

# *Lecture 7: Optical waveguides*

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Types of guiding structures:

- Planar waveguides (integrated optics)
- Fibers (communications)

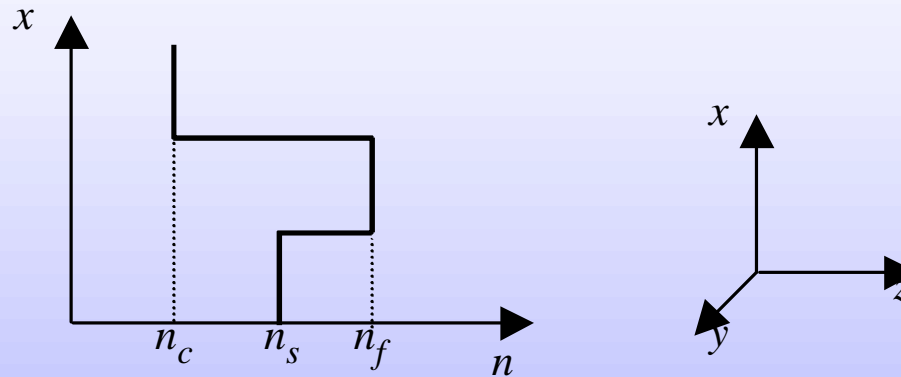
Theory:

- Rays and field approach
- Various shapes and index profiles

Attenuation and dispersion

Coupling of light into the waveguide

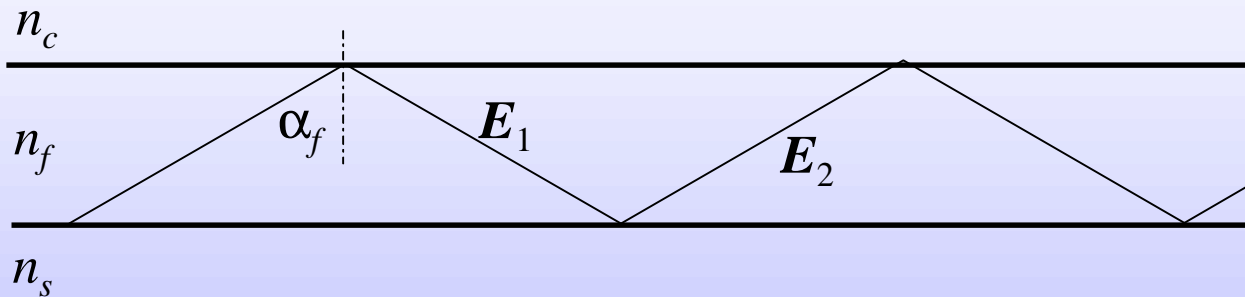
# Homogeneous planar waveguide



Series of total internal reflections (angle of incidence  $\alpha_f$ ):

$$\sin \alpha_f > \frac{n_s}{n_f} > \frac{n_c}{n_f}$$

# Transverse resonance condition



$$E_1 = E_{10} \exp(i\omega t - ikn_f (-x \cos \alpha_f + z \sin \alpha_f))$$

$$E_2 = E_{20} \exp(i\omega t - ikn_f (x \cos \alpha_f + z \sin \alpha_f))$$

$$E_1(x=h) e^{i\delta_c} = E_2(x=h)$$

$$E_2(x=0) e^{i\delta_s} = E_1(x=0)$$



Phase change  
during total reflection

# Waveguide equation

Continuity of  $E_1$  and  $E_2$  lead to:

$$\exp(-2ikn_f h \cos \alpha_f + i\delta_c + i\delta_s) = 1$$

Waveguide dispersion equation:

