Electromagnetism of Continuous Media — Optics

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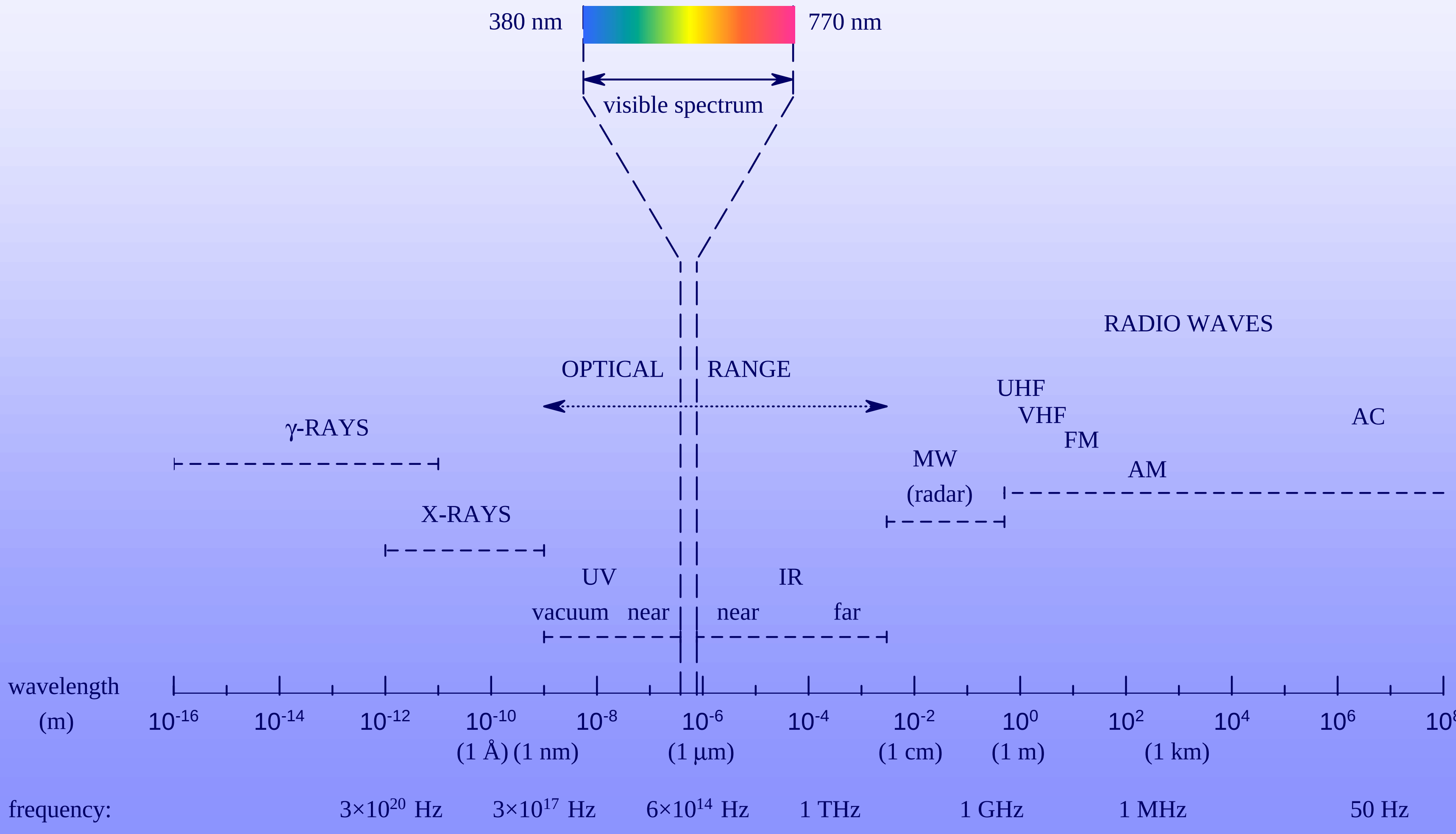
Course for 3rd year of University studies (Université Paris-Nord)

Series of 14 lectures in Department of Dielectrics, IOP

Topics of Interest

• Layered systems and optical coatings:	in detail
• Birefringence and polarization optics:	in detail
• Dispersion and pulse propagation:	yes
• Electro- and magneto-optics:	yes
• Nonlinear optics:	basics
• Waveguides and fibers:	basics
• Inhomogeneous media:	basics
• Coherence:	marginal
• Diffraction, gaussian beams:	no

