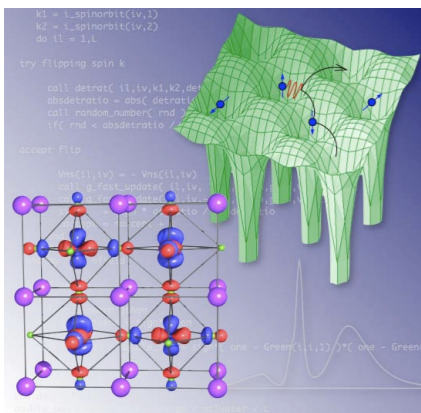
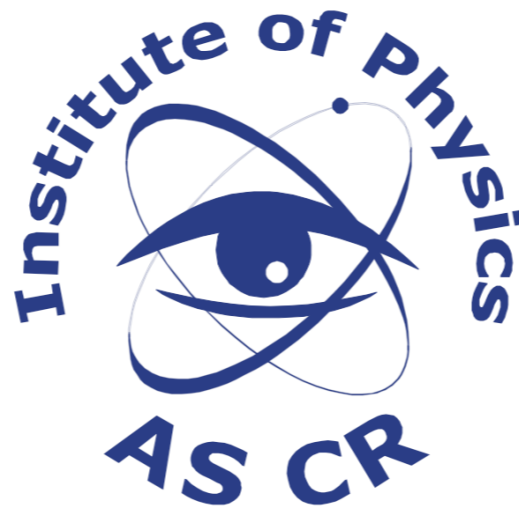


Surprising effect of electronic correlations in band insulators

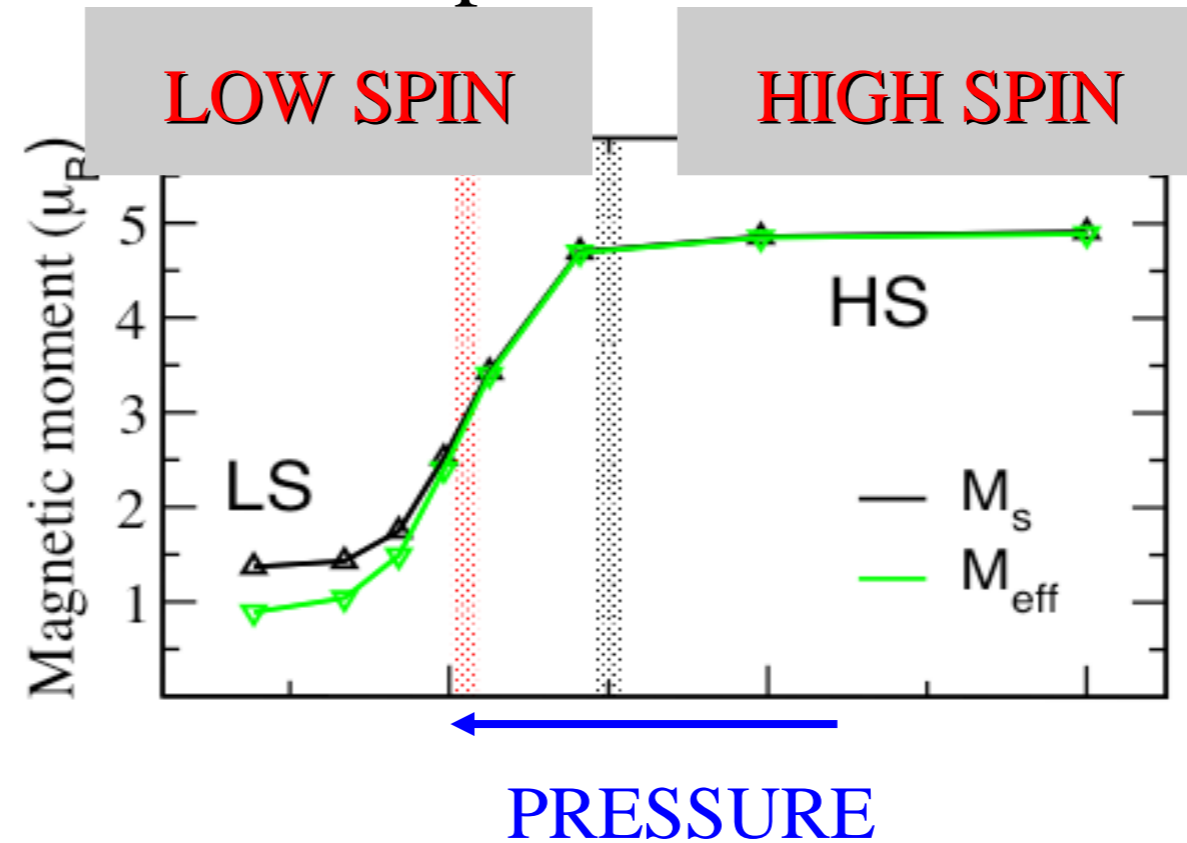
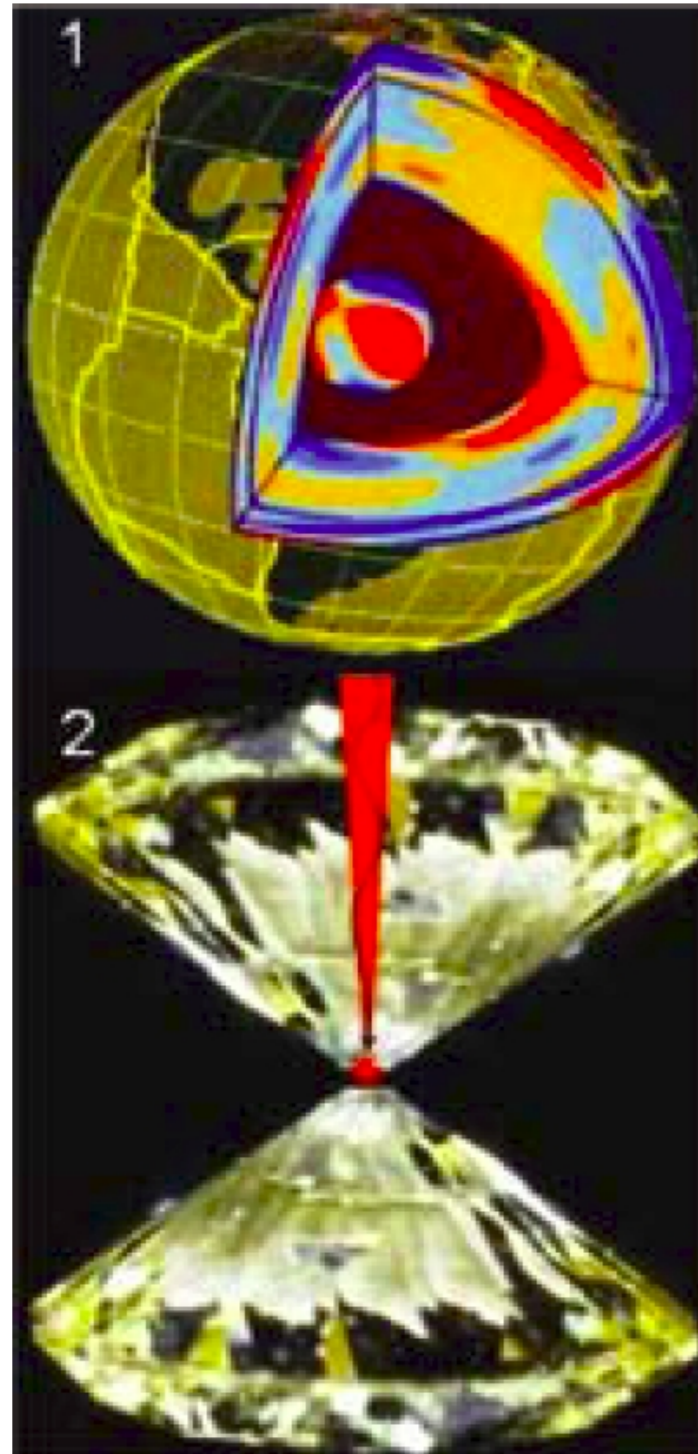
Jan Kuneš and Vlastimil Křápek