$$2kn_f h \cos \alpha_f = \delta_c + \delta_s + 2\pi m$$

Solution numeric or graphic

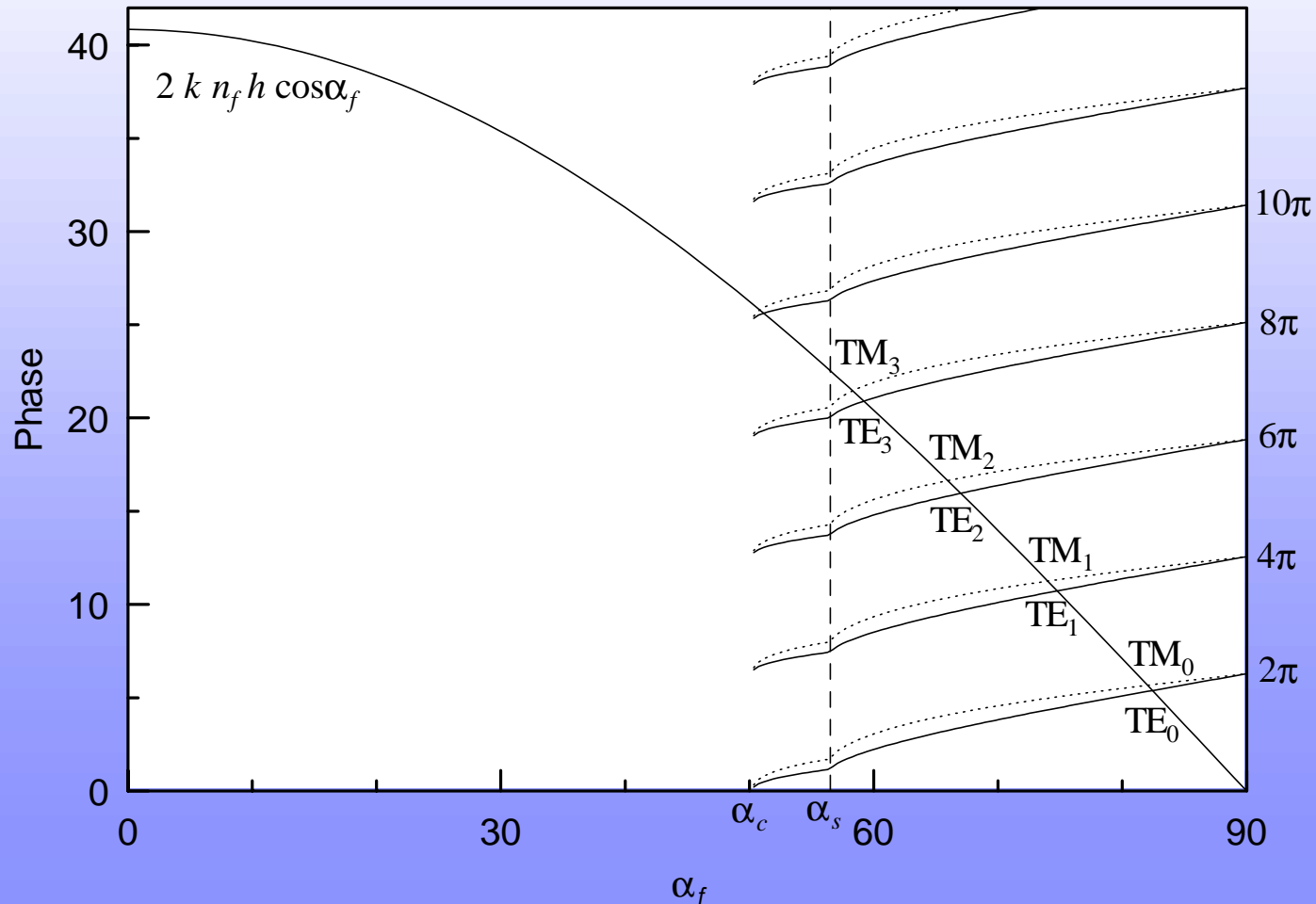
$kh$  increase  $\rightarrow$  number of modes increase

symmetric waveguide  $\rightarrow$  at least one guided mode

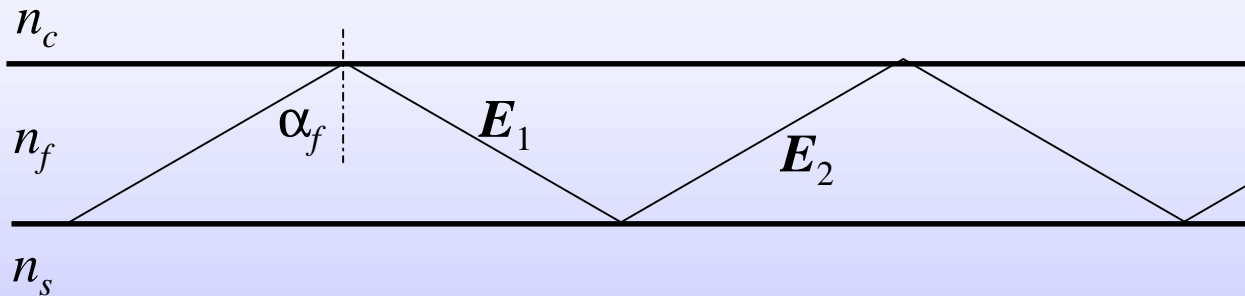
non-symmetric waveguide, small  $kh$   $\rightarrow$  no guided mode