Electromagnetic Spectrum



Notation conventions

$$\times \equiv \wedge$$

$$\boldsymbol{curl\ E} \equiv \boldsymbol{rot\ E} \equiv \nabla \times \boldsymbol{E} \equiv \nabla \wedge \boldsymbol{E}$$

$$\boldsymbol{div\ E} \equiv \nabla \cdot \boldsymbol{E}$$

Maxwell Equations

$$\nabla \wedge \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0$$

$$\nabla \wedge \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{j}$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{\partial}{\partial t}\int \boldsymbol{B} \cdot d\boldsymbol{S}$$

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \frac{\partial}{\partial t}\int \boldsymbol{D} \cdot d\boldsymbol{S} + \boldsymbol{J}$$

$$\oint \boldsymbol{D} \cdot d\boldsymbol{S} = Q$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{S} = 0$$

$$\boldsymbol{E}_{2t} - \boldsymbol{E}_{1t} = 0$$

$$\boldsymbol{H}_{2t} - \boldsymbol{H}_{1t} = \boldsymbol{j}_s$$

$$D_{2n} - D_{1n} = \sigma$$

$$B_{2n} - B_{1n} = 0$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} = \mu_0 \boldsymbol{H} + \boldsymbol{M}$$

Wave equation

$$\nabla^2 \boldsymbol{E} - \frac{N^2}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0$$

$$\nabla^2 \boldsymbol{H} - \frac{N^2}{c^2}\frac{\partial^2 \boldsymbol{H}}{\partial t^2} = 0$$

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$$\boldsymbol{H} = \boldsymbol{H}_0 e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

Maxwell Equations

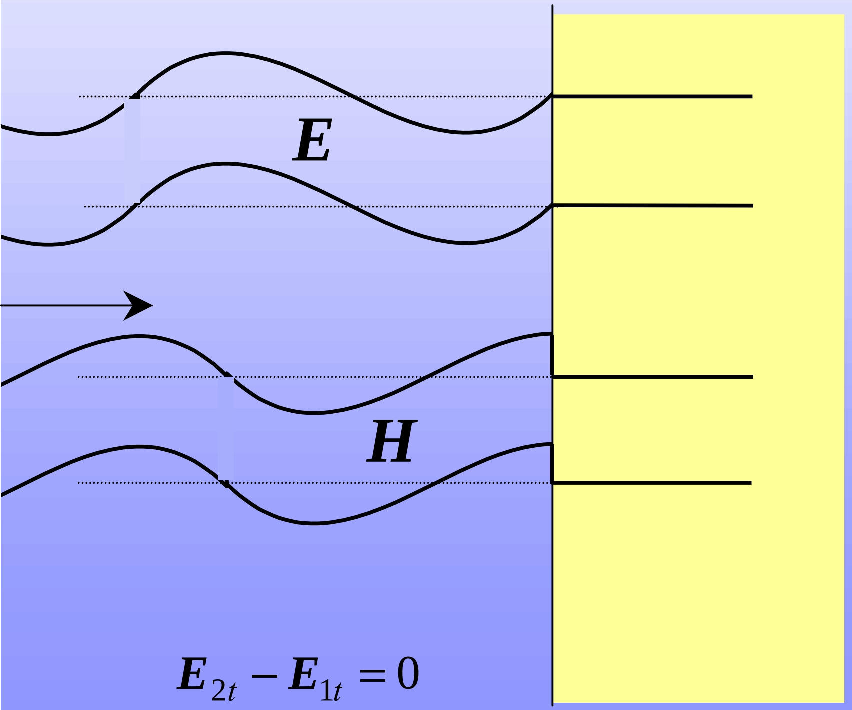
$\nabla \wedge \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0$	Faraday (induction)	$\boldsymbol{E}_{2t} - \boldsymbol{E}_{1t} = 0$
$\nabla \wedge \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{j}$	Ampère (induction)	$\boldsymbol{H}_{2t} - \boldsymbol{H}_{1t} = \boldsymbol{j}_s$
$\nabla \cdot \boldsymbol{D} = \rho$	Coulomb (electric charges)	$D_{2n} - D_{1n} = \sigma$
$\nabla \cdot \boldsymbol{B} = 0$	No magnetic charges	$B_{2n} - B_{1n} = 0$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