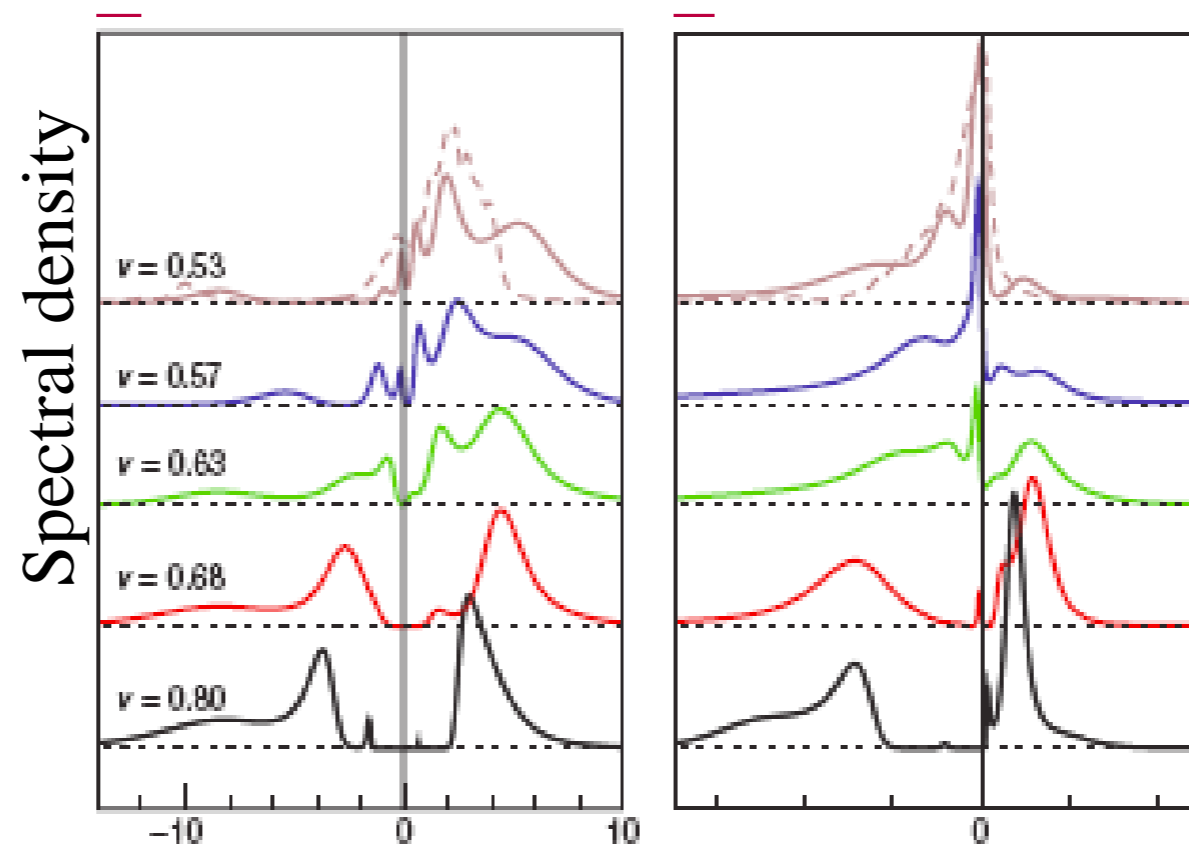
DFG FOR 1346
Dynamical Mean-Field Approach with Predictive Power for Strongly Correlated Materials
&
GA CR P204/10/0284