Graphic solution

# Graphic solution of the waveguide equation



# Longitudinal propagation



Longitudinal propagation constant:

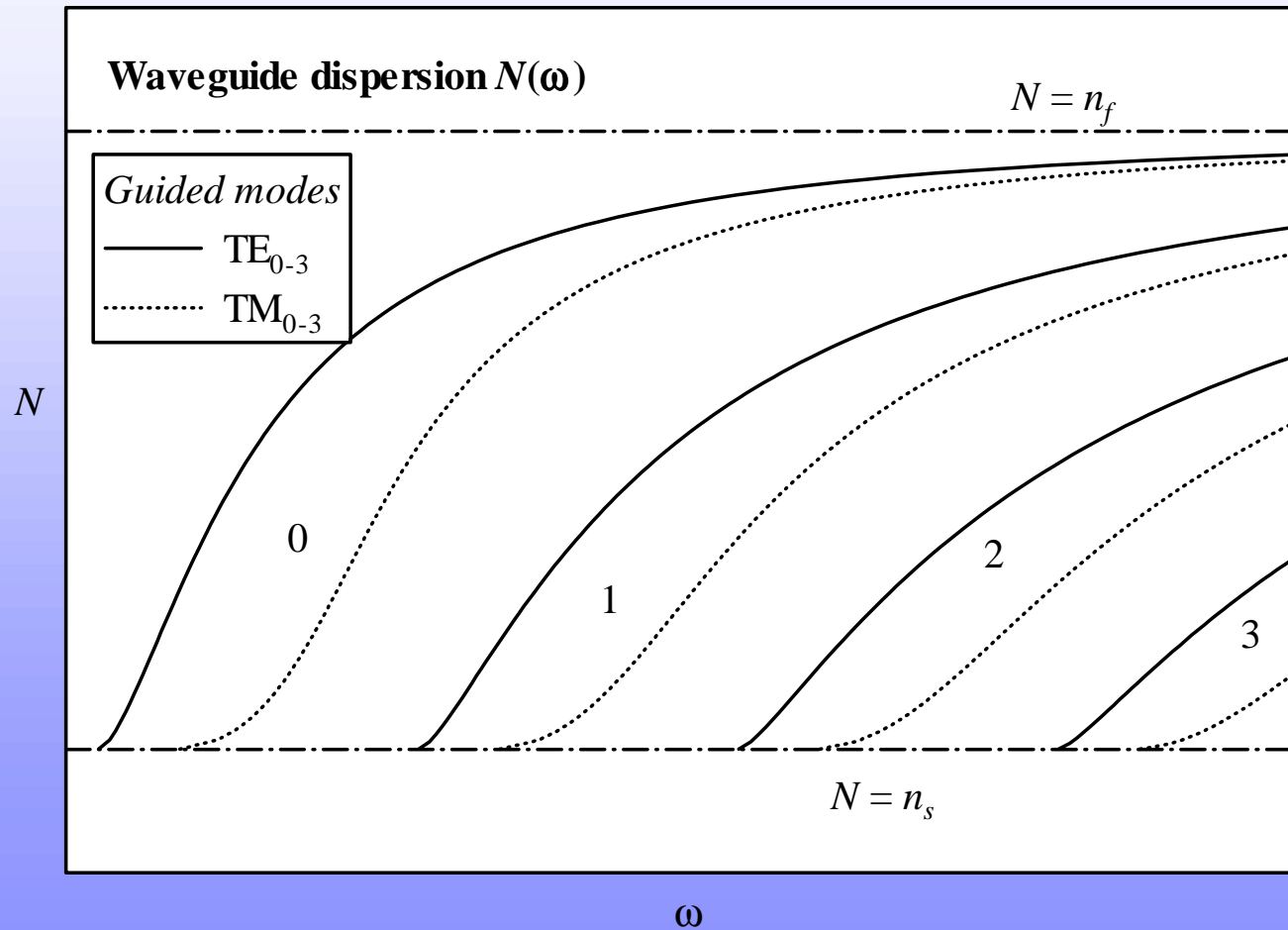
$$\beta = kn_f \sin \alpha_f = \frac{\omega}{c} n_f \sin \alpha_f (\omega)$$

