it clear what the temperature actually is and what it is not. Therefore, before suggesting our solution of this interesting puzzle, we are critically revising the definition of fundamental concept of phenomenological thermal physics, i.e. that of the temperature.

2. The concept of temperature

Astonishingly, a satisfying definition of the phenomenological physical quantity called temperature is lacking in the literature. There exist, of course, definitions in terms of statistical physics, which are, however, related to the phenomenological quantity measured by a macroscopic device, thermometer, only by rather superficial considerations. Taking further into account that practically all actual temperature measurements are performed by means of thermometers and not by statistical analysis of bodies, treated as statistical ensembles of elementary particles and excitations, an urgent need for a good phenomenological definition of temperature is thus evident.

It should be stressed here that the temperature is not a primary concept but, as was for the first time with sufficient plausibility shown by Mach [14], that the nearest experimentally accessible structure behind is so-called **hotness manifold** (“Mannigfaltigkeit der Wärmestände”) the elements of which are **hotness levels** which are one-to-one related with experimentally observable **thermoscopic states**. As was shown in our recent paper [15], the existence of such a set of thermoscopic states may be proved using an operational definition specifying the properties of a measuring device (thermoscope) together with the

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measurement conditions. The most salient among these conditions is that of **thermal equilibrium** [16] that is enabled by a **thermal (diathermic) contact** of the thermoscope with a body under investigation and the existence of which may be checked independently by so-called **correlation test**.¹ It is an important consequence of the introduction of thermoscopes that we can experimentally observe the topological order of hotness levels as an order of quite another real physical quantity called **thermoscopic variable** (e.g. length of mercury thread, resistance and thermoelectric voltage). As these thermoscopic variables are real continuous quantities defined in certain closed intervals, this property is transferred by the said one-to-one relation also into the hotness manifold. The sewing-up of the different sets of thermoscopic states corresponding to the different thermoscopic variables, however, was shown not to be possible without exploitation of another physical entity, **fixed (thermometric) points**. The fixed point is a body prepared according to a definite prescription for which it is known that the thermoscope in diathermic contact with this body indicates reproducibly the well-defined thermoscopic state. The fixed points such as boiling point of helium, melting point of water, boiling point of water, and melting point of platinum (all at normal atmospheric pressure) are examples well-known from practical thermometry [17]. Because of the correspondence between the fixed points and some of thermoscopic states, the set of fixed points is ordered and, because fixed points are real bodies, countable. Furthermore, an important empirical fact that the limitations put on the construction of the fixed points are not known² makes plausible the assumption that for each fixed point there is another “hotter” or another “colder” one and that there is an inter-lying fixed point between any two fixed points, sounds quite reasonably. Just such a property enables one not only to sew-up together the different overlapping parts of the hotness manifold corresponding to various thermoscopic variables but simultaneously provides a dense countable subset in the hotness manifold ensuring that this set is linear [18], and thus fully equivalent to the set of the real numbers (real axis).

Now we are ready to approach to the following definition: **temperature is any continuous one-to-one order-preserving mapping of hotness manifold on a simply connected subset of real numbers**. It is evident from this definition that we have an enormous liberty to choose a particular temperature scale which thus rests entirely upon a convention. Only the hotness formally represented by hotness manifold has a right to be regarded as an entity existing in the Nature (cf. [14]). From this point of view our question, i.e. how the temperature, which is in fact a result of some arbitrary mapping, behaves by relativistic transformations, sounds as an ill-defined assignment. Indeed, the solution of the problem will depend on the particular way how the temperature is constructed and not only on the objective constraints. Moreover, it is quite clear that the rationality or irrationality of choice of the temperature scale will be decisive for further performance and

intelligibility of theory of thermal effects. Therefore, for the solution of our problem a detailed analysis of temperature concept in use is quite essential.

On the more-or-less historical and practical grounds, in other words, on the basis of fully arbitrary anthropomorphic criteria [15], a special mapping of the hotness manifold was chosen on an ordered subset of real numbers called the **ideal (perfect) gas scale**. The equation controlling the behavior of the ideal gas, which is a hypothetical substance or concept rather than a real thing, reads

$$pV = nRT, \quad (3)$$

where p and V are, respectively, the pressure and the volume of the ideal gas which alternatively play the role of thermoscopic variables. As the hypothetical thermoscope is considered a conventional gas thermometer [19] filled with n moles, $n > 0$, of ideal gas. The constant R on the right side of Eq. (3), which need not be Lorentz invariant, has then a form of product $R = kN$ where k and N are Boltzmann's and Avogadro's constants, respectively. The ideal gas temperature scale T defined by means of Eq. (3) has some remarkable properties. For example, as both quantities p and V have a natural lower bound $= 0$, the temperature T has also this lower bound and thus belongs to the class of **absolute temperatures** [20] for which one assumes that always $T > 0$. Notice that the possible value $T = 0$ is already excluded by our definition of temperature, because due to the absence of the lowest hotness level in the hotness manifold any continuous one-to-one order-preserving transformation has inevitably to map its improper point (i.e. $-\infty$) just on the point corresponding to absolute zero. **Nernst's law** of unattainability of absolute zero of temperature [21] is thus together with its consequences intrinsically involved in this definition of temperature.

3. Lorentz transformation of temperature

Already having at our disposal the prescription defining the temperature in mathematical terms, it is in principle possible to perform the Lorentz transformation of the left-hand side of Eq. (3) and to obtain in this way the formula for the relativistic transformation of temperature. The physical reasoning behind such a computation would not be, however, very transparent and convincing because the gas thermometer is rather a difficult device. Moreover, in order to be able to also analyze other temperature measuring methods (using e.g. platinum resistance, black-body radiation and thermoelectric voltage) in a sufficient generality the methodical approach is more relevant.

Fortunately, the properties of hotness manifold enable one to make the following fairly general considerations. First of all, it is evident that in order to not to violate the principle of relativity the behavior of bodies realizing fixed thermometric points has to be the same in all inertial frames (cf. [22]). For example, it would be absurd to admit an idea that the water violently boiling in its rest system can simultaneously³ look calm if observed from another relatively moving inertial system. In other words, any fixed point has to correspond to the same hotness level regardless of the inertial frame used for the observation. Assigning, by means of some convention, to each body realizing the fixed point a certain “inventory entry”, the resulting, by pure convention established list of numbers cannot be changed by a mere transfer from one inertial system to another.

¹ A correlation test is the procedure frequently used in practical thermometry which can be in general terms described as follows. Let us have in an inertial frame two systems, A and B, separated by a macroscopically firm material partition defining their common boundary. Such a partition is called diathermic if the changes of system A induce the changes of system B and vice versa.

² The temperatures observed range from $\sim 10^{-10}$ K (Low Temperature Lab, Helsinki University of Technology) up to $\sim 10^9$ K (supernova explosion) without any traces that the ultimate limits were actually reached. Speculative upper limit provides only the so-called Planck temperature $T_P = \sqrt{(hc/G)} \times (c^2/k) \approx 1.417 \times 10^{32}$ K, hypothetically corresponding to the first instant of Big Bang and depending on the assumption that the constants involved are really universal. Therefore, the conjecture presented in this paper, i.e. that the hotness manifold has no upper or lower bound, is obviously valid at least for all phenomena already known.

³ Notice that we have to do here with the essentially time-independent stationary process, where the Lorentz transformation of time plays no role. Let us also recall that the pressure, controlling e.g. boiling point of water, may be proved independently to be Lorentz invariant [23].

For example, using, thus, as an operational rule for stocktaking of fixed points formula (3) (in SI units with $R = 8.3145 \text{ J/K mol}$) and assigning to the triple point of water an inventory entry 273.16 K, we obtain an ordered table of fiducial points of ideal gas scale (similar to the ITS [17]) which must be valid in all inertial frames. As the set of fixed points provides a dense subset (skeleton) in continuous hotness manifold, such a Lorentz-invariant table can be extended and detailed as we like and consequently, any hotness level can be, by means of this table, approximated with arbitrary accuracy. Due to the continuity of prescription (3) the whole **ideal gas (Kelvin) scale T is then inevitably Lorentz invariant**.

The invariance of Kelvin scale has, however, a very interesting and far reaching consequence. Let us make the following “Gedankenexperiment” with two identically arranged gas thermometers both filled with one mole of ideal gas which are in two relatively moving inertial systems in diathermic contact with the same fixed point bath (for definiteness, with triple point of water) placed in their own frames. The pressure in both devices must be the same, because, as can be proved quite independently the pressure is Lorentz invariant [23]. Therefore, we can write

$$p = p_0, \quad (4)$$

$$T = T_0, \quad (5)$$

where index 0 is related, as above, to the quantities measured in the a-priori chosen rest system. Taking now the well-known Lorentz transformation of volume into account, we obtain from Eq. (3) the following series of equations:

$$pV = p_0 V_0 \sqrt{(1 - v^2/c^2)} = RT = R_0 T_0 \sqrt{(1 - v^2/c^2)}, \quad (6)$$

from which a somewhat astonishing relation immediately follows:

$$R = R_0 \sqrt{(1 - v^2/c^2)}. \quad (7)$$

The physical meaning of this formula is really far reaching. Taking into account, namely, that R is an entropy unit, Eq. (7) must simultaneously represent the transformation formula for entropy in general. This is, however, in severe contradiction with **Planck's Ansatz** claiming that the entropy is Lorentz invariant. We have to recall here that this Ansatz, serving as a starting point of numerous considerations in relativistic thermodynamics, has never been proved with sufficient exactness but from the beginning it was mere an intuitive conjecture [24]. (It was namely argued that the entropy has to be invariant, because it is the logarithm of a discrete number of states which is “naturally” Lorentz invariant.) Nevertheless, admitting once the relativistic invariance of temperature, we have to reject Planck's conjecture as unsound and particularly, we can also no more treat the various units of entropy, e.g. gas constant R and Boltzmann's constant k , as universal constants.

4. Distant measurement of temperature

The very task of the special theory of relativity is to study the transformation laws connecting the experimental results of observers in different inertial systems performing the same measuring operations. Frequent types of such measurements are so-called “**distant measurements**” the aim of which is to determine the physical quantity belonging to a certain moving inertial system by means of measurements made at the rest system. The operational methods for distant measurement of e.g.

length, time, and intensity of fields are generally known. In case of temperature, however, due to its special physical nature we encounter some peculiar difficulties. The main problem, which is intuitively not quite obvious, is the principal impossibility to establish the thermal equilibrium between two relatively moving inertial systems. Namely, the relative movement of systems A and B (see footnote 1) prevents one from answering without ambiguity, on the basis of correlation test, the question whether the common boundary is diathermic or not, which makes any judgment on the thermal equilibrium quite questionable. Indeed, it is clear that the boundary between two relatively moving systems has to move at least with respect to one of them. In such a case, however, the interaction between these systems can exist even if the boundary is non-diathermic (adiabatic). For example, the moving boundary can exert a pressure on one of the systems without changing the state of the other and/or a charged system A surrounded by a metallic envelope, regardless of the fact whether it is diathermic or adiabatic, can induce dissipative equalization currents in system B without affecting the charge distribution inside system A. In order to exclude such cases, the temperature of any body must be measured only by means of a thermometer which is in the rest with respect to the body, and this operation cannot be, in principle, performed by a relatively moving observer⁴ (cf. also [25]). Hence the temperature cannot be the subject of a direct distant measurement in principle. It can only be the result of local measurement and subsequent data transfer into another inertial system. (If possible, the digital mailing of the data would be the best choice.) Of course, as the theoretical basis for the determination of temperature of moving objects Eq. (5), expressing the Lorentz invariance of temperature, has to be simultaneously taken into account. The operational rules for distant measurement of temperature may then be formulated as follows:

- (1) Bring the measured body in diathermic contact with the thermometer in their common inertial rest frame.
- (2) Reconstruct in the relatively moving inertial system the reading of the thermometer applying transformation rules relevant to the thermoscopic variable used.

Obviously, such a two-step procedure should ensure the consistent results even if different thermometers are used. If properly chosen and correctly transformed, namely, the thermoscopic variable must reproduce the same thermometer reading in any inertial system.