TM oxides under high pressures

Simultaneous moment collapse metal-insulator transition



MnO
 α -Fe₂O₃

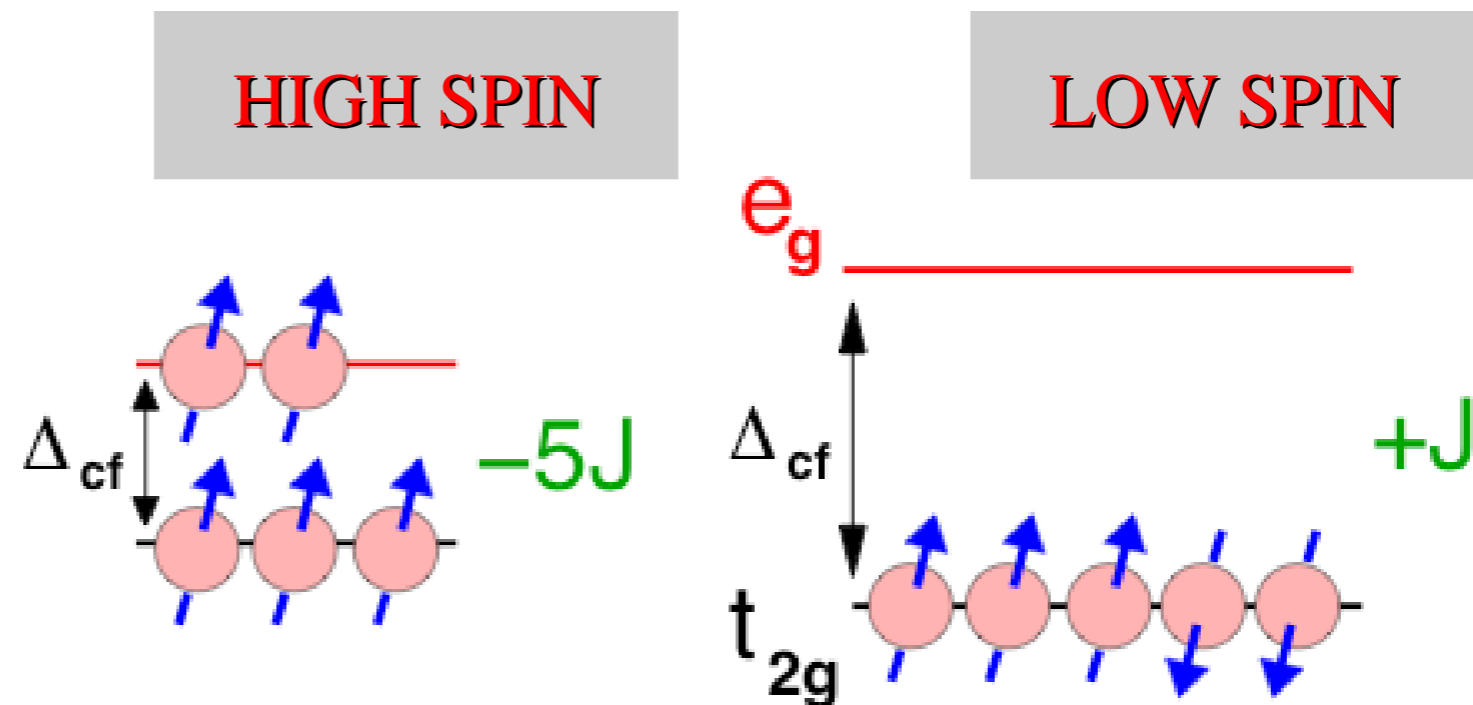
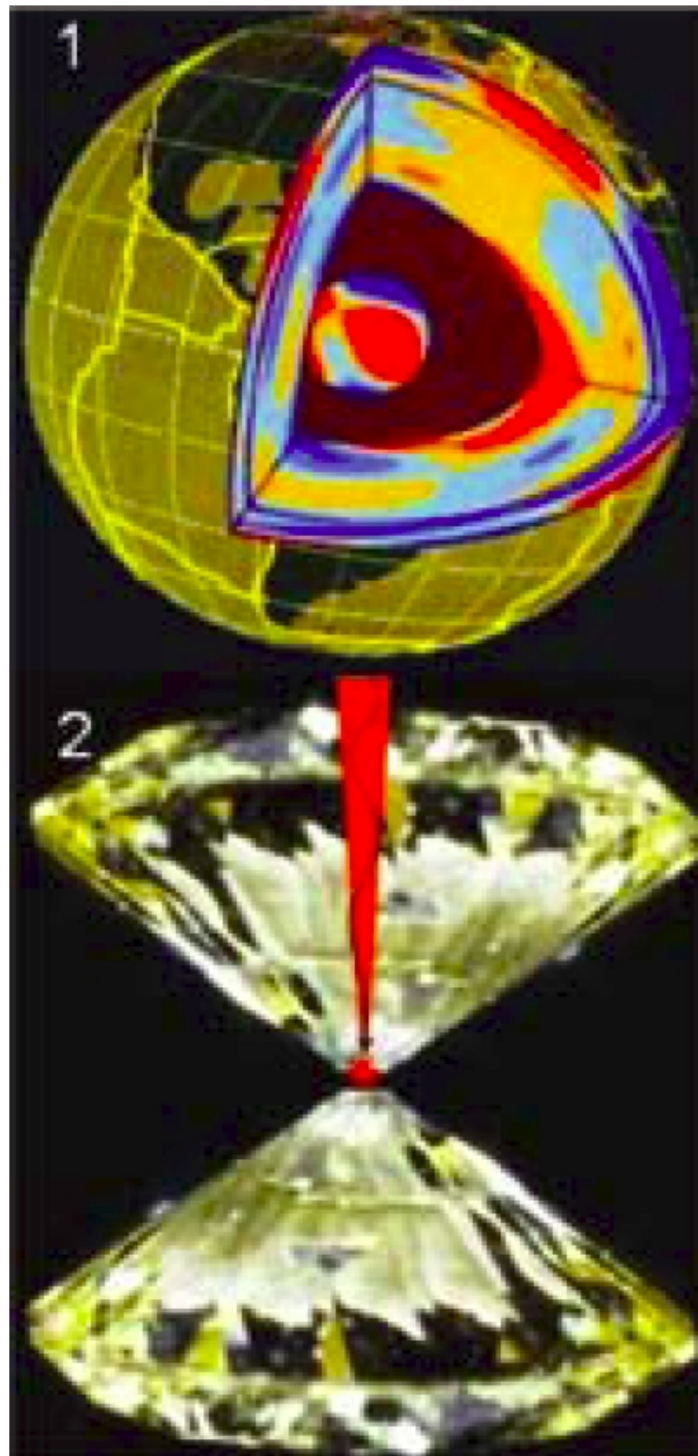