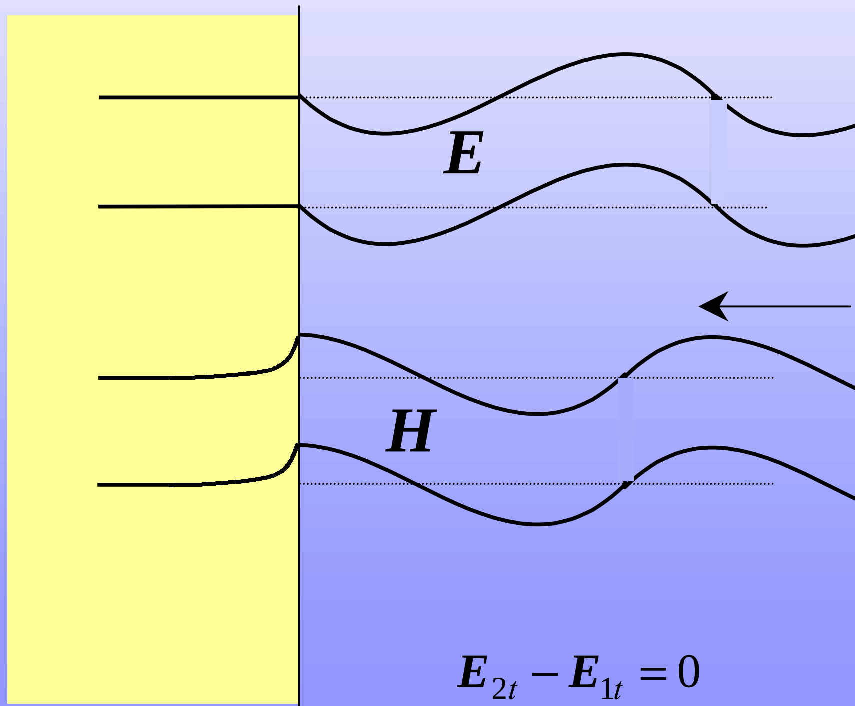
$$\boldsymbol{B} = \mu \boldsymbol{H} = \mu_0 \boldsymbol{H} + \boldsymbol{M}$$

Discontinuity at the interface

- Ideal conductor ($\sigma = \infty$)
- Real conductor ($\sigma < \infty$)



$$\boldsymbol{E}_{2t} - \boldsymbol{E}_{1t} = 0$$
$$\boldsymbol{H}_{2t} - \boldsymbol{H}_{1t} = \boldsymbol{j}_s$$