In order to illustrate the application of the above-formulated rules and especially to exemplify the importance of proper choice of thermoscopic variable, the temperature measurement by means of optical pyrometry has been chosen. This example is very instructive because it clearly shows, beside the general features of this technique, some of its deceptive aspects as well. Moreover, it is also closely related to the problem of Mosengeil–Ott's antinomy mentioned in the title.

Optical pyrometry is in its simplest form based on the so-called **Wien's displacement law** which may be written in terms of frequency as

$$T_0 = \hbar \omega_{M0} / 2.82 k, \quad (8)$$

where ω_{M0} is the frequency corresponding to the maximum of equilibrium distribution of black-body radiation [27]. As a thermometer serves the cavity with a small opening attached to

⁴ Interestingly enough, the similar assumption that the temperature measurement is possible just only if the thermometer is in rest with respect to the measured system is as self-evident, without any proof, used in the recent theoretical literature, see e.g. [26].

the measured body, or alternatively, simply the surface of the body itself which is assumed to be “black”. The emitted light is then in the rest system of the body analyzed by means of a spectrometer and the frequency ω_{M0} corresponding to the maximal radiation power is determined. After that the temperature T_0 of the body can be immediately determined using Eq. (8). If we investigate the radiation of black-body thermometer, being, observers in another relatively moving inertial system, the thermoscopic variable ω_M should be, according to our rule (2), transformed into original ω_{M0} . It is a well-known fact that the relativistic transformation of frequency is reduced to the multiplication by Doppler’s factor $K(v, \theta)$ i.e.

$$\omega_{M0} = \omega_M K(v, \theta), \quad (9)$$

where v is the relative velocity of the motion and θ the angle between axis of motion and the direction of observation [28]. Writing then Doppler’s factor in *scriptio plena* and substituting the resulting ω_{M0} into relation (8), we obtain a formula which is normally used for the analysis of **relict radiation** [29], namely

$$T_0 = T \{1 - (v/c) \cos \theta\} / \sqrt{1 - v^2/c^2}. \quad (10)$$

It is apparent at first glance that for measurement performed in the direction perpendicular to the relative velocity vector (i.e. for $\theta = \pi/2$) formula (10) becomes identical with Ott’s relation (2). How can be, however, this result reconciled with our assertion that the temperature is Lorentz invariant? We claim that this discrepancy is due to the improper choice of thermoscopic variable. We have, namely, tacitly made an incorrect assumption that the shape of distribution of black-body radiation is Lorentz invariant. As was, however, convincingly shown e.g. by Boyer [30] the only Lorentz invariant part of Planck’s distribution is that represented by the so-called zero-point temperature independent term $(h/2\pi^2c^3)\omega^3 d\omega$. Just in contrast, the temperature-dependent term of black-body radiation distribution observed from a moving inertial system is skewed a little bit losing, thus, the affinity to Planck’s function. Consequently, the frequencies corresponding to the maxima of radiation in the rest and moving frames are no more connected by means of Eq. (9) and cannot thus serve as a “good” thermoscopic variable. Instead, in order to determine parameters of Planck’s distribution belonging to the rest system (the temperature T_0 involved) it must be reconstructed from the complete distribution observed in the moving system. In other words, in case of optical pyrometry the role of the thermoscopic variable cannot play a single point but the distribution as a whole.

5. Conclusions

Analyzing anew the concept of phenomenological temperature, we have been able to demonstrate that the Mosengeil–Ott’s antinomy is actually an artefact, while the Kelvin temperature has to be Lorentz invariant. It has been further shown that in such a case the extensive variable conjugated with the temperature, i.e. entropy, can no more be Lorentz invariant (violation of Planck’s Ansatz). Besides, operational rules for distant measurement of temperature were formulated and applied to the practically important case of optical pyrometry.

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Shadows over the speed of light

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Abstract

In this paper, we discuss some of the consequences of the CGPM (1983) definition of meter and, in particular, we discuss giving the speed of light an exact value. It is shown that this act touches the fundamental paradigms, such as the second postulate of the special theory of relativity (STR), the c -equivalence principle and the method of time synchronization. In order to fill the arising logical gaps, we suggest, among others, to weaken the second postulate of STR to a form directly confirmed by experiments and make new measurements of Maxwell's constant with accuracy comparable with that of the speed of light.

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(Some figures may appear in color only in the online journal)

1. Introduction

Shortly after the discovery of advanced experimental techniques, such as the cavity resonance method and laser interferometry in the 1970s, enormous progress was made in increasing the accuracy of the measurement of the speed of light. Using a 1960 definition of meter in terms of a particular spectral line of krypton-86, and a newly constructed laser interferometer, a group at the National Bureau of Standards, Boulder, CO (1972, [1]), obtained for the speed of light a value $c = 299\,792\,456.2 \pm 1.1 \text{ m s}^{-1}$, which was ~ 100 times less uncertain than the values accepted previously. As similar systematic experiments conducted at that time in competing laboratories provided results of comparable or of even better accuracy, the 15th Conférence Générale des Poids et Mesures (CGPM) held in 1975 recommended the use of a value $c = 299\,792\,458 \text{ m s}^{-1}$ for the speed of light. The results of terrestrial measurements of the speed of light carried out by various techniques during the period 1907–74 are summarized in figure 1. The convergence of the measured values of c to a certain constant in the last two decades is really remarkable. Due to this very fact and because of the inadequacy of the system of units for some metrological experiments, the 17th CGPM (1983) decided also to redefine the meter [2] in the following way: 'The meter is the length of the path traveled by light in vacuum during a time interval of $1/299\,792\,458$ of a second.' The consequence of this definition is that the speed of light became an exact constant, namely

$$c = 299\,792\,458 \text{ m s}^{-1}. \quad (1)$$

Apparently, such a far reaching decision was strongly influenced by the general acceptance of the special theory of relativity (STR) according to which the speed of light is a fundamental universal constant preserving its numerical value in all inertial systems. It is an important property of this definition that it is applicable, due to the form of the Lorentz transformations of time and length, not only to the measurements of length in a rest system but also to other inertial systems. The authors of the reform thus believed that the improvement of experimental techniques will not affect the value of c , but instead it will allow us a more precise realization of the meter.

In addition to the quite evident advantages of the introduction of speed of light into physics and metrology in the form of a fundamental constant having an exact value, we also see some weak points related to such an act. A discussion of the controversial weak points summarized below is the subject of this paper:

- the kinematic origin of the magnetic force and the nature of Maxwell's constant b ,
- Maxwell's equations, c -equivalence principle,
- transformation properties of Maxwell's equations, Bessel–Hagen invariants,
- kinematics of light rays and the criticism of the second postulate of the STR and
- the CGPM (1983) definition of meter versus the G de Bray scenario.

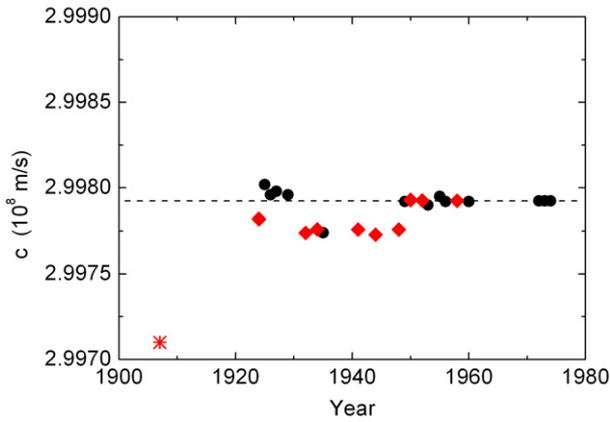


Figure 1. Results of the measurement of speed of light as performed during the period 1907–74. Symbols: ●, optical experiments using visible light; ◆, experiments using electromagnetic waves of lower frequency. The point * at 1907 corresponds to the measurement of Maxwell’s constant b by Rosa and Dorsey [3]. The CGPM (1975) value of $c = 299\,792\,458\text{ m s}^{-1}$ is depicted by a dashed line.

2. Origin of magnetic forces and Maxwell’s constant b

The electromagnetic field ‘*an sich*’ is a typical ‘as if’ entity. The very existence of the electromagnetic field apart from the ‘ponderable’ matter is questionable since its experimental investigation without the interaction with material objects is impossible. The electric and magnetic bodies thus behave ‘as if’ there would be something mediating their interactions and to be honest, nobody knows what the electromagnetic field is or what it should be. Setting aside, however, such epistemological questions, let us recall the experimental facts and theoretical assumptions that are the source of our knowledge of the electromagnetic field.

The present theory of electromagnetic field, essentially due to J C Maxwell, was developed by a generalization of three experimentally established laws, the Coulomb law, Ampère’s law (or its equivalent the Biot–Savart law) and the Faraday induction law. Only afterwards an experimentally confirmed hypothesis of ‘current of displacement’ was added to these laws. Let us for a while turn our attention to the first couple of these laws. The Coulomb law defines the electrostatic force F_E exerted in an empty space by a point charge Q_1 on another one Q_2 placed at a distance R . Both the point charges are assumed to be in rest with respect to the observer. The analytical form of the law then reads

$$F_E = -Q_1 Q_2 / (4\pi \epsilon_0 R^2). \quad (2)$$

The proportionality factor ($1/4\pi\epsilon_0$) is arbitrary in the sense that it depends exclusively on the choice of the system of units. (In our case where the SI system is used, the force is measured in newtons, the distance in meters and the charge in coulombs.) Another important feature of the Coulomb law (2), not apparent at first glance, is that it describes an instantaneous action at a distance (*actio in distans*) along the actual line connecting point charges [4]. Just the same property, i.e. the instantaneous action at a distance along the connecting lines, is given also to a magnetic force between two straight

parallel thin wires (‘filaments’) bearing currents I_1 and I_2 , respectively. Their mutual attraction per length L is controlled by the so-called Biot–Savart law, namely

$$F_M = \mu_0 I_1 I_2 L / (2\pi R), \quad (3)$$

where R is the distance between the wires. In a consistent system of units where the current units are defined in terms of charge units, the constants in formulae (2) and (3) cannot be independent. This fact together with another fundamental link between laws (2) and (3) can be elucidated by means of a simple thought experiment. Let us first rewrite the Coulomb law (2) for a system having identical geometry as that used for the formulation of the Biot–Savart law (3). The repulsion between segments of length L of two straight parallel filaments uniformly charged with densities $\gamma_1 = Q_1/L$ and $\gamma_2 = Q_2/L$ which are placed at distance R apart is given by the formula [5]

$$F_E = -\gamma_1 \gamma_2 L / (2\pi \epsilon_0 R). \quad (4)$$

Let us now imagine that the charged system starts to move with the velocity v in the direction parallel to the wires. Obviously, for an observer in the rest system the charged filaments moving relatively to velocity v represent the currents $I_1 = v\gamma_1$ and $I_2 = v\gamma_2$. Comparing thus formulae (3) and (4), we obtain for the dimensionless ratio of the magnetic and the electric force the relation

$$F_M / F_E = -\epsilon_0 \mu_0 v^2. \quad (5)$$

The immediate physical interpretation of our thought experiment and formula (5) is the following: the magnetic force is nothing but the electric force observed from a relatively moving system. Since the ratio F_M / F_E is dimensionless, it is possible to introduce a normalizing factor having physical dimension of the speed, namely

$$b = 1 / \sqrt{(\epsilon_0 \mu_0)}. \quad (6)$$

This quantity, sometimes called Maxwell’s constant, enables one to express the total force of electromagnetic origin $F_{EM} = F_E + F_M$ in a compact form

$$F_{EM} = F_E (1 - v^2 / b^2), \quad (7)$$

from which it follows that the magnetism is with respect to electrostatic interaction only a second-order effect. In accordance with formula (6), one of the quantities ϵ_0 and μ_0 can be chosen arbitrarily, while the other one or b should be determined experimentally. For example, putting in the SI system of units $\mu_0 = 4\pi \times 10^{-7}\text{ H m}^{-1}$ (exactly), the ϵ_0 or b must be determined by an independent experiment enabling a direct comparison of electric and magnetic forces acting at a distance or a comparison of the units that are related to these forces.