METAL

INSULATOR

TM oxides under high pressures

Simultaneous moment collapse metal-insulator transition

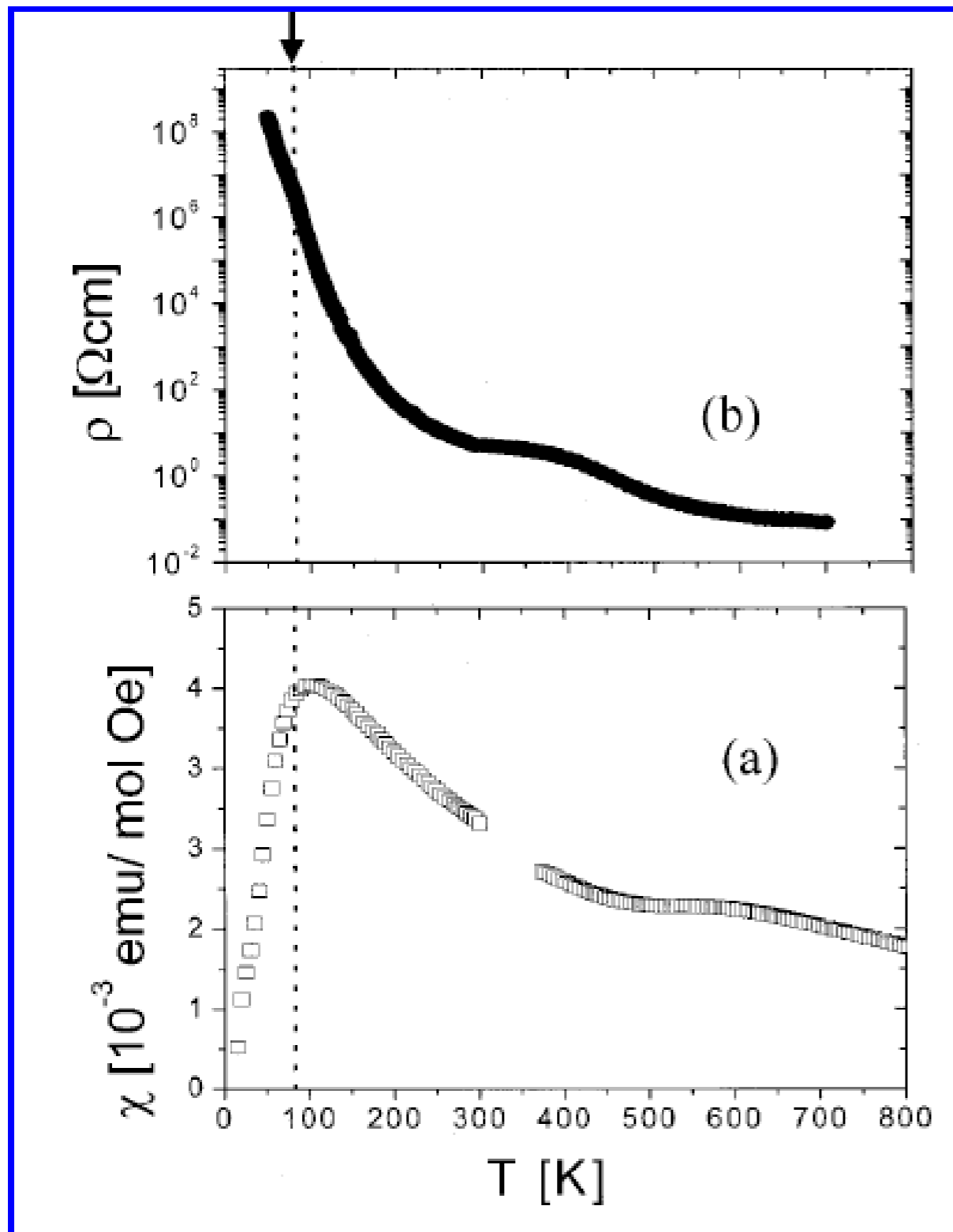


What happens right at the transition?

Outline

- LaCoO_3
- Dynamical mean-field theory
- HS-LS transitions in 2-band model
- HS-LS order on bipartite lattice
- Blume-Emery-Griffiths model for fermionic systems
- From cobaltites to manganites
- Conclusions