Effective refractive index:

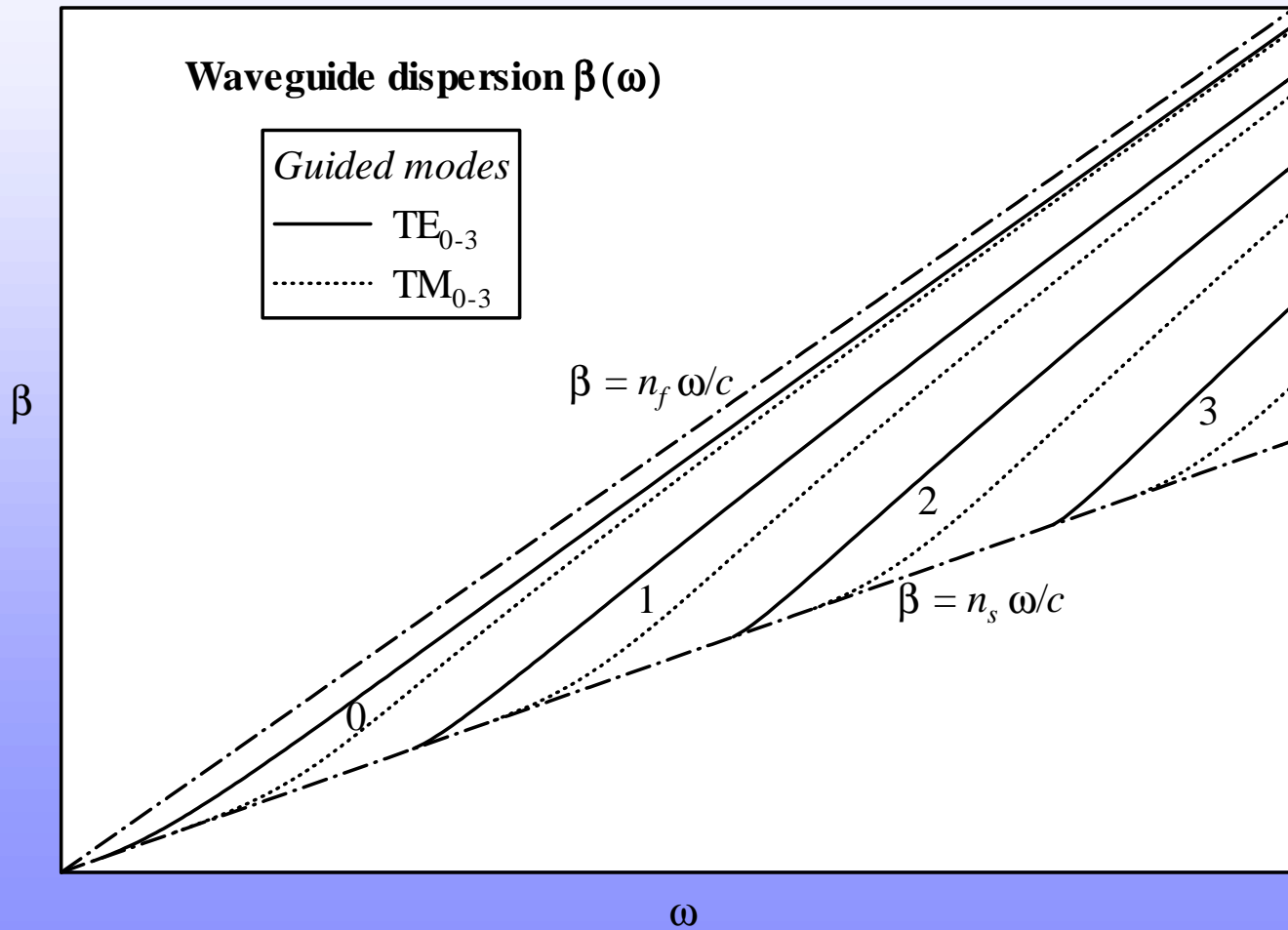
$$N = \frac{\beta}{k} = n_f \sin \alpha_f$$