Until now we have not used, with full awareness, the considerations belonging to the STR. In spite of it, we obtained conclusions and formula (7) which are, with the proviso that $b \equiv c$, traditionally attributed exclusively to the STR [6]. For example, it is stressed in many textbooks that the magnetism is a purely relativistic effect operating even at

negligibly small relative velocities. Argumentation in favor of this statement is mostly based on the fact that it is possible, by means of the kinematic principle of relativity, to derive from the electrostatic Coulomb law the Ampère's law (or its equivalent the Biot–Savart law) and the Faraday induction law as well [7–9]. Nevertheless, as we have already seen above, the conclusion that the magnetism is a consequence of the relative motion of electric charges with respect to the observer must be more general, because it can be made without any reference to the postulates of the STR. Moreover, in the said relativistic derivations based on the Coulomb law its intrinsic element, the immediate *actio in distans* was tacitly used, i.e. an assumption which is absolutely at odds with the gist of the STR, which is conceptually a field theory postulating the finiteness of the speed of interactions.

The effect of non-instantaneous interaction is, however, treated also in the frame of classical pre-Maxwellian theories, e.g. in [10] or especially in excellent Riemann's posthumous paper [11] and in a more advanced and complete form in papers of Liénard and Wiechert [12, 13], appearing already at the turn of the century. These authors assumed that the electric potential or electric forces acted with a delay corresponding to the transmission of the signal over the distance R . The actual time of interaction is thus given by the relation

$$t = \tau + R/b, \quad (8)$$

where τ is the local time at the source of the electric force and b is the speed of electromagnetic interaction. The potentials generated in this way are called retarded (Liénard–Wiechert's) potentials. Accordingly, instead of the distance R the product KR must be inserted into formulae (2)–(4), where K is the correcting ‘Doppler factor’ given by [14]

$$K = \partial t / \partial \tau = 1 + (1/b) \partial R / \partial \tau = 1 - (\mathbf{v} \mathbf{R}_0) / b, \quad (9)$$

where \mathbf{v} is the velocity vector and \mathbf{R}_0 the unit vector in the direction of the considered field point in the rest system. Let us apply this rule to the Coulomb law (2) using a somewhat special assumption that the vector \mathbf{v} is parallel to the vector \mathbf{R}_0 . The Coulomb law corrected by such a special Doppler factor then reads

$$F_{EM} = - (Q_1 Q_2 / 4\pi \epsilon_0 R^2) (1 - v^2/b^2), \quad (10)$$

where the symbol F_{EM} is used for the retarded Coulomb force. Obviously, the physical content of this formula is essentially the same as that of formula (7). Interestingly enough, formula (10) is also identical to a central relationship of pre-Maxwellian Wagner's electrodynamics (written for the particular case of zero acceleration) [10]. According to this theory, b represents a speed limit at which the electric force is just compensated for by the invoked magnetic force (‘... die Constante b stellt dabei diejenige relative Geschwindigkeit vor, welche die elektrischen Massen Q_1 und Q_2 haben und behalten müssen, wenn gar nicht mehr auf einander wirken sollen’ [10]; as we are convinced, this old idea, i.e. that the electromagnetic field decouple from electric charges in the cases when its relative speed attains the value b , has an underestimated importance).

Regardless of the fact that we have derived the ‘relativistic’ formulae (7) and (10) ‘classically’ in a

very elementary way and under rather special simplifying assumptions, it became clear that the existence of the magnetic interaction cannot be treated exclusively as a relativistic effect as is usually done. In our view, a more adequate statement is that the STR accounts for the magnetism just because it belongs to a wider class of theories implicitly involving the concept of non-instantaneous interaction. Note in this connection that the experimentally established phenomenological laws (3) and (4), formulated in terms of instantaneous *actio in distans*, do not pretend to explain the magnetism but only to describe the real observations made in the rest system. Therefore, although the algebraic operations with formulae (3) and (4) lead to relationship (7), they are not directly applicable to the systems moving relatively to the rest system.

Let us now turn our attention to the properties of the electromagnetic field decoupled from ponderable matter.

3. Maxwell's equations of the electromagnetic field

The structure of a hypothetical entity, an electromagnetic field in an empty space, is controlled by reduced Maxwell's equations derived from experimentally observed laws mentioned above. In bi-vector SI notation they can be written in a marvelously symmetric form due to Silberstein [15]:

$$\text{rot } \mathbf{\Lambda} = (i/b) \partial \mathbf{\Lambda} / \partial t, \quad (11)$$

$$\text{div } \mathbf{\Lambda} = 0, \quad (12)$$

where the electromagnetic complex bi-vector is defined as

$$\mathbf{\Lambda} = \mathbf{E} + i b \mathbf{B}, \quad i = \sqrt{(-1)}. \quad (13)$$

From these equations, a number of interesting conclusions can be drawn on the basis of purely mathematical deductions. For example, it can be proved that the discontinuity in the electromagnetic field is, in the absence of material bodies, either longitudinal stationary or constitutes a transversal vortex wave propagating with the velocity b [16]. Since the theory based on equations (11)–(13) describes only the field itself (the so-called *reine elektromagnetische Wellen*, i.e. ‘pure electromagnetic waves’ by Silberstein) and not the interaction with ordinary matter, the origin of such discontinuities is outside the scope of the theory. It cannot be thus, e.g., concluded that the electromagnetic wave excited by an oscillating electric dipole spreads from the source with velocity b . In many cases, the speed of energy transfer represented by the flux of Poynting's vector is appreciably slower than b [17, 18]. The transfer speed coincides with b only in the case of ‘purely electromagnetic’ conjugate waves for which identically

$$\mathbf{E}^2 = b^2 \mathbf{B}^2. \quad (14)$$

However, condition (14) takes place practically only in the radiation zone, i.e. in the region that has sufficiently departed from all material bodies (sources, lenses, mirrors, etc). It can be stated quite generally that the speed of discontinuity of the electromagnetic field in the vicinity of ponderable matter is diminished and the speed b can be achieved only

if the electromagnetic field is sufficiently decoupled from the material objects. As we believe, this effect might be responsible for the systematically lower observed values of the speed of light in the cases when microwaves or millimeter waves were used (see figure 1). The relative extent of zones where condition (14) is not exactly fulfilled is appreciably larger than that in experiments using visible light.

Nevertheless, on the basis of the substantial agreement between the magnitudes of c and b , Maxwell proposed (not without skepticism and in understandable ignorance of the facts just mentioned!) his famous ‘Dynamical Theory of the Electromagnetic Field’ [19]. However, Einstein has gone much farther when, by postulating that in all inertial systems the kinematic ‘speed of the ray of the light in vacuum is constant, being independent of movement of emitting body’ (the second postulate of STR; [20]), he has implicitly assumed that c is a universal constant identical to Maxwell’s constant b , i.e.

$$c = b. \quad (15)$$

This relation, considered in the frame of the STR as self-evident, is now regarded to be an independent postulate, called the *c-equivalence principle* [21]. This principle has never been checked experimentally with sufficient accuracy, and therefore it bears a somewhat philosophical character. Note that the last electromagnetic measurements of Maxwell’s constant b which were carried out in the year 1907 (asterisk in figure 1; [3]) provide a value of b differing essentially from that of c ($b = 299\,710\,000 \pm 20\,000 \text{ m s}^{-1}$, while the exact value of $c = 299\,792\,458 \text{ m s}^{-1}$). Later, due to the general acceptance of the STR and of its second postulate the experimentalists concentrate their efforts rather on the accuracy of the kinematic speed of light measurements, while the difficult and not very exact electric measurements of b were considered as pointless. It has to be stressed here, however, that such experiments had nothing to do with light and that b does not primarily represent the speed of light [22]. Restrained distinction between the electromagnetic waves and the light is felt also from Maxwell’s comment concerning the nature of experiment for the determination of constants b and c [19]: ‘The value of b was determined by measuring the electromotive force with which the condenser of known capacity was charged, and then discharging the condenser through a galvanometer, so as to measure the quantity of electricity in it in electromagnetic measure. *The only use made of light in the experiment was to see the instruments.* The value of c found by M. Foucault was obtained by determining the angle through which a revolving mirror turned, while the light reflected from it went and returned along a measured course. *No use whatever was made of electricity and magnetism.*’

Maxwell’s constant b being the only parameter controlling the behavior of a pure electromagnetic field in vacuum is of primary significance in all considerations concerning this important physical entity. We are convinced that, as such, it cannot be identified on the basis of more or less justified conjectures with another quantity (the speed of light, *c-equivalence principle*) without a sufficient experimental confirmation. However, recent measurements of the constant b having comparable accuracy as that of the speed of light are lacking. The knowledge of this constant

would thus either fill the logical gap in the present theory or will provide a new experimental material for its further development.

4. Transformation properties of the electromagnetic field

Of special interest and profound physical significance are the transformation properties of reduced Maxwell’s equations (11)–(13) describing a pure electromagnetic field. As was convincingly shown by Bateman [23], reduced Maxwell’s equations are invariant with respect to the group of spherical wave transformations, which are in fact a generalized evocation of Huygens’ principle. Unifying in a way usual in the STR temporal and spatial coordinates into Minkowski’s space of four dimensions, it has been further shown that the spherical wave transformations can be there represented by a 15-element conformal mapping group. This group, actually identical with that studied by Sophus Lie, provides 15 so-called Bessel–Hagen invariants [24]. It was shown only later that the number of group elements and corresponding independent invariants could be reduced to just 11 [25]. However, even after such a reduction, the number of elements of this group exceeds the number of elements in the group of the Lorentz transformations as introduced in the frame of the STR [14, 26]. The group of these transformations known from mathematics as ‘la transformation par directions réciproques’ differs from that of Sophus Lie (or Bessel–Hagen), in fact, only by one element, namely by the scale transformation. The very physical meaning of this transformation can be specified as the independence of physical processes in the electromagnetic field from the absolute scale or dimensions in which these processes take place or, alternatively, as the absence of an intrinsic length in the electromagnetic field. In contrast to the pure electromagnetic field, in the world of ponderable matter the intrinsic length scales do exist; recall e.g. the Compton length of the electron, which is $\lambda_C = 2\pi\hbar/m_e c$. It is probably a fundamental fact that the existence of intrinsic length scale in any system is conditioned by the presence of ponderable matter [27]. Recall in this connection Mach’s conjecture claim that ‘the matter creates the space’, particularly that it defines shapes, points, etc, so that the operative geometry cannot do without material bodies. On the other hand, any ‘as if’ entity, such as electromagnetic field, is for such a purpose quite insufficient. From this point of view the CGPM (1983) definition of meter sounds somewhat paradoxical; the unit of length is based on the entity for which the concept of length is absolutely irrelevant.

Comparing now the physical content of reduced Maxwell’s equations of electromagnetic field with that established within the frame of the STR, one can immediately recognize that the limitations due to the introduction of the second postulate of the STR, from which the group of Lorentz transformations takes its origin, are quite serious. The second postulate eliminating the scale transformations consequently excludes the existence of some important Bessel–Hagen invariants, such as the analogue of spin of the Coulomb field. This prevents us, e.g., to come to some results of Dirac’s quantum theory independently. It should also be noted that the existence of invariance of some Maxwell’s

equations under the time-independent conformal mappings was already discovered and used a long time ago for the solution of electrostatic problems (cf [5]). From this point of view the introduction of the Lorentz transformations into the theory must be regarded as a kind of compromise usable simultaneously for ponderable matter and electromagnetic radiation, mechanics and optics [26]. Unfortunately, such a compromise leading to a strict exclusion of some potential properties of the electromagnetic field from theoretical considerations must be classified as inconsistent.