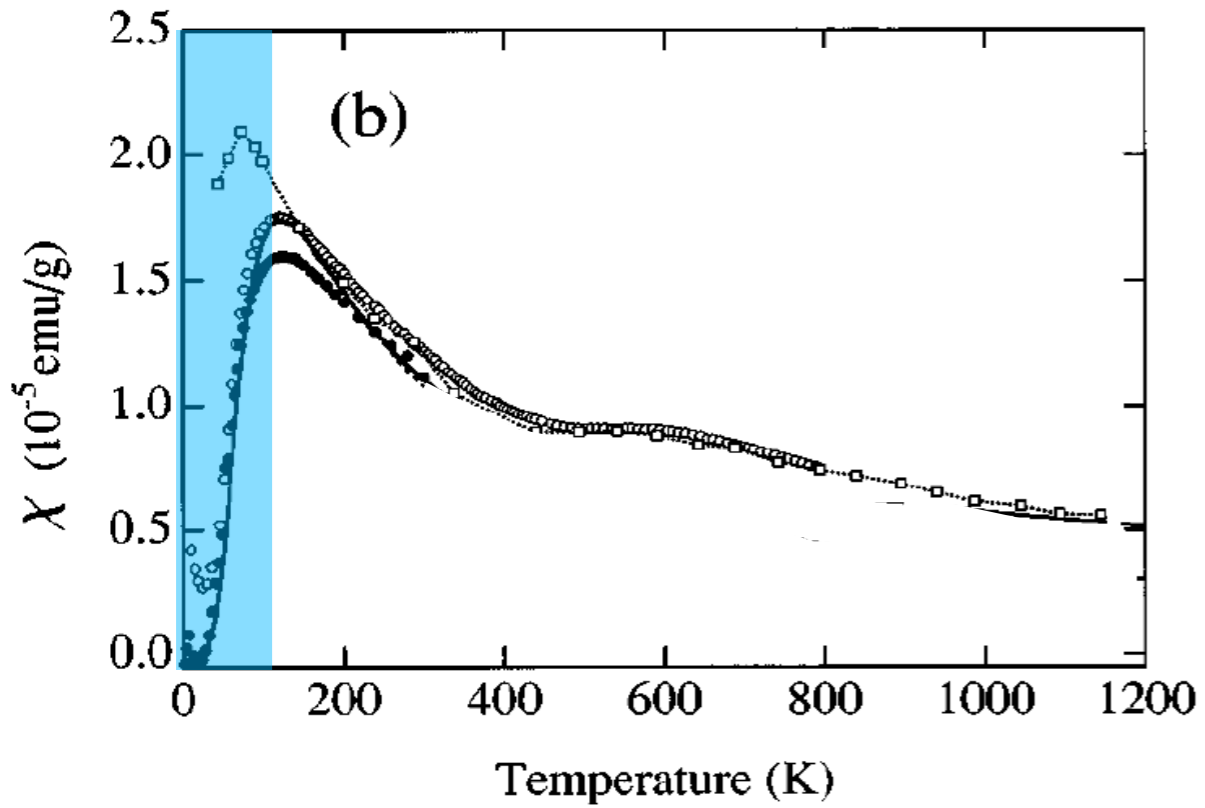
LaCoO₃



Resistivity

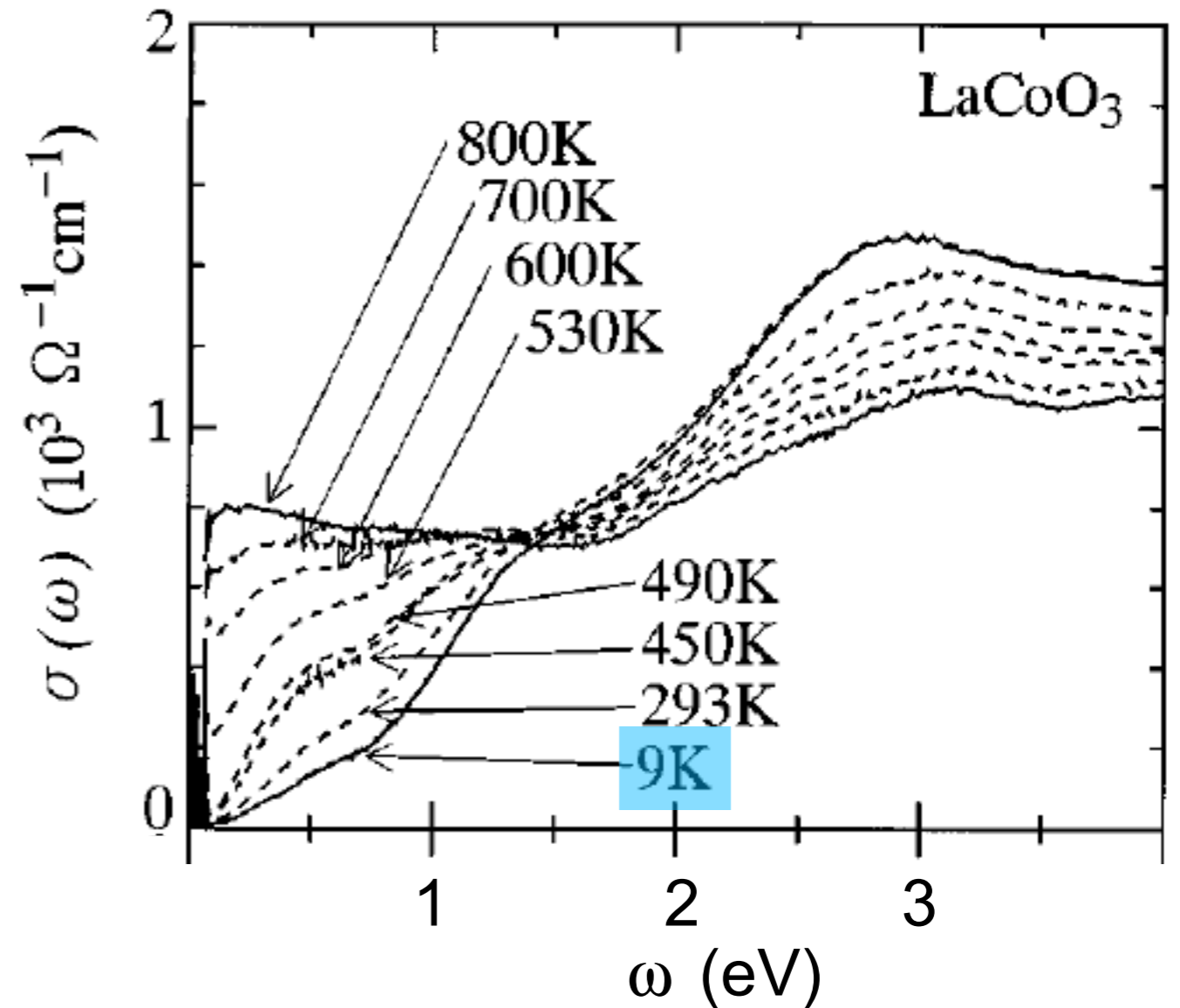
Magnetic susceptibility

LaCoO₃



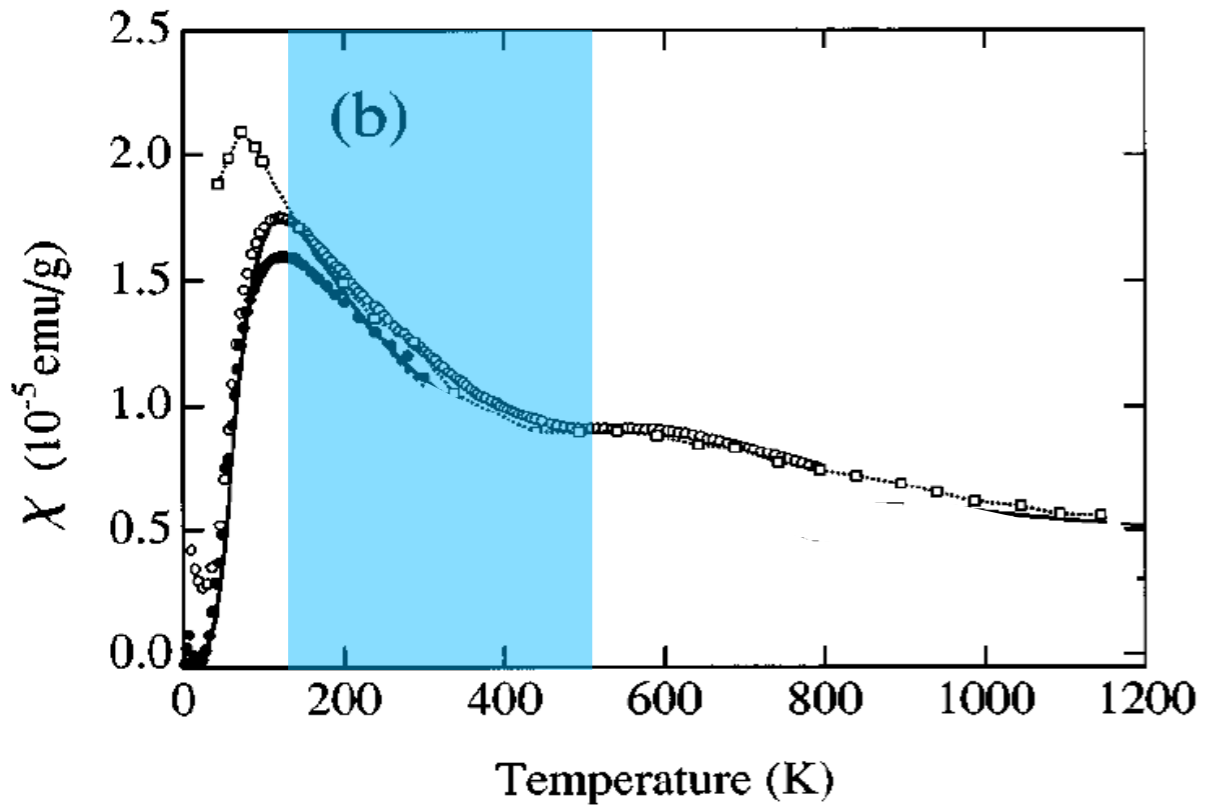
Saitoh et al. *Phys. Rev. B* **55**, 4257 (1997)