Waveguide dispersion curves:  $\beta(\omega)$  or  $N(\omega)$

# Waveguide dispersion



# Waveguide dispersion



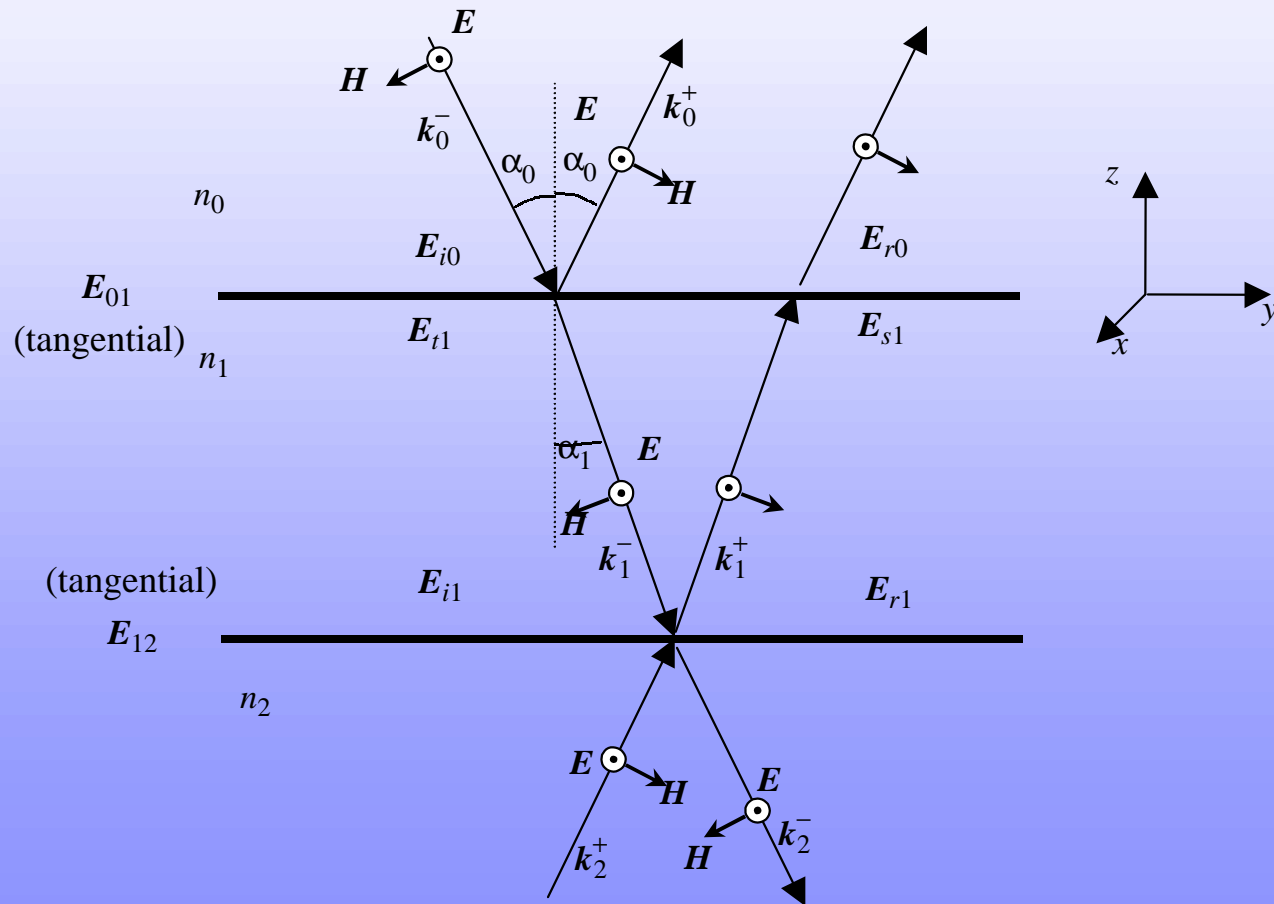


# Wave approach to waveguides

Linear combinations of the basis functions should satisfy the continuity conditions at the interfaces

- One-dimensional planar waveguide: harmonic plane waves
- Rectangular waveguide: analytic solution does not exist
- Circular waveguide: Bessel functions
- Circular waveguide with parabolic  $n$ -profile: Gaussian beams

# Planar waveguide



# Attenuation in waveguides

Mechanisms:

- Intrinsic

Residual absorption ( $\text{SiO}_2$ , impurities:  $\text{OH}^-$ )

Rayleigh scattering (proportional to  $\omega^4$ )