5. Kinematic considerations

Prior to the discussion of the kinematic aspects of the speed of light, it is necessary to distinguish between the so-called two-way and one-way speed of light [28]. In the two-way speed time-of-flight experiment, one must first trace out a closed path for the ray of light. By clocks placed at the starting point, one then measures the time during which the front of the light ray returns back. The ratio of the length of the closed path to the return time then defines the two-way speed of light. The time-of-flight measurement of the one-way speed of light, and of course that of the one-way speed of any other physical entity, requires an accurate synchronization of the pair of clocks placed at the initial and terminal points of the testing path which is topologically a line segment. However, for the realization of the synchronization procedure either as prescribed by the STR [26, 28–31] or by using ‘infinitesimally slow’ transfer of clocks [33], it is necessary *a priori* to assume the isotropy and the constancy of the one-way speed of light. Obviously, such a clear case of circular reasoning can hardly be cured by a convention giving to the one-way speed of light a certain exact value, such as (1). It would be more reasonable to admit as an experimental fact that the measurement of one-way speeds is principally impossible and that the only two-way time-of-flight speed measurement can be realized using a closed testing path, which can do without synchronization of departed clocks. From the epistemological point of view thus the one-way speed of light must be excluded from our considerations and the second postulate of the STR, in the above form, should be abandoned as an empty proposition. The difficult idea of the light spreading from the source into all directions with the same (one-way) constant speed c regardless of inertial system from which it is observed as introduced by the second postulate of the STR, will thus be replaced by a concept of a light which returns along the closed path (loop) to the starting point within the time just corresponding to the ratio = (length of the loop/ c). Of course, in such a case one can easily imagine that the speed of light in moving inertial systems may differ in various points of space and directions, but only in such a way that the time-of-flight along all isometric loops would be the same because of some kind of ‘compensation’ during the backward movement. In other words, in such a version of the second postulate it is assumed that *the two-way speed of light measured along different loops in different inertial frames would represent exactly the same constant*. Simultaneously, it is just the condition restricting a set of admissible space–time transformations reformulated in terms of two-way speed of light. Making such an assumption, it is

among others immediately clear that the Michelson–Morley experiment in which a split light ray is conveyed along two isometric loops has to provide a well known ‘negative’ result, or, directly confirms the ‘two-way speed version’ of the second postulate.

Basing on these ideas, it starts to be apparent that the most problematic operation in measuring speeds is the synchronization of clocks at departed stations. Therefore, let us mention here the standard technique for the determination of one-way delay time t_A necessary for propagation of light signal from point ‘1’ to another point ‘2’ which seemingly circumvents the synchronization of the clocks [32]. (This technique was also used, e.g. in famous OPERA experiment [34] where superluminal speed of neutrino was reputedly indicated.) Split light signal is sent from point ‘1’ via two alternative paths A (direct) and B (slightly longer) to point ‘2’, where the time difference $t_A - t_B$ is measured between arrivals of both signals. After that the total time $t_A + t_B$ is measured, which is necessary for the propagation of the signal from ‘1’ to ‘2’ along the path A and back to point ‘1’ along the path B. The time t_A , which we are interested in, can be then computed by an obvious formula

$$t_A = \frac{1}{2}[(t_A + t_B) + (t_A - t_B)]. \quad (16)$$

Evidently, the method is fully based on the assumption that the time t_B needed for the signal propagation along the path B from ‘1’ to ‘2’ is exactly the same as that for backward propagation from ‘2’ to ‘1’. Such an assumption is, however, equivalent to the validity of the second postulate of STR, a statement for which the experimental evidence is in fact lacking. Admitting instead of the two-way speed version of the second postulate confirmed by the experiments, the times corresponding to the signal propagation in opposite directions may generally differ, i.e. $t_B(1 \rightarrow 2) \neq t_B(2 \rightarrow 1)$, which makes formula (16) useless. In such a case the results of speed measurements based on it will be inevitably spurious. This example clearly illustrates that the second postulate of STR is not only redundant but that it can be also harmful.

The redundancy and arbitrariness of the second postulate of the STR was recognized by a group of Chinese researchers who formulated a theory of relativity which can do without second postulate intimately related to the speed of light [35]. This is so-called ‘Taiji relativity’. (The term ‘Taiji’ is a central concept of ancient Chinese cosmology having a meaning of ultimate principle existing before the creation of the Universe. As such, it is akin to the European term *apeiron*—ἀπειρον.) It claims among others that the relativity of time and the universality of speed of light are not physical entities inherent in nature but human conventions imposed upon it. The Taiji theory is in fact essentially based on the experimentally confirmed two-way speed version of the second postulate of the STR as formulated above, according to which the two-way speed must be isotropic in all inertial frames.

The fact that only the two-way speed of light has physical meaning has its consequences also for the realization of the meter according to its CGPM (1983) definition. The authors of this definition and recommendations for the practical realization of the meter normal (*mise en pratique* [2]), namely, tacitly count with the possibility of one-way measurement of

the speed of light and with their full awareness admit the postulates of the STR. The shape of the test path need not be thus specified and the time measurement, particularly the synchronization of clocks, is done according to the ‘accepted good practice’ which follows the rules of STR. Astonishingly, it is further assumed that the measurements are restricted to the lengths which are sufficiently short to be negligible for the effects belonging to the scope of general relativity (e.g. gravitation). Such an assumption logically challenges the fact that the universal constant c is ‘universal’ and ‘constant’. It is thus apparent that the critical reconsideration of the definition and of allied problems is necessary.

Taking into account the fact that the most accurate arrangements for the optical measurements of the speed of light operate as a two-way laser interferometers of Fabry–Perot type where the test path is enclosed between two parallel semi-permeable mirrors, the normal of length can be quite naturally realized just by the distance between these two plains. However, the distance defined in this way can be converted into terms inherent to the CGPM (1983) definition only by a consequent use of the second postulate of STR. The CGPM (1983) definition thus makes this postulate conceptually indispensable although it has for the practical realization of the normal of length no significance.

6. G de Bray scenario

As was already mentioned above, according to the CGPM (1983) definition of meter and to the corresponding *mise en pratique*, the measurements should be restricted to the lengths which are ‘sufficiently’ short in order not to be influenced by gravitation etc. Beside the space-related effects, however, there can possibly also be the time drift of the value of c . To this effect turned attention G de Bray who after the careful revision of experiments made by various researchers between the years 1849–1933 concluded that *the earlier observations give a higher speed of light* [36]. Such a behavior of the ‘pseudoconstant’ c has been attributed by the author to the apparent decrease of the unit of length caused by the expansion of the Universe. If the radius of the Universe doubles every K years, then the measured velocity of light will be halved every K years, that is, the velocity will be proportional to $c \propto (1/2)^{\tau/K}$, where τ is here time in years. Of course, an enormous increase of accuracy of measurements and the constancy of the speed of light during the second half of the 20th century made the G de Bray scenario highly improbable. Nevertheless, to be correct, the long-term drift of c can be hardly excluded or proved on the basis of say around ten measurements made during two decades only (see figure 1). Since the influence of cosmological effects belonging to the scope of general relativity cannot be *a priori* excluded even for terrestrial observations, the CGPM (1983) definition anticipating in fact the absolute impossibility of G de Bray scenario sounds rather bold and not very wise. Moreover, as we have seen above, the CGPM (1983) definition is not a plain technicality but it contains hidden assumptions which can make the analysis of experiments confused and difficult.

7. Conclusions

In the present contribution we have discussed, as we believe, a legitimate question, what the consequences of the CGPM (1983) definition of meter are, especially of the fixation of the speed of light at the exact value $c = 299\,792\,458\text{ m s}^{-1}$. This question has important aspects which, remaining in the shade of the STR, are discussed in the current literature only marginally. However, as we are convinced, their omission may lead and led to the incorrect conclusions or to the masking of real effects. Therefore, these aspects should be analyzed critically and repeatedly. Some of them were discussed in this paper with the following conclusions.

First of all, we have summarized the arguments in favor of the idea that the magnetic force is a manifestation of the electric force in relative movement. We have shown that such a picture is independent of STR and of its second postulate and that the square of Maxwell’s constant b controlling the ratio of the electric and the magnetic force in any particular situation is not necessarily related to the speed of light, c .

Although we are not convinced that the c -equivalence principle is not valid ($c \neq b$), we have stressed that its experimental confirmation is at present lacking. Consequently, there is a serious logical gap in the electromagnetic theory of light and all related theories as well. According to our meaning, it is thus quite necessary to repeat the measurements of b with an accuracy comparable with that of the speed of light in the near future.

Considering the description of the electromagnetic field, we have turned our attention to the fact that the transformation group of Maxwell’s equations is larger than the Lorentz group, in other words that there are invariants (Bessel–Hagen invariants) and properties of the electromagnetic field that are outside the scope of the STR. This fact may have a number of important consequences which should not be omitted only on the basis of a teleological argument, i.e. because they do not fit well the STR.

In the paragraph concerning the kinematic problems we have pointed out the principal impossibility of measuring the one-way speed of light and other physical entities. Therefore, we have suggested a ‘two-way speed’ version of the second postulate of the STR which fits well the known experimental observations and in some cases effectively eliminates the difficulties with synchronization of clocks. Some features of this approach are shared by the so-called Taiji theory of relativity which left out the second postulate of the STR completely. As we have further shown, ignorance of the principal impossibility of measuring the one-way speeds may lead to serious experimental errors and misinterpretations.

The CGPM (1983) definition anticipates the absolute impossibility of the scenario of G de Bray’s type, i.e. of a slow drift of the speed of light, although such a drift is compatible with the predictions of, e.g., the general theory of relativity. We are therefore convinced that the measurements of the speed of light should remain the subject of astrophysical experimental research in future, in spite of the fact that they already lost their formal sense.

Last but not least, although we criticized, in this paper, among others, one of the cornerstones of modern physics, the second postulate of the STR, our aim was by no means to

belittle the merits and great efforts of scientists such as W Voigt, J Larmor, H A Lorentz, H Poincaré and A Einstein who established the STR more than 100 years ago but only to direct attention to the problems which should be solved. Let us therefore add a quotation from the paper of one of the creators of the STR (Poincaré [32]) which elucidates our attitude: ‘Good theories are flexible. . . . Specious arguments have no effect on them, and they also triumph over all serious objections. However, in triumphing they may be transformed.’

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2. PHENOMENOLOGICAL MEANING OF TEMPERATURE

Jiří J. Mareš

Dolus latet in universalibus

2.1. Introduction

It is difficult to imagine the immense effort which was necessary for the transformation of ancient theory of thermal phenomena based on the peripatetic concepts of *Antiperistasis* and *Doctrine of Four Elements* (coldness, hotness, dryness, humidity) [1,2] into the modern system based on measurable quantities having definite magnitudes and physical dimensions, which would be convenient for effective mathematical processing. Path-breaking statement “*Coldness is nothing but the absence of hotness*” which can be found in the Galileo’s correspondence with his pupils [3] and denying the very fundamentals of Aristotelian physics, was probably even more heretical than his famous statement concerning the rotation of Earth. It should be noticed here that the research into the thermal phenomena is enormously difficult in comparison with e.g. the research into the mechanics because the concepts involved are of very abstract nature. For example, it took an appreciable time, more than ~150 years, till the “intensive” and “extensive” aspects of heat were in a pregnant way introduced into the thermal physics by J. Black [4], where they substituted a vague term of “heat” closely related to our sensations. Another serious obstacle in the development of thermal physics is its universality. Considering namely any process in the nature, one immediately recognizes that it is controlled or at least influenced by the “thermal” conditions in which it takes place. Therefore the theory of thermal phenomena has to be inevitably quite universal and consequently very prone to misinterpretations and far reaching persistent errors (according to our motto in the heading, “*Deception is concealed in the universality*”). Keeping thus in mind just this last important circumstance, we have made an attempt to outline anew the logical structure of the phenomenological thermal physics which is, as we are convinced, far from being well established. In this short review we have focused our attention mainly on one crucial concept of the thermal physics, namely on the temperature.

What does the temperature actually mean? It is a classically simple question astonishingly lacking an appropriate answer. The answers, namely, which can be found in the textbooks on thermodynamics, are only hardly acceptable without serious objections. For illustration, let us give a few typical examples here and the reader himself can easily find others in the current literature. To the most hand-waving belong the statements such as that “the temperature is known from the basic courses of physics” or even “temperature is known intuitively”. More frankly sounds the practical formulation that “the temperature is reading on the scale of thermometer” throwing doubt, perhaps without awareness, upon the very existence of temperature if the thermometer is not at our disposal. In contrast to it, rather philosophical is the statement that “temperature is a physical property of a system that underlies the common notions of hot and cold” which, nevertheless, rather specifies what the temperature should be, giving no idea of what it actually is. To a high scientific standard pretends the definition telling us that “on the macroscopic scale the temperature is the unique physical property that determines the direction of heat flow between two objects placed in thermal contact” being in fact an explanation *obscurum per obscurius* which transforms the temperature problem to the problem of flow of something even more uncertain. In order not to

fall, however, into an unfertile and non-constructive criticism we stop now with examples the purpose of which was only to illustrate the state of affairs and turn our attention to the other aspects of the problem.