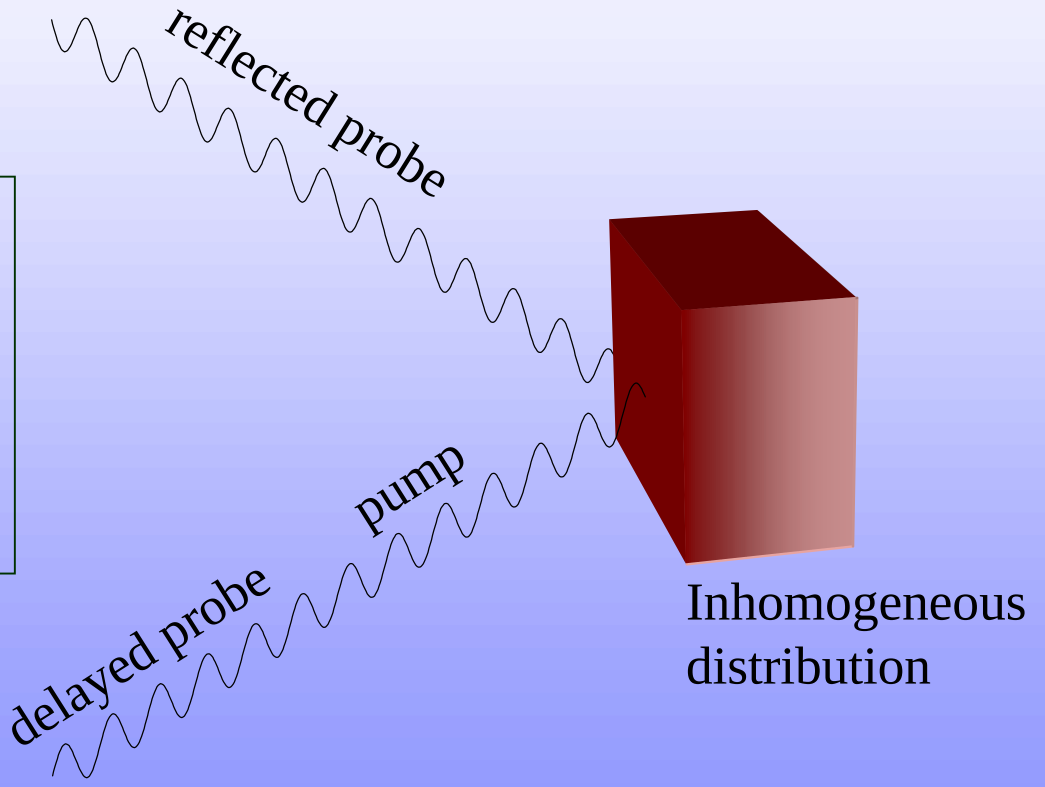


inhomogeneity

$$\boldsymbol{E}_{2t} - \boldsymbol{E}_{1t} = 0$$
$$\boldsymbol{H}_{2t} - \boldsymbol{H}_{1t} = 0$$

Time-resolved reflectivity

- Pump pulse:
  high density of free carriers
  (non-equilibrium state)
- Probe pulse:
  monitoring of the
  permittivity change



Properties of the permittivity

- dispersive [$\varepsilon = \varepsilon(\omega)$] — not dispersive ($\varepsilon$ does not depend on ω)
- absorbing (ε is complex) — not absorbing or transparent (ε is real)
- inhomogeneous [$\varepsilon = \varepsilon(\boldsymbol{r})$] — homogeneous ($\varepsilon$ does not depend on $\boldsymbol{r}$)
- anisotropic (ε is a second rank tensor) — isotropic (ε is a scalar)
- nonlinear [$\varepsilon = \varepsilon(\boldsymbol{E},\boldsymbol{H})$] — linear ($\varepsilon$ does not depend on the fields)

$$\boldsymbol{D}(\omega) = \varepsilon(\omega)\,\boldsymbol{E}(\omega) = \varepsilon_0 \boldsymbol{E}(\omega) + \boldsymbol{P}(\omega)$$

$$\boldsymbol{P}(\omega) = \varepsilon_0 \chi(\omega)\,\boldsymbol{E}(\omega)$$

$$\varepsilon(\omega) = \varepsilon_0 \left(1 + \chi(\omega)\right)$$

Spectral decomposition

Real notation:

- Monochromatic wave $a(t) = |A|\cos(\omega t + \alpha) = \mathrm{Re}\{Ae^{i\omega t}\}$, with $A = |A|e^{i\alpha}$

- Polychromatic wave $a(t) = \sum_k |A_k|\cos(\omega_k t + \alpha_k)$

$$a(t) = \int_0^{\infty} |A(\omega)|\cos(\omega t + \alpha(\omega))\, d\omega$$

Complex notation:

- Monochromatic wave $a(t) = Ae^{i\omega t}$ **or** $a(t) = \frac{1}{2}\left(Ae^{i\omega t} + A^* e^{-i\omega t}\right)$

- Polychromatic wave $a(t) = \int_0^{\infty} A(\omega) e^{i\omega t}\, d\omega$ **or** $a(t) = \frac{1}{2}\int_{-\infty}^{\infty} A(\omega) e^{i\omega t}\, d\omega$

Calculation of the field products

- If only mean values are needed (energy density and flow):

$$\langle a(t)b(t)\rangle = \frac{1}{T}\int_0^T |A|\cos(\omega t+\alpha)|B|\cos(\omega t+\beta) = \frac{1}{2}|AB|\cos(\alpha-\beta)$$

in complex notation: $\langle a(t)b(t)\rangle = \frac{1}{2}\mathrm{Re}\left\{AB^*\right\}$

- If instantaneous values are needed (nonlinear optics):

$$a(t) = \frac{1}{2}\left(Ae^{i\omega\omega} + A^* e^{-i\omega\omega}\right) \qquad b(t) = \frac{1}{2}\left(Be^{i\omega t} + B^* e^{-i\omega t}\right)$$

Maxwell equations for the spectral components

$$\nabla \wedge \boldsymbol{E} = -i\omega \boldsymbol{B}$$
$$\nabla \wedge \boldsymbol{H} = \boldsymbol{j} + i\omega \boldsymbol{D}$$
$$\nabla \cdot \boldsymbol{D} = \rho$$
$$\nabla \cdot \boldsymbol{B} = 0$$