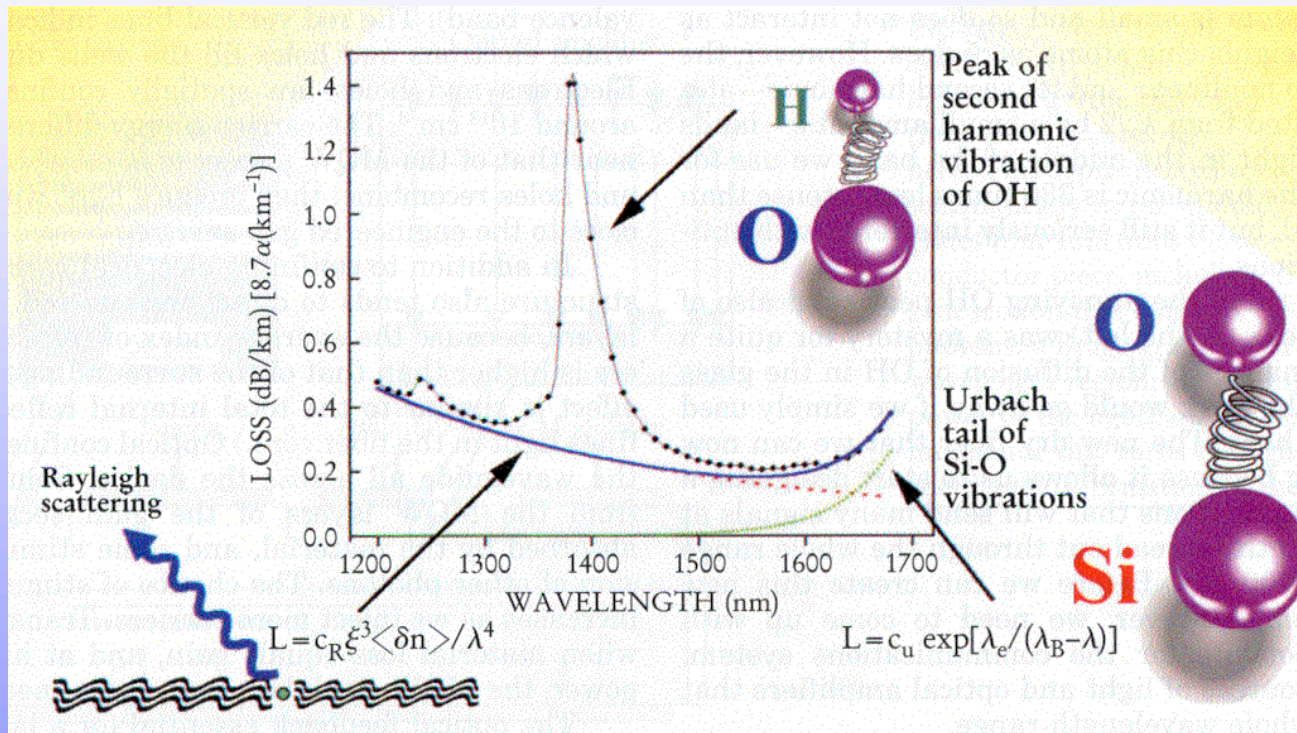
- Extrinsic

- Large inhomogeneities (fabrication of the fiber)
- geometrical irregularities (curvature, surface defects...)
- Losses at the fiber input and output (Fresnel reflection, aperture...)

$\text{SiO}_2$  fibers: 0.2 dB/km with  $\lambda = 1.55 \mu\text{m}$

$$\alpha[\text{dB}] = 10 \log\left(\frac{I_1}{I_2}\right)$$

# Attenuation in waveguides



# Dispersion

Broadening of pulse during the propagation (limits the rate of the information transfer)

1. Modal dispersion (tens of ns per km)

Single mode fibers, gradient index profile

$$\frac{\Delta\tau_1}{L} = \frac{N_{\max}}{c} - \frac{N_{\min}}{c} \approx \frac{n_f - n_s}{c}$$

2. Material dispersion (hundreds of ps per km)

It vanishes for SiO<sub>2</sub> at 1.27 μm

$$\frac{\Delta\tau_2}{L} = -\frac{\lambda}{c} \frac{d^2 n_f}{d\lambda^2} \Delta\lambda$$

3. Waveguide dispersion ( $N$  depends on  $\omega$ )

(tens of ps per km)

$$\frac{\Delta\tau_3}{L} = -\frac{\lambda}{c} \frac{d^2 N}{d\lambda^2} \Delta\lambda$$