Of course, there is no doubt that the temperature is a central concept of thermal physics and therefore a lot of researchers feeling the lack of a good definition were trying, in different ways, to bring the temperature concept on the safer grounds. For example, the authors preferring the axiomatic approach are, as a rule, inclined to assume that the temperature is a primitive concept which need not be, in principle, derived from other presumable more primitive ideas. Unfortunately, the experimental determination of temperature in any particular case requires performing a lot of non-trivial operations which should be substantiated by its definition. Thus such a shift from the operative physical definition to the metaphysical one, very comfortable for theoreticians, makes an inexplicable obscure ritual any actual temperature measurement performed by experimentalists.

For researchers who consider the thermal physics being nothing but an outgrowth of statistical mechanics, the equilibrium temperature of a system can be (up to an arbitrary factor) defined as an inverse of the derivative of natural logarithm of the statistical weight of its actual macroscopic state with respect to the energy of the system. Nevertheless, practically no actual measurement of temperature is directly linked with the evaluation of statistical data related to a certain ensemble of particles and excitations but it is as a rule performed by means of various forms of thermometers converting the temperature to another measurable physical quantity. It is thus evident that prior to the identification of the temperature defined in the frame of statistical theory with the phenomenological temperature, the latter has to be satisfactorily defined first.

The purpose of this contribution is thus to re-examine a fundamental concept of thermal physics, *temperature*, partly from the historical and partly from a logical point of view. In order to avoid very general abstract considerations the exposition is confined practically only to fluids, i.e. mechanical systems for which are the constitutive relations simple and well known. The mathematical structure of the precursor of temperature, *hotness manifold*, is, as far as we know, for the first time discussed in terms of *elementary set theory*. In the exposition of the subject the emphasis is put on experiment and on the generalization of empiric data while mathematical proofs of some statements has reader confer with special references already given.

2.2. Phenomenological conjugate variables

The very task of any mathematical theory of material systems is to establish general rules for the further treatment of *empirical constitutive relations* describing the state of a body in terms of suitably chosen parameters. We do not mean here the parameters specific for the description of thermal effects but just the parameters already introduced in other branches of physics and generally known as the *phenomenological variables* [5]. The existence of such variables, playing the roles of macroscopic conditions which are compatible with a huge number of parameters describing each microscopic component of the body, is clearly a matter of experience. Usually, there is an appreciable number of various phenomenological variables determining the state of the body but fixing a chosen one by external means, this number can ever be diminished by one. Continuing such a procedure the number of significant phenomenological variables can be finally reduced to two. It is a fact worth to noticing that the two-parameter system is the simplest model of a real system because it enables one to construct a meaningful constitutive relation and, simultaneously, it is apt for straightforward generalization, e.g. by adding another pair of independent variables. In order to ensure easy perceptibility of mathematical description of two-parameter systems a special form of

phenomenological variables was found to be desirable. It is a well known fact that the terms entering the energy balance equations in mechanics and electrodynamics have a canonical form which may be characterized by means of the following dimensional relation

$$[\text{Energy}] = [X] \times [Y], \quad (1)$$

where square brackets mean the physical dimension of the quantities enclosed. As the energy is an extensive quantity, it is favourable for first phenomenological variable to choose also an extensive quantity, say X. In such a case, however, the second parameter has to be inevitably an intensive quantity, Y [6]. Such a couple of quantities obeying relation (1) is then called a couple of *conjugate variables*. (For example, the list of the most frequently used conjugate parameters is given in Table 1.) The existence of the intensive and extensive “aspects” of heat which was already recognized by J. Black [4] is thus in this context the discovery of primary importance for the formalization of theory of heat and its compatibility with other branches of physics. His “intensity of heat” and “matter of heat” can be, namely, quite naturally assigned to a certain couple of conjugate variables, which may be tentatively called „temperature „, and „heat“. Formal compatibility of these two quantities with the system of quantities already introduced in other branches of physics is thus only a matter of proper choice of suitable operative definitions and units.

Table 1. Selected conjugate variables

Extensive quantity	X	Intensive quantity	Y	Energy term E
Volume	V	pressure	p	pV
Momentum	G	velocity	v	vG
Charge	Q	el. potential	φ	φQ
Mass	M	grav. pot.	γ	γM

2.3. Diathermic partition, thermal equilibrium

The idea to treat real system in terms of conjugate variables enable one to introduce some fundamental concepts of thermal physics in a quite systematic way and, somewhat astonishingly, without a priori reference to the thermal phenomena *per se*, particularly to the quantities of temperature and heat [7]. The important role plays here the so called *correlation test*. It is the procedure frequently used in the practical thermometry which enables one to check whether the thermometer is in proper thermal contact with the measured body. Simultaneously, it provides the basis for the following operational definition of diathermic and adiabatic partitions (walls), viz: Let us have two systems characterized by couples of conjugate variables (X,Y) and (X',Y'), respectively, and separated by a macroscopically firm material partition (wall) defining their common boundary. Such a partition is called *diathermic* if the changes of the variables (X,Y) induce the changes of the variables (X', Y') and vice versa (“diathermic“ originates from Greek *διά* = through, *θερμός* = warm). A concept complementary to that of diathermic partition is the *adiabatic* partition (from Greek *α-* = negation, *διά* = through, *βαίνειν* = to go) which prevents the thermal contact of neighbouring bodies, i.e. ensures their thermal insulation. Obviously this concept is a limiting one, depending to an appreciable extent on the accuracy of the correlation test. The adiabatic

partition is, namely, any diathermic partition for which the experimental proof of its ability to realize the thermal contact by the said test failed.

Using then the language of two-parameter description, the general definition of the equilibrium state as known from other branches of physics can be extrapolated also to the region of thermal phenomena. Let us recall first what the equilibrium state means e.g. in mechanics. Standard formulation for the two-parameter system reads: Any state of a body in which the conjugate coordinates X and Y remain constant so long as the external conditions are unchanged is called *equilibrium state*. Combining then this definition with that of the diathermic partition we can immediately define the concept of thermal equilibrium, which already belongs to the scope of thermal physics, namely:

If two bodies being in diathermic contact are both in equilibrium state, they are in *thermal equilibrium*.

2.4. Thermoscope

The first devices known as thermoscopes have appeared during the later Renaissance in connection with the first edition of Latin translation of Hero's "*Pneumatica*" by F. Commandino Urbinate (1575) [8]. The influence of this book dealing with various unexplained natural phenomena and curious contrivances worked by air, water or steam was

so general that it is almost impossible to tell, whether a particular device was directly derived from Hero's descriptions or whether it is an original invention. Therefore, one can find in the literature a long series of "inventors of thermometer", e.g. Cardano, Galileo, Sanctorius, Besson, de la Porta, Drebbel, Fludd, Leurechon, Ens, Harsdoerfer, Kirchner etc. [9], but to decide about the priority of any one of them is a very difficult task. Quite early it was recognized that these devices, having various forms of fluid or air dilatometers, enable the objectification of the subjective feelings of hot and cold. Almost simultaneously appeared an idea that the thermal states of bodies, which are in common terms described by means of ordered series of terms cold, cool, tepid, warm, hot, could be characterized by the ordered series of thermoscope readings as well. The substitution of *thermoscope* for human *sensations* in experiments leads finally to the conviction that the thermal state of bodies or of environment can be characterized by thermoscope readings incomparably better than by means of sensation in itself.

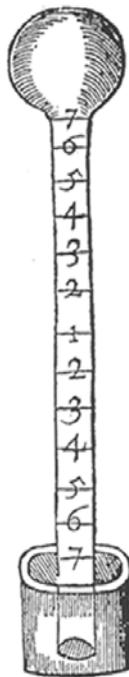


Fig. 1. A typical early thermoscope provided with an arbitrary scale. Glass tube and bottom vessel were filled with wine, bulb thus contained a mixture of air with water and alcohol vapours. (According to [10].)

The thermoscope was thus promoted to the device apt to indicate with "absolute exactness" the thermal state of vicinal bodies. Such a (of course, very optimistic!) belief reflects also famous Fludd's quotation [10] "Weather-glass (i.e. thermoscope) became a mighty weapon in the Herculean fight between Truth and Falsehood" (see Fig. 1).

2.5. Thermoscopic states

In order to continue this story in a more contemporary and systematic way we will call *thermoscope* any two-parameter system in which one of the conjugate parameters, say Y , can be fixed, $Y = Y_0$. It is further assumed that thermoscope can be brought into a diathermic contact with other bodies and that it is sufficiently “small” in comparison with these bodies in order to not appreciably disturb their thermal equilibrium. The second conjugate real parameter X , which is called in this connection *thermoscopic variable*, is generally of quite a diverse physical nature and dimension. It may be length, volume, resistance, voltage, frequency and many others. In order to distinguish formally among various thermoscopic variables, differently constructed thermoscopes and physical conditions under which they operate, a small Latin index is used. Applying this convention, reading $X_k(P)$ of the k -th thermoscope which is in diathermic contact with a body under investigation defines the *thermoscopic state* P of the body. The corresponding set of the thermoscopic states which can be observed in this way is then marked as \mathbf{H}_k . Notice that the readings X_k are related to the thermoscope while the indicated thermoscopic state as e.g. $P \in \mathbf{H}_k$, already relates to the body. It is a matter of fact that the phenomenological parameters were introduced into classical mechanics and electrodynamics as continuous quantities covering certain closed intervals of real axis. Therefore, according to our definitions, such a property is transferred also to the thermoscopic variables X_k . We will thus assume that the numerical values of quantity X_k also continuously cover a certain closed interval \mathbf{I}_k , operation range of the k -th thermoscope, which is a proper part of the set of real numbers, \mathbf{E}_1 . In usual symbols we can thus write¹:

$$X_k \in \mathbf{I}_k \subset \mathbf{E}_1. \quad (2)$$

If it is further for every couple $P, Q \in \mathbf{H}_k$,

$$P \neq Q, \Leftrightarrow X_k(P) \neq X_k(Q), \quad (3)$$

the set \mathbf{H}_k can be ordered in accordance with the intrinsic order already existing in real interval $\mathbf{I}_k \subset \mathbf{E}_1$. Simultaneously, condition (3) ensures the existence of one-to-one mapping between the sets \mathbf{I}_k and \mathbf{H}_k and we can thus define order (\prec, \succ) in \mathbf{H}_k by the following equivalences:

$$\begin{aligned} P \prec Q &\Leftrightarrow X_k(P) < X_k(Q) \\ P \succ Q &\Leftrightarrow X_k(P) > X_k(Q) \\ P = Q &\Leftrightarrow X_k(P) = X_k(Q) \end{aligned} \quad (4a-c)$$

Moreover, we have a liberty of choosing the “arrow” indicating the order because the symbols \prec, \succ on the right side of relations (4a,b) can be changed, according to our need, into $>, <$.

It is apparent that conditions (4a-c) provide basis for the construction of a primitive temperature scale. Indeed, unambiguous assignment of a certain value $X_k(P)$ to every state $P \in \mathbf{H}_k$ is nothing but realisation of a local *empirical temperature scale* in terms of thermoscopic variable X_k . It is, however, an important fact belonging rather to the scope of epistemology, that although the empirical scales enable one to characterize the thermal states

¹ We do not distinguish here the physical quantity and its corresponding numerical value. See also [6].

of bodies, for the revealing of the very nature of physical quantity called temperature are almost useless and further development of more involved concepts is thus necessary.