Optical conductivity



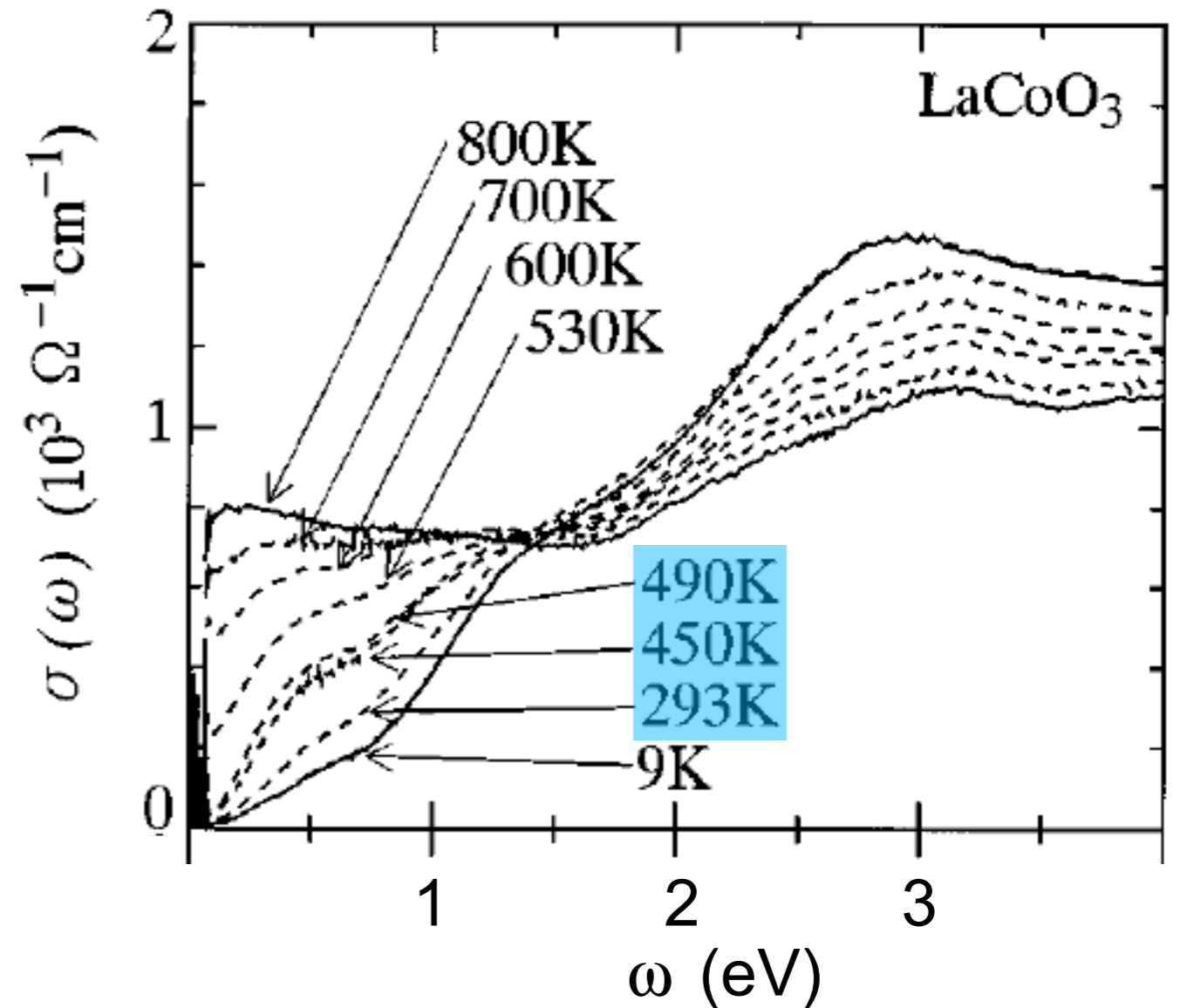
Tokura et al. *Phys. Rev. B* **58**, R1699 (1998)

LaCoO₃



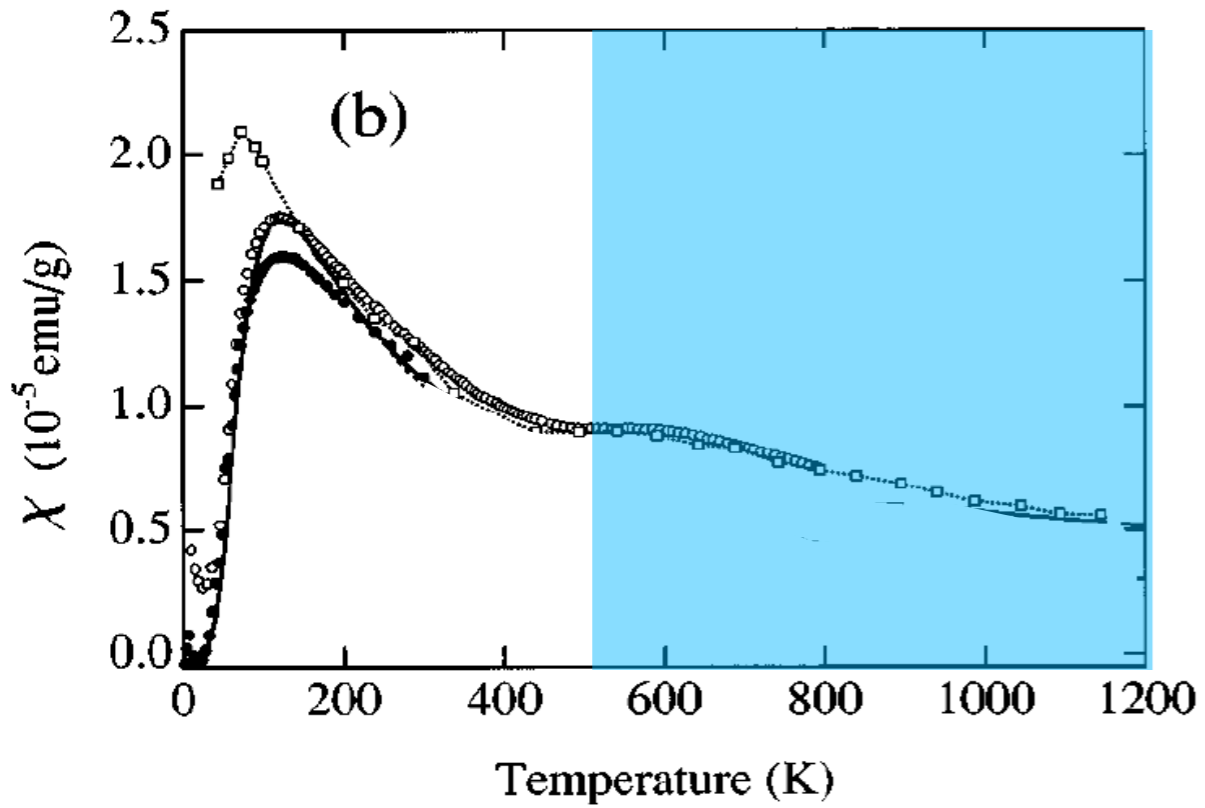
Saitoh et al. *Phys. Rev. B* **55**, 4257 (1997)