Introduction of the generalized permittivity:

Total current (free + polarization charges):

$$\boldsymbol{j}_{TOT} = \sigma \boldsymbol{E} + i\omega \boldsymbol{D} = i\omega \left(\varepsilon - \frac{i\sigma}{\omega} \right) \boldsymbol{E} = i\omega \hat{\varepsilon} \boldsymbol{E}$$

We define a new ***D*** which involves the free charges:

$$\hat{\boldsymbol{D}} = \hat{\varepsilon} \boldsymbol{E}$$

Third Maxwell equation becomes:

$$\nabla \cdot \hat{\boldsymbol{D}} = \nabla \cdot (\varepsilon - i\,\sigma/\omega) \boldsymbol{E} = \rho + \frac{1}{i\omega} \nabla \cdot \boldsymbol{j} = 0$$

$$\nabla \wedge \boldsymbol{E} = -i\omega \boldsymbol{B}$$
$$\nabla \wedge \boldsymbol{H} = i\omega \boldsymbol{D}$$
$$\nabla \cdot \boldsymbol{D} = 0$$
$$\nabla \cdot \boldsymbol{B} = 0$$

Conservation of energy

All conservation laws have the general form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{j} = \boldsymbol{P}$$

Work of the field per unit of time exerted on the charges (Lorentz force):

$$q(\boldsymbol{E} + \boldsymbol{v} \wedge \boldsymbol{B}) \cdot \boldsymbol{v} = q\boldsymbol{v} \cdot \boldsymbol{E} \rightarrow \boldsymbol{j} \cdot \boldsymbol{E}$$

From Maxwell equations:

$$\left.\begin{array}{ll} \nabla \wedge \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} & / \cdot \boldsymbol{H} \\ \nabla \wedge \boldsymbol{H} = \boldsymbol{j} + \dfrac{\partial \boldsymbol{D}}{\partial t} & / \cdot \boldsymbol{E} \end{array}\right\} \quad \boldsymbol{E} \cdot \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{H} \cdot \frac{\partial \boldsymbol{B}}{\partial t} + \nabla \cdot (\boldsymbol{E} \wedge \boldsymbol{H}) = -\boldsymbol{j} \cdot \boldsymbol{E}$$

$$\frac{\partial U}{\partial t} + \nabla \cdot \boldsymbol{S} = -\boldsymbol{j} \cdot \boldsymbol{E}$$

Conservation of energy: continued

$$\boldsymbol{E}\cdot\frac{\partial \boldsymbol{D}}{\partial t}+\boldsymbol{H}\cdot\frac{\partial \boldsymbol{B}}{\partial t} \quad + \quad \nabla\cdot\left(\boldsymbol{E}\wedge\boldsymbol{H}\right) \quad = \quad -\boldsymbol{j}\cdot\boldsymbol{E} \quad \longleftrightarrow \quad \frac{\partial U}{\partial t}+\nabla\cdot\boldsymbol{S}=-\boldsymbol{j}\cdot\boldsymbol{E}$$

Linear non-dispersive (and non-absorbing) media:

$$U=\tfrac{1}{2}\left(\boldsymbol{E}\cdot\boldsymbol{D}+\boldsymbol{H}\cdot\boldsymbol{B}\right)$$

Absorbing medium: for a monochromatic wave the energy change corresponds to the heat:

$$Q=\frac{\omega}{2}\,\mathrm{Im}\left(\varepsilon^{*}\right)\boldsymbol{E}\boldsymbol{E}^{*}=\omega\,\mathrm{Im}\left(\varepsilon^{*}\right)\left\langle E^{2}\right\rangle$$

Medium with small dispersion: quasi-monochromatic wave:

$$\left\langle U\right\rangle=\frac{1}{4}\left(\frac{d(\omega\varepsilon)}{d\omega}\boldsymbol{E}\cdot\boldsymbol{E}^{*}+\mu_0\boldsymbol{H}\cdot\boldsymbol{H}^{*}\right)=\frac{1}{2}\left(\frac{d(\omega\varepsilon)}{d\omega}\left\langle\boldsymbol{E}^{2}\right\rangle+\mu_0\left\langle\boldsymbol{H}^{2}\right\rangle\right)$$