There is another requirement ensuring the objectivity of the above conception. It is so called *Principle of indifference* according to which different thermoscopes k, j operating in the common range of thermoscopic states should distinguish any two different states $P \neq Q$,

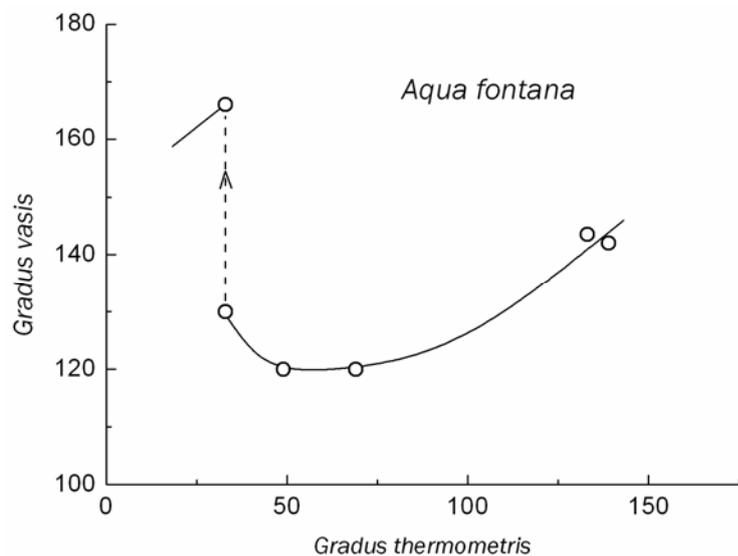


Fig. 2. Dulong-Petit plot of an alcohol-filled thermometer (Gradus thermometris) vs. readings on dilatometer (Gradus vasis) filled with fresh water showing the so called water anomaly. Data were obtained by Accademia del Cimento about 1660. [2,12]

monotonic. Interestingly, very similar method was used much earlier by savants of *Accademia del Cimento* who discovered in this way, just above the freezing point of water its anomaly (i.e. non-monotonic Dulong-Petit plot with respect to the other at that time known thermoscopes, see Fig. 2) [12], which excludes the water from being in this range a suitable thermometric substance.

2.6. The use of fixed points for calibration of thermoscopes

An appreciable irreproducibility of early thermoscopes was a serious obstacle for development of non-peripatetic thermal physics. There were attempts to improve the situation by making exact copies of a standard instrument and by sending them to the various laboratories where they were intended to serve as secondary standards [12]. This, theoretically correct approach had nevertheless lot of practical limitations. It required a really high reproducibility of glass-blowing and preparation of glass and thermometric substances. Consequently, standard “thermometers” were very expensive and the transport of such delicate instruments over the long distances was quite risky. Therefore an important qualitative step toward the scientific thermometry was done when the so called *fixed thermometric points* were discovered and became in general use. The fixed point is called a body prepared by a definite prescription revealing by some observable qualitative property its physical state (e.g. boiling point of helium, melting point of water, melting point of platinum – all at normal atmospheric pressure) and which being in thermal equilibrium with other bodies defines unambiguously their thermoscopic state. Fixed points thus may serve as a mean for realization of fiducial points on empirical temperature scales corresponding to the

$P, Q \in (\mathbf{H}_k \cap \mathbf{H}_j)$, regardless of their construction, thermometric substances, variables X and other physical conditions used. A procedure worked out by Dulong and Petit [11] is used in practical thermometry for the comparison of different empirical temperature scales. So called Dulong-Petit plot is a locus of readings X_k of one thermometer versus readings X_j of another thermometer both being in thermal contact with the same body (thermal bath). Evidently, in terms of such a plot the Principle of indifference may be formulated simply as follows: Two empirical temperature scales agree with the Principle of indifference just if their Dulong-Petit plot is



Fig 3. Thermoscope invented by Professor Santorius, serving for determination of “*temperatura*” of human body, about 1612. Instrument was made of glass and filled with wine. The bulb having diameter of golf ball was during the measurement inserted into patient’s mouth.

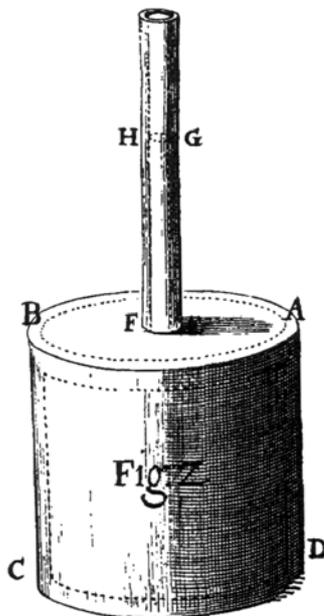


Fig. 4. Dilatometer designed by R. Hooke for realization of standard temperature scale based on single fixed point [13].

thermoscopes of different construction and different physical nature of thermoscopic variable. Probably the first who recognized the existence of a fixed point was Santorius, professor of medicine in Venetia (see Fig. 3). Studying by means of a simple thermoscope the “*temperatura*” of many patients he recognized that the reading of thermoscope was practically the same in all cases where the patient was healthy.

Very soon a lot of other fixed points, sometimes quite curious, were discovered and introduced into the practical thermometry. At this place it is interesting to mention that very likely the first savant who suggested the use the freezing and boiling points of water as fiducial points of temperature scale was Ch. Huygens. In his letter from January 2, 1665 addressed to the first president of Royal Society of London, R. Moray, he wrote: “*Je vous remercie du thermomètre...Il seriot bon de songer à une mesure universelle et déterminée du froid et du chaud,... et puis prenant pour commencement le degré de froid par le quel l’eau commence à geler, ou bien le degré de chaud de l’eau bouillante, a fin que sans envoyer de thermomètres l’on peut se communiquer les degrez du chaud et du froid, qu’on auriot trouvé dans les experiences, et les consigner a la posterité.*” From the last sentence of this quotation it is quite apparent that the aim of his suggestion was to substitute the unpractical sending of the thermometers for comparison of their scales by much more versatile calibration method.

Historically, there are essentially two methods used for the graduation (calibration) of thermoscopes by means of fixed points. These methods exploit either single or at least two fixed points. The first one is likely due to R. Hooke [13]. His specially designed dilatometer consisted of a cylindrical vessel with the cylindrical tube projecting out of its top (see Fig. 4). The tube had an inner diameter equal to one-tenth of the large vessel and was marked in intervals equal to one-tenth of its depth. One division („degree“) on the tube thus represented an expansion (contraction) of one-thousands above (below) the volume of the fluid at a chosen fixed point, e.g. freezing point of water, giving thus an objective (i.e. reproducible in different places and times) measure of thermal state. Belief in the universality of this method is

based on a tacit assumption that a dilatometer and thermometric substance used can reveal all in the nature accessible thermal states.

The first significant attempt at application of two-point calibration of thermometers belongs to members of *Accademia del Cimento* (1657-1667) [14]. They divided into three hundred degrees the interval between two marks indicating somewhat vague “fixed points”, namely, the greatest summer heat and the most severe winter cold. Two-point calibration

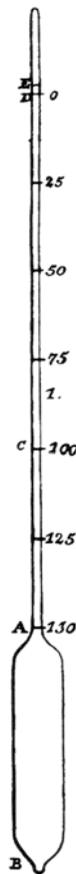


Fig. 5. An example of graduation of a thermometer by means of two-fixed point's method. Figure depicts a thermometer provided with the original “reverted” centigrade scale (Celsius, 1742). It is also shown the uncertainty of boiling point of water (=zero) which is due to fluctuations of barometric pressure [15].

technique but based on more reliable fixed points, melting point of ice and boiling point of water (incidentally the fiducial points which were proposed much earlier by Huygens!) were used also by C. Rinaldini (1693) for realization a his 12-degree thermometric scale. This well-done temperature scale was never in general use, however, a half century later, in hands of Celsius [15], became essentially the same scale the basis of our centigrade thermometric system. On the problem of graduation of thermometers worked independently Danish astronomer O. Rømer, and especially his immediate follower, German instrument maker G. D. Fahrenheit, who had used large deal of Rømer's know-how [16]. Accordingly, Fahrenheit accepted as a lower fixed point mixture of salt and ice (0°F) and normal body temperature (96°F) as the upper one. The freezing point of water (32°F) was retained as a subsidiary calibration point [17]. Enormous care was devoted to the purification of thermometric substance, mercury, improvements of glass-blowing procedure and exact specification of conditions for realization of fixed points. As a result, Fahrenheit's thermometers were much admired throughout the scientific community for their accuracy and extraordinary reproducibility and became thus for a long time a thermometric standard. In 1742 Swedish astronomer A. Celsius created a two-fixed point's temperature scale whereby zero represented the boiling point of water and one hundred represented the freezing point of water (see Fig. 5). In his lectures announced the observations showing that melting point of ice was effectively unaffected by pressure. He also determined with remarkable precision how boiling point of water varied as a function of atmospheric pressure and proposed that zero on his temperature scale (= boiling point of water) would be calibrated at the mean barometric pressure at mean sea level (standard atmosphere). As his scale was intended for meteorological observations where the temperatures are always below the temperature of boiling point of water, the scale was, in order to avoid the changes of the sign, reverted in comparison with the contemporary centigrade scale. Two years later, however, the famous Swedish botanist C. Linnaeus changed the Celsius's scale in order to be more convenient for the use in his greenhouse, constructing the thermometric scale where zero represented the melting point of ice and 100 represented boiling point of water. As the “centigrade” system was compatible with the philosophy of

metric centesimal system the Celsius's scale became later to the scale almost exclusively used on the Continent [15].

2.7. Mach's postulates

Importance of fixed points for thermometry is, however, not confined only to the calibration of thermoscopes but as was recognized not before the end of 19th century their theoretical significance is much more general. Quite interestingly, it has been originally taken for a self-evident empirical fact that it is always possible to find in an operation range of any thermoscope a sufficient number of fixed points enabling calibration of a local empirical scale, sewing it up with other overlapping empirical scales and to check, by means of the Dulong-Petit plots, the validity of Principle of indifference. The choice of fixed points for such purposes was quite pragmatic, respecting especially their easy realization, suitable distribution and stability (see e.g. Table 2). The very fact that such a liberty of choice can only be a consequence of the existence of enormous (if not infinite) number of fixed points falling into any interval of thermoscopic states remained for a long time quite unnoticed. Similar fate, i.e., being effectively undiscovered, has also the fact that the fixed points can be always found out of any interval of thermoscopic states.

Table 2. Selection of fixed points of the ITS-90

Substance	State	T ₉₀ /K
Cd	S	0.5190
Zn	S	0.8510
Al	S	1.1796
In	S	3.4145
Pb	S	7.1996
e-H	T	13.8033
Ne	T	24.5561
O ₂	T	54.3584
Ar	T	83.8058
Hg	T	234.3156
H ₂ O	T	273.1600
Ga	M	302.9146
In	F	429.7485
Sn	F	505.0780
Zn	F	692.6770
Al	F	933.4730
Ag	F	1234.9300
Au	F	1337.3300
Cu	F	1357.7700

S – superconductivity transition

T – triple point

M – melting point

F – freezing point

(both at a pressure of 101325 Pa)

These and other experimentally observed properties of fixed points have been generalized by means of method of incomplete induction, the reasoning according to which the conclusion related even to the infinite number of cases is drawn from the knowledge of a finite number of cases provided that they, without exception, imply the same conclusion. Such a type of generalization of experience resulting into certain verities or postulates is quite analogous to that made e.g. prior the axiomatic construction of Euclidean geometry.

First of all, as every fixed point defines unambiguously a certain thermoscopic state and because the set of thermoscopic states \mathbf{H}_k is ordered by means of relation (\prec, \succ) , it can be assumed that the *set of fixed points* $(\mathbf{F} = \cup \mathbf{F}_k, \text{ where } \mathbf{F}_k \text{ are related to } \mathbf{H}_k)$ can be also ordered just according to this relation. Giving to such an idea a physical meaning, we can say that calibration of empirical scales by means of fixed points is nothing but an ordering of fixed points. We can thus postulate:

1) *The set of fixed points \mathbf{F} is ordered by means of relation (\prec, \succ) .*

The generalizations of experience with experimental establishment of new fixed points and of making their inventory list lead then to the following three postulates:

2) *To every fixed point $P \in \mathbf{F}$ exists at least one fixed point Q such that $Q \succ P$.*

3) *To every fixed point $R \in \mathbf{F}$ exists at least one fixed point S such that $S \prec R$.*

4) *For every couple of fixed points $P \prec R$ there exists at least one interlaying point Q such that the relations $P \prec Q$ and $Q \prec R$ are simultaneously valid.*

There is another remarkable empirical property of the set of thermoscopic states closely related to that of fixed points which can be generalized as follows (see Fig. 6).