Optical conductivity

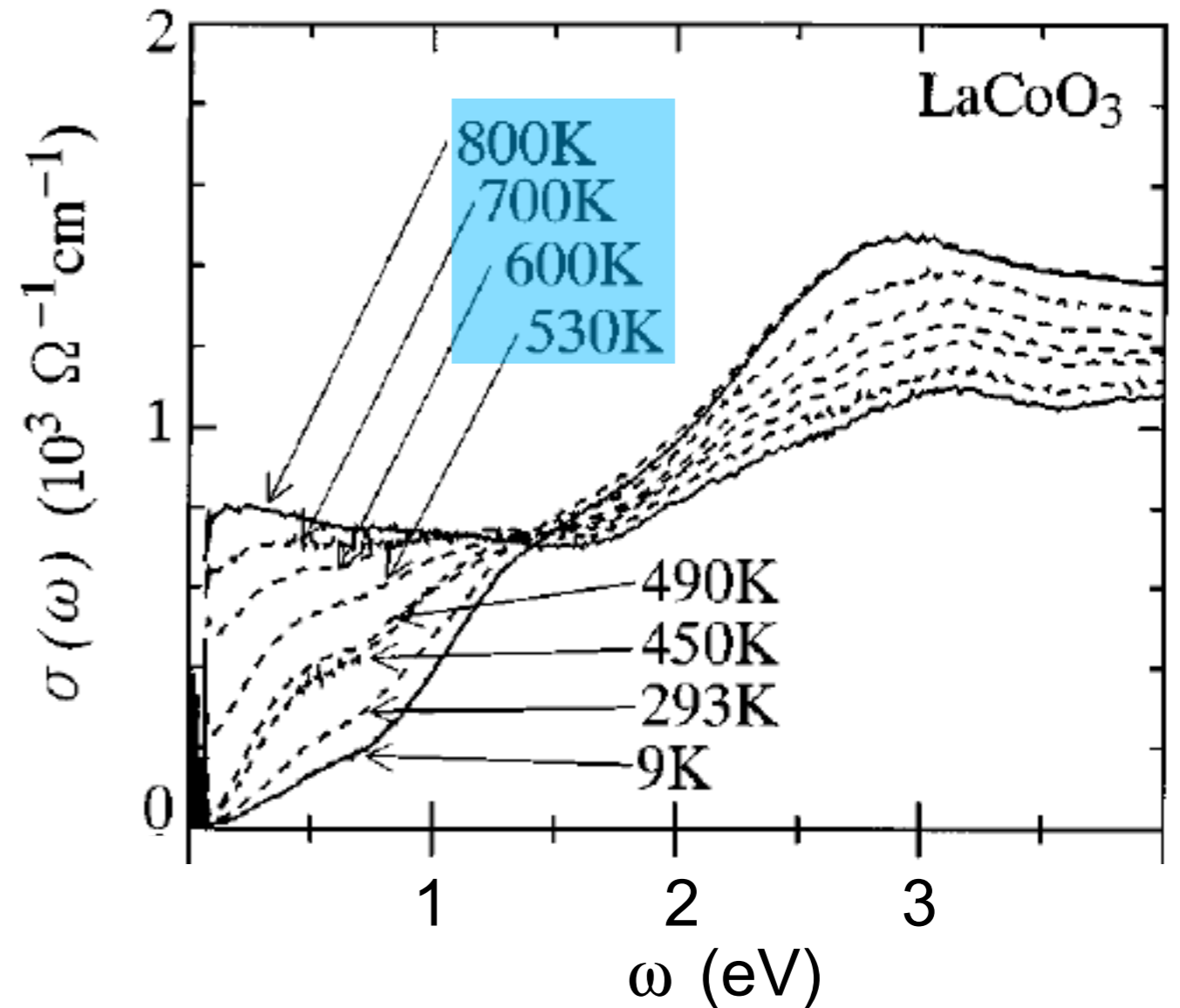


Tokura et al. *Phys. Rev. B* **58**, R1699 (1998)

LaCoO₃



Optical conductivity

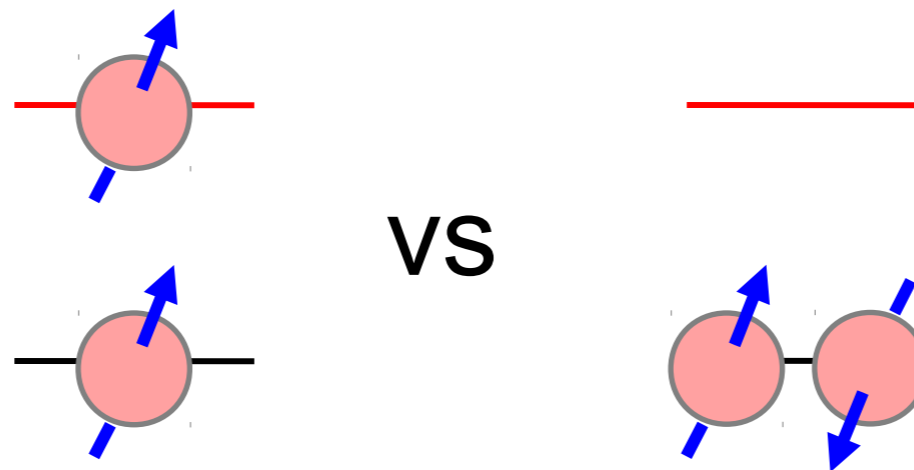


Saitoh et al. *Phys. Rev. B* **55**, 4257 (1997)