5) *If a body changes its thermoscopic state from the state corresponding to a fixed point A to the point represented by a fixed point E , it must inevitably pass all the interlaying states corresponding to say fixed points B, C and D .*

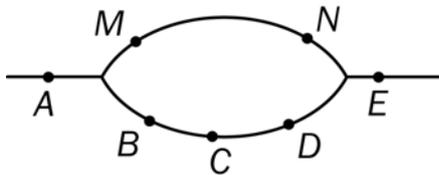


Fig.6. *Illustration to 5th Mach's postulate. If there is a thermal process connecting thermoscopic states A and E via states B, C, D , the alternative path $AMNE$ is then excluded [11].*

*the alternative path e.g. through the points M and N disjunctive with the path involving the points B, C and D is excluded. As these propositions received their first explicit formulation in the hands of E. Mach [11], we suggest calling them tacitly *Mach's postulates*. The formulation of these postulates enabled Mach to introduce intuitively the concept of *hotness manifold* (*Mannigfaltigkeit der Wärmestände*) which is the ultimate experimentally accessible concept representing the thermal states of bodies.*

2.8. Mathematical structure of hotness manifold

It is a matter of historical fact that formulation of Mach' postulates (1896) and establishment of Cantor's set theory (1895) were practically contemporaneous events. That is probably why the mathematical structure of hotness manifold has not been fully appreciated and, as far as we know, has never been systematically analyzed from the point of view of the set theory. In terms of this theory (e.g. [18]) Mach's postulates may be interpreted in the following way. Taking first into account the fact that the realizations of fixed points are real bodies, their number must be either finite or equivalent to the set of natural numbers, i.e. \mathbf{F} must be countable. In the 4th Mach's postulate one can easily recognize the definition of dense sets belonging to Cantor's theory; from this we can immediately conclude that \mathbf{F} is also dense.

Postulates 2 and 3 then mean that the set \mathbf{F} has no upper or lower bound. Any ordered countable dense set is, however, called rational series or set of rational numbers. We can thus summarize, the set of fixed points \mathbf{F} is equivalent to an unbounded set of rational numbers.

The mathematical structure of *hotness manifold* \mathbf{H} , which is a union of all sets of thermoscopic states \mathbf{H}_k , $\mathbf{H} = \cup \mathbf{H}_k$, is not as simple as the structure of \mathbf{F} . It is necessary first to make clear the operational method (i.e. a method related to the experimental procedures which can be really performed) enabling sewing-up the overlapping sets of thermal states and matching of corresponding empirical temperature scales. Let us assume that two sets of thermal states, \mathbf{H}_k , \mathbf{H}_{k+1} , overlap, i.e. that $\mathbf{H}_k \cap \mathbf{H}_{k+1} \neq \emptyset$. In order to realize this fact in experiment one has to find a fixed point $R \in \mathbf{F}$ belonging to both these sets, i.e. $R \in \mathbf{H}_k$, $R \in \mathbf{H}_{k+1}$. Theoretically the possibility of such an operation is ensured by 4th Mach's postulate. For the sake of definiteness and without loss of generality we can further construct the subsets $\mathbf{H}'_k \subset \mathbf{H}_k$, and $\mathbf{H}'_{k+1} \subset \mathbf{H}_{k+1}$ in such a way that $Q \prec R$ for every $Q \in \mathbf{H}'_k$ and $P \succ R$ for every $P \in \mathbf{H}'_{k+1}$. Evidently, the empirical temperature scale for thermal states from $\mathbf{H}_k \cup \mathbf{H}_{k+1} = \mathbf{H}'_k \cup \mathbf{H}'_{k+1}$ corresponds below R to empirical scale in \mathbf{H}_k and above R to that in \mathbf{H}_{k+1} . Moreover, in order to assign the same value of empirical temperature to the common point R , it is necessary to make formal changes at least in one of empirical scales. Applying the procedure just described and simultaneously looking for new fixed points and for new physical effects enabling the construction of new kinds of thermoscopes, we can build a chain of \mathbf{H}_k 's more and more extending the region of accessible thermal states. We are obliged to Professor Mach for belief that such a procedure is limited only by our skills². Taking now into account the fact that every \mathbf{H}_k is equivalent to a real interval $\mathbf{I}_k \subset \mathbf{E}_1$ it is obvious that \mathbf{H}_k is a continuous set. Furthermore, fixed points, e.g. such as R , are then nothing but rational cuts in sets \mathbf{H}_k and \mathbf{H}_{k+1} [Hu]. Analyzing these circumstances we can conclude that the properties of the hotness manifold $\mathbf{H} = \cup \mathbf{H}_k$ discussed above can be put in the form of two axioms already well-known from the set theory, namely

Dedekind's axiom: If \mathbf{H}_1 and \mathbf{H}_2 are any two non-empty parts of \mathbf{H} , such that every element of \mathbf{H} belongs either to \mathbf{H}_1 or to \mathbf{H}_2 and every element of \mathbf{H}_1 precedes every element of \mathbf{H}_2 , then there is at least one element $R \in \mathbf{H}$ such that:

- i) any element that precedes R belongs to \mathbf{H}_1 ,
- ii) any element that follows R belongs to \mathbf{H}_2 .

Axiom of linearity: The hotness manifold \mathbf{H} contains countable subset $\mathbf{F} \subset \mathbf{H}$ in such a way that between any two points $P \prec Q \in \mathbf{H}$ there is a point $R \in \mathbf{F}$ such as $P \prec R$ and $Q \succ R$.

As the second axiom ensures that the requirements of 5th Mach's postulate are automatically satisfied, the couple of Dedekind's axiom and axiom of linearity may be regarded as a concise reformulation of Mach's postulates. The simultaneous validity of both these axioms, however, defines in set theory the class of sets which are equivalent to the set of

² Speaking in terms of temperature, the temperatures observed range from $\sim 10^{-10}$ K (Low Temperature Lab, Helsinki University of Technology) up to $\sim 10^9$ K (supernova explosion) without any traces that the ultimate limits were actually reached. Speculative upper limit provides only the so called Planck temperature $T_p = \sqrt{(\hbar c/G)} \times (c^2/k) \approx 1.417 \times 10^{32}$ K, hypothetically corresponding to the first instant of Big Bang and depending on the assumption that the constants involved are really universal. Therefore the conjecture referred to as Mach's postulates 2 and 3, i.e. that the hotness manifold has no upper or lower bound, is obviously operating at least for all phenomena already known.

real numbers \mathbf{E}_1 . The mathematical structure of hotness manifold may thus be summarized as follows:

Hotness manifold (a set of all accessible thermoscopic states) \mathbf{H} is a set topologically equivalent to the set of all real numbers (real axis) \mathbf{E}_1 . It contains a countable, dense and unbounded subset of all fixed points $\mathbf{F} \subset \mathbf{H}$, realizing the skeleton of \mathbf{H} .

As we have seen above, the construction of this manifold is based on well-defined operational methods specifying conditions and procedures necessary for determination or reestablishment of a particular thermoscopic state. Manifold \mathbf{H} is just the experimentally accessible entity enabling one to judge how hot or cold the bodies are³. Therefore, it is this entity which is right to be regarded as an entity objectively existing in the Nature and representing the universal Platonic idea behind the usual concept of temperature, in philosophical jargon, the “temperature *an sich*”. Of course, as the set \mathbf{H} has no intrinsic *metric properties* it yields directly no physical quantity. Introduction of corresponding physical quantity, tacitly called temperature, thus requires special definitions which obviously have to take into account all the properties of hotness manifold.

2.9. Definition of temperature

As has been shown in previous paragraphs, the temperature and even the hotness manifold cannot be taken for primary concepts of thermal physics but are in fact the subjects of somewhat convoluted constructions. In the hierarchy of conceptual basis of thermal physics, however, the concept of temperature plays the role subordinate to that of hotness manifold \mathbf{H} , which is characterized just only by its topological properties. On the other hand it is quite clear that hotness manifold alone is not sufficient for the development of quantitative theory of thermal effects. For such a purpose, namely, a regular physical quantity [6] is necessary. The introduction of such a quantity called temperature into the thermal physics, which would simultaneously preserve all essentials of hotness manifold, may be based on the following definition:

Temperature is any continuous one-to-one order-preserving mapping of hotness manifold on a simply connected open subset of real numbers.

From this definition it is evident that we are at enormous liberty of choosing a particular temperature scale which thus rests entirely upon a convention. It is further quite clear that the rationality or irrationality of the choice of the temperature scale (i.e. of the said continuous one-to-one order-preserving mapping) will be decisive for further performance and intelligibility of theory of thermal effects. Obviously, in order to be able to continue with the development of the theory we have to make a decision concerning the temperature scale being simultaneously aware that such a decision has a character of provisional working hypothesis which may have in the future unexpected consequences. Traditionally the safest guide for introduction of new concepts into the science is the so called *anthropomorphic principle*. This principle taking into regard previous experience, commonly accepted patterns of thinking, human feeling of reality and other practical or cultural aspects usually generates

³ Interestingly enough, admitting the scheme above we are in fact transferring the mathematical structure of continuous quantities X_k introduced in other branches of physics into the thermal physics; e.g. without such a step the hotness manifold would not be continuous. Shortly in Hilbert's terminology, just through this gate enter thermal quantities to Cantor's Paradise governed by actual infinity and continuum [19].

intelligible, almost self-evident theories, which can be further developed and easily applied. On the other hand, a theory not respecting the anthropomorphic principle is, as a rule, very abstract, incomprehensible, and full of antinomies and with vague relation to reality. Its further development requires enormous effort and makes it thus an esoteric doctrine accessible only for limited audience.

As an example of successful application of anthropomorphic principle in thermal physics, may serve just the choice of universal thermometric scale. On the more-or-less historical and practical grounds, in other words, on the basis of arbitrary anthropomorphic criteria, a special mapping of the hotness manifold on the set of positive real numbers known as the International Kelvin Temperature Scale (T) was established. There are two alternative ways how to approach this scale, both based on a certain idealization procedure. The first one uses an idealization of a real substance (*perfect gas*), while the second one results from the idealization of a real physical process (*Carnot's reversible cycle*).

2.10. Ideal (perfect) gas temperature scale

The first approach mentioned above is based on the idealization of the most salient common features of constitutive relations of real gases. The behavior of majority of real gases is, namely, almost the same in cases where the gases have sufficiently low density. This fact was used for the definition of the perfect gas and later for the construction of the *ideal (perfect) gas temperature scale* T. The equation controlling the behavior of the ideal gas, which is a hypothetical substance or concept rather than a real thing, reads:

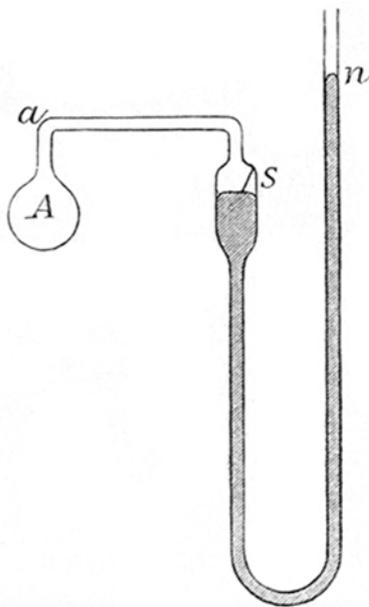


Fig. 7. A schematic view of a simple constant volume gas thermometer which simultaneously represents an operational device for realization of ideal gas temperature scale [21].

$$T = pV / nR, \quad (5)$$

where p and V are respectively the pressure and the volume of the ideal gas which may both alternatively play the role of thermoscopic variables. As the hypothetical thermoscope a conventional gas thermometer [21] filled with n moles, $n > 0$, of ideal gas is considered (see Fig. 7). The constant R on the right side of equation (5), has then a form of product $R = k N$ where k and N are Boltzmann's and Avogadro's constants, respectively (in SI system of units $k = 1.38 \times 10^{-23}$ J/K, $N = 6.02 \times 10^{23}$ mol⁻¹). Formula (5) defining the temperature T by means of constitutive properties of non-existing substance, perfect gas, reveals remarkable symmetry with respect to quantities p and V . We can thus exploit anyone of

these two quantities as a thermoscopic variable keeping the other constant. Comparing these two cases it must be inevitably:

$$T_p = T_v = T \quad (6)$$

where T_p and T_v are temperatures of a body (e.g. corresponding to temperature of a certain fixed point) determined by means of constant pressure and constant volume method,

respectively. The exact realization of condition (6) in experiments with real gases and with prescribed high accuracy (typically of order of 0.1%) is very difficult if not impossible. However, Berthelot [22] devised a simple mathematical method based on plausible assumptions which enables to extrapolate experimental data obtained on real gases at finite pressures to the case corresponding to the ideal gas and finally determine also the value of T satisfying conditions (6). From these facts it is thus apparent that the ideal gas temperature scale can be experimentally realized in the range where the gaseous phase of real gases exists.

2.11. Absolute temperature

The ideal gas temperature scale T defined by means of equation (5) has some other remarkable properties. For example, as both quantities p and V have a natural lower bound equal to zero, the temperature T has also this lower bound. In this connection it is interesting to mention considerations of French savant G. Amontons (1703) [23,24] who made extensive experiments with constant volume gas thermometer filled with dry air. By means of extrapolation of a regular “centigrade” temperature scale, constructed between boiling and freezing points of water, down to the zero pressure he obtained a value of -240°C (contemporary measurements would provide the value near -273°C). As for him it was absolutely inconceivable that the temperature of any body in the Universe could decrease below the point in which the activity of such a “vivacious” substance as the air disappears, he started to call this hypothetical temperature *l’extrêm froid*. Interestingly, quite independently but on the basis of similar experiments, introduced *absolute zero of temperature* (*zéro absolu de la chaleur*) also Clément and Désormes [25]. Nevertheless, not diminishing huge merits of these distinguished men of science, their considerations were not correct in principle. Indeed, as was already mentioned, every thermoscope enables to order the thermoscopic states only locally, within a certain well-defined range. The extrapolations out of this range are not permissible. Moreover, technical collapse of a particular type of thermoscope (e.g. due to the freezing of air in its bulb) cannot serve as a proof for non-existence of other types of thermoscopes possibly working at lower temperatures. On the other hand the definition of temperature gives us a sufficient freedom to introduce the scale with absolute zero not on the basis of some recognized “Law of Nature” but simply on the basis of our decision. Such a scale, namely, may have besides compatibility with equation (5) some advantages, e.g. numerous problems with sign can be avoided, the evaluation of integrals depending on temperature is simplified etc. The definition of absolute temperature⁴ scale then reads [20]:

Any temperature scale which is chosen in such a way that their functional values have highest lower bound equal to zero (i.e. T is always positive), is called absolute temperature scale and the corresponding temperatures are called absolute temperatures.

Notice that the possible value $T = 0$ (equal to highest lower bound) is already excluded by our definition of temperature, because due to the absence of the lowest hotness level in the hotness manifold any continuous one-to-one order-preserving transformation on the set with lower bound = 0, has inevitably to map its improper point (i.e. $-\infty$) just on the point corresponding to absolute zero. *Nernst’s law* of unattainability of absolute zero of temperature (“Third Law of Thermodynamics”) [26] is thus together with its consequences intrinsically

⁴ Notice that this definition of absolute temperature scale differs from that due to Lord Kelvin who related the adjective “absolute” rather to the independence of temperature scale of thermometric substance than to the existence of lower bound of temperature values.

involved in this definition of temperature and, consequently, needs no additional, sometimes very curious, justifications or “proofs” [27].

2.12. Carnot’s theorem

Reasonably chosen temperature function which maps the hotness manifold on a subset of real numbers should be, as was already mentioned in paragraph 2.2., conformal with other terms entering the energy balance equation. In such a case will temperature (intensive quantity, T) and heat (extensive quantity, ζ) make up a couple of conjugate variables obeying dimensional equation (1) i.e.

$$[\text{Energy}] = [T] \times [\zeta].$$

The principal possibility to write down the thermal energy term just in this form was confirmed by early experiments on the development of mechanical work by means of heat engines. In spite of the fact that these experiments were backed by a rather primitive technique (e.g. temperatures were measured by roughly calibrated mercury thermometers and heat by the weight of burned coal) being thus of doubtful accuracy, their analysis enabled S. Carnot to introduce some new theoretical concepts and draw out definite conclusions. In the present context play among new Carnot’s concepts the most important roles two idealizations of real thermal process taking place in the heat engine, namely, the *cyclic process* and *reversible process*. By cyclic process (*cycle*) is meant any thermal process in which initial and final physical state of the heat engine are the same. The reversible process is then a thermal process in which heat engine works without wastes of heat. For heat engines utilizing the cyclic reversible process (so called *ideal heat engines*) was Carnot able to formulate a theorem which in its archaic version reads⁵ [28]

“The motive power of heat is independent of the agents set at work to realize it; its quantity is fixed solely by the temperatures of the bodies between which, in the final result, the transfer of caloric is done.” (S. Carnot, 1824)

Of course, from the modern point of view is Carnot’s theorem rather a desideratum than piece of scientific knowledge. (Remarkable is also a somewhat inconsistent use of heat and caloric as synonyms!) On the other hand, it has a form of the energy balance postulate we are searching for. Indeed, if we, namely, transform the theorem into mathematical symbols we can write it in terms of finite differences [29]

$$\Delta L = \zeta F'(t) \Delta t, \quad (8)$$

where ζ means the quantity of heat regardless of the method of its measurement, ΔL is the motive power (i.e. useful work done by heat engine) and Δt is the difference between empirical temperatures of heater and cooler. The unknown dimensionless function $F'(t)$ called *Carnot’s function* should be for concrete empirical scale determined by experiment. As the

⁵ It should be stressed here that there can be found in the literature a lot of various arbitrarily changed forms of “Carnot’s theorem” or “principle” which are not equivalent one to each other and which essentially differs in their very content from the original formulation. As was thus quite correctly pointed out by distinguished researcher into the vapour turbines and president of Royal Society, H. L. Callendar [29], the original oldest Carnot’s formulation of his principle is at the same time the best one.

gained work ΔL has a dimension of energy and as this energy must be for reversible cycle equal per definition to the thermal energy of heat ζ supplied to the ideal heat engine, we can conclude that the terms suitable for insertion into the energy balance equation have to have a form of products $\zeta \Delta t$ properly modified by Carnot's function.

2.13. Kelvin's proposition

A revolutionary step toward the definition of temperature scale independent of particular type of thermometer and thermometric substance was made at 1848 by Lord Kelvin [30]. He proposed to treat Carnot's theorem not as a heuristic statement deduced from experiments of rather a limited accuracy but as a fundamental postulate of absolute validity. He further pointed out that the very purpose of Carnot's function is to modify or correct the difference of temperatures measured in a particular empirical temperature scale in such a way that it could serve as a exact proportionality factor between work, ΔL , and heat, ζ . As this factor has to be according to Carnot's postulate the same for all substances, Kelvin, inverting the logics of reasoning, suggested to define a universal (in his terminology "absolute", see footnote ⁴) temperature scale just by *prescribing* a proper analytical form of Carnot's function. For example, giving to Carnot's function the simplest permissible analytical form, namely, $F'(*T) = 1$ (so called caloric gauge⁶) we are in fact defining a new temperature scale $*T$ in terms of which equation (7) reads:

$$\Delta L = \zeta (*T_2 - *T_1). \quad (9)$$

It is immediately seen that using such a definition of temperature scale the energy terms have the desired form of a product of two conjugate variables ζ and $*T$. Interestingly enough, equation (10) is simultaneously a fundamental relation of caloric theory of heat (cf. [31]). Accordingly, the heat is from phenomenological point of view a kind of substance or fluid, caloric (*calorique*, *Wärmestoff*, *теплородъ*, *teplík*), which being dissolved in all bodies is responsible for their thermal state. It is treated as an indestructible fluid (recall that the only method how to get rid of heat is to convey it away), which is created in every irreversible processes such as rubbing, chemical reactions, burning, absorption of radiation and eating during which "something" simultaneously disappears for ever. The properties of so defined quantity are thus very near to the concept of heat in a common sense [32,33]. Taking further into account the structure of equation (9) we can also conclude that the development of moving force in an ideal heat engine is not connected with some actual consumption of heat as is claimed in thermodynamics but rather with its transfer from hotter body to a colder one (water-mill analogy [28]). At the same time equation (9) defines an entropy-like unit of heat fully compatible with the SI system which may be, according to *Callendar's* suggestion, appropriately called "Carnot" (Abbreviation "Ct") [29]. 1 Ct is then that quantity of heat which is in a reversible process capable of producing 1 J of work per 1 K temperature fall.

In the present context is, nevertheless, far more important another aspect of equation (9). Accordingly, namely, the temperature difference $*T_2 - *T_1$ between two bodies used e.g. as "heater" and "cooler" of an ideal heat engine, is identical with the ratio $\Delta L/\zeta$ where both of

⁶ Due to the later general acceptance of Joule-Mayer's postulate claiming the equivalence of work and heat it was no more possible to choose freely Carnot's function which had inevitably the form J/T (thermodynamics' gauge), where J is so called mechanical equivalent of heat [31]. Fortunately, this fact does not influence our further considerations but rather gives an opportunity to the exercise in elementary algebra interpreted in textbooks as a "proof" of mechanical origin of the Kelvin temperature scale.

these quantities are measurable in principle; ΔL by means of standard methods well-known from mechanics and ζ e.g. by amount of fuel consumed by heating the heater or, if the cooler is kept at 0 °C, by amount of ice melted during the cycle. It is quite obvious that such a technique of temperature measurement, although possible in principle, is rather a curiosum which would be very difficult to realize with sufficiently high accuracy in practice. The idea of this method is, however, of primary importance for theory. Obviously, due to Carnot's postulate equation (9) has to be valid for any ideal heat engine regardless of its construction and working substance used. Analyzing thus one representative case of ideal heat engine general conclusions can be made. For example, if we imagine an ideal heat engine driven by perfect gas and working in cycles which consist of two isothermal and two isochoric reversible processes, the useful work can be easily computed, provided that the temperatures are measured in terms of perfect gas scale. The result of such a computation reads

$$\Delta L = nR \ln(V_2/V_1) (T_2 - T_1), \quad (10)$$

where V_1 and V_2 are limits of volume between which the engine operates. It is apparent at first glance that this last equation is fully congruent with equation (9) with the proviso that the heat (measured in entropy units, e.g. Ct) transferred from heater to cooler per cycle is given by $\zeta = nR \ln(V_2/V_1)$. The congruence of these equations means that the system of units can be always chosen in such a way that scales $*T$ and T will be identical [6]. Expressing this fact more physically we can say:

Measurement of temperatures by means of ideal gas thermometer is equivalent to the measurement of temperatures by means of ideal heat engine.

The theoretical significance of this theorem is enormous because it enables one to relate without ambiguity the ideal gas (Kelvin) temperature scale to the temperatures defined by other types of ideal heat engines e.g. “*gedanken*” reversible cycles in systems controlled by electric, magnetic or electrochemical forces. Besides, it should be stressed that this theorem, although it is based on arbitrary assumptions, is by no means accidental. The idealization of constitutive relation of real gases and idealisation involved in Carnot's postulate have the same anthropomorphic roots, namely, the feeling that the thermal dilatation of bodies must be linearly dependent on their thermal state. Incidentally, in the range between 0°C and 100°C are the air scale and the mercury temperature scale, prevailingly used in experiments related to establishment of Carnot's theorem, almost identical.

2.14. Scholion

The concept of fundamental quantity of thermal physics, *temperature*, is defined as an arbitrary one-to-one order preserving continuous mapping of so-called *hotness manifold* on a certain simple connected open subset of real numbers. The hotness manifold \mathbf{H} representing all in the Nature existing thermoscopic (thermal) states is the only entity accessible to direct physical observation. The set \mathbf{H} , which was proved to be topologically equivalent to the set of all real numbers (real axis), contains a countable, dense and unbounded subset of all *fixed points* $\mathbf{F} \subset \mathbf{H}$. Any fixed point is realized by means of a specially prepared body which defines just one thermal state. The properties of set \mathbf{F} and its relation to the manifold \mathbf{H} are specified by means of Mach's postulates. As has been finally shown, the special mapping of \mathbf{H} on the set of all positive real numbers known as the *International Kelvin Temperature Scale* T was chosen on the grounds of two anthropomorphic idealizations providing a concordant result, namely, ideal substance, *perfect gas* and ideal process in heat engine, *reversible cycle*.

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