Tokura et al. *Phys. Rev. B* **58**, R1699 (1998)

Two-band Hubbard model

$$\begin{aligned}
 H = & \sum_{i,\sigma} ((\Delta - \mu)n_{i,\sigma}^a - \mu n_{i,\sigma}^b) + \sum_{\langle ij \rangle, \sigma} (t_{aa} a_{i,\sigma}^\dagger a_{j,\sigma} + t_{bb} b_{i,\sigma}^\dagger b_{i,\sigma}) \\
 & + U \sum_i (n_{i,\uparrow}^a n_{i,\downarrow}^a + n_{i,\uparrow}^b n_{i,\downarrow}^b) + (U - 2J) \sum_{i,\sigma} n_{i,\sigma}^a n_{i,-\sigma}^b \\
 & + (U - 3J) \sum_{i,\sigma} n_{i,\sigma}^a n_{i,\sigma}^b
 \end{aligned}$$

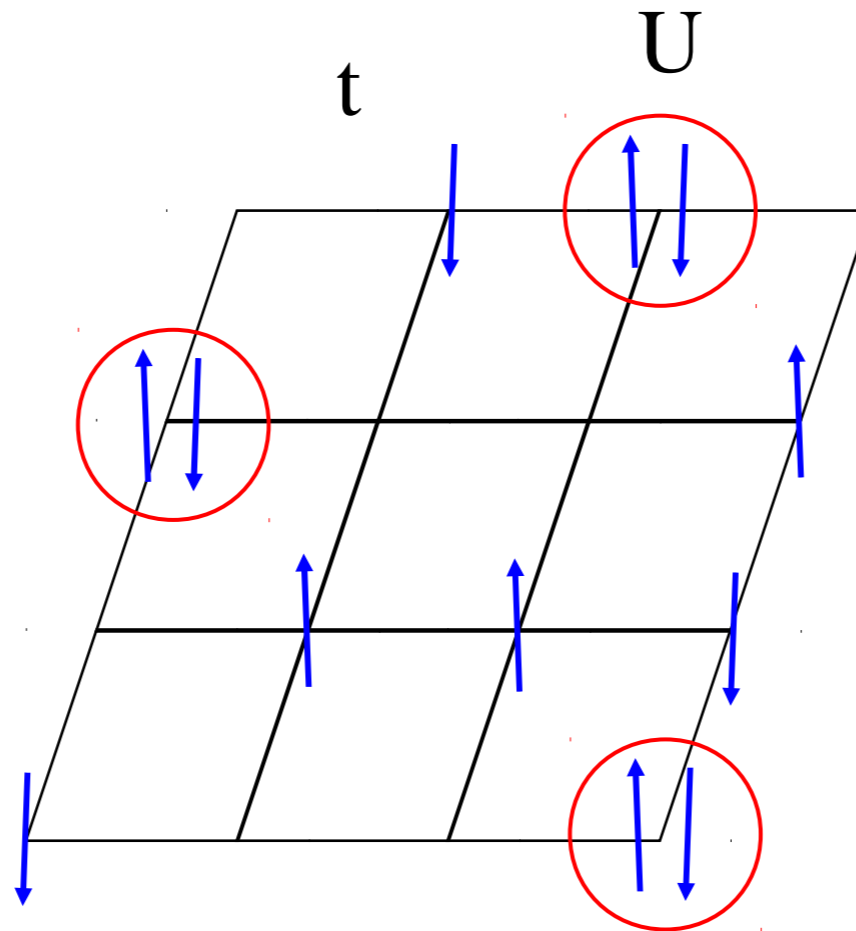


Dynamical mean-field theory

Hubbard model in $d=\infty$

Hamiltonian = Hopping + Local interaction

$$H = \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Large dimension limit - classical Heisenberg model

$$Z = \sum_{\{s_i\}} \exp(-S)$$

Cavity construction: $S = S_0 + \Delta S + S_{(0)}$ $\Delta S = \sum_i J_{0i} s_0 s_i$

Expansion in 'hybridization':

$$Z = Z_{(0)} \sum_{s_0} \exp(-S_0) \left(1 + \sum_i J_{i0} \langle s_i \rangle_{(0)} s_0 + \frac{1}{2} \sum_{i,j} J_{i0} J_{j0} \langle s_i s_j \rangle_{(0)} s_0^2 + \dots \right)$$

Cumulant expansion:

$$Z = Z_{(0)} \sum_{s_0} \exp(-S_{\text{eff}})$$

$$S_{\text{eff}} = h s_0 + \sum_i J_{i0} \langle s_i \rangle_{(0)} s_0 + \frac{1}{2} \sum_{i,j} J_{i0} J_{j0} \left(\langle s_i s_j \rangle_{(0)} - \langle s_i \rangle_{(0)} \langle s_j \rangle_{(0)} \right) s_0^2 + \dots$$

Scaling:

$$J = \frac{J^*}{d}$$

Dynamical mean-field theory

- Weak coupling expansion

In $d=\infty$ limit only local contributions to the self-energy survive!

$$\Sigma(\mathbf{k}, \omega) \equiv \Sigma(\omega)$$

$$\Sigma = \Phi[G_{\text{loc}}]$$

- Cumulant expansion (cavity construction)

In $d=\infty$ limit the action reduces to interacting site subject to “time dependent” external field equivalent to fermionic bath

*Georges and Kotliar,
PRB 45, 6479 (1992)*

DMFT

Weiss molecular field

$$H = \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$H = \sum_{i,j} J_{ij} S_i S_j + h \sum_i S_i$$

$$G_{ii}(\tau) = -\langle T c_i(\tau) c_i^\dagger(0) \rangle$$

$$s_i = \langle S_i \rangle$$

$$H_{\text{loc}} = (\varepsilon - \mu) \sum_{\sigma} c_{\sigma}^\dagger c_{\sigma} + U n_{\uparrow} n_{\downarrow} + \sum_{r\sigma}^{\sigma} \eta_r b_{r\sigma}^\dagger b_{r\sigma} + \sum_{r\sigma} V_r c_{\sigma}^\dagger b_{r\sigma} + c.c.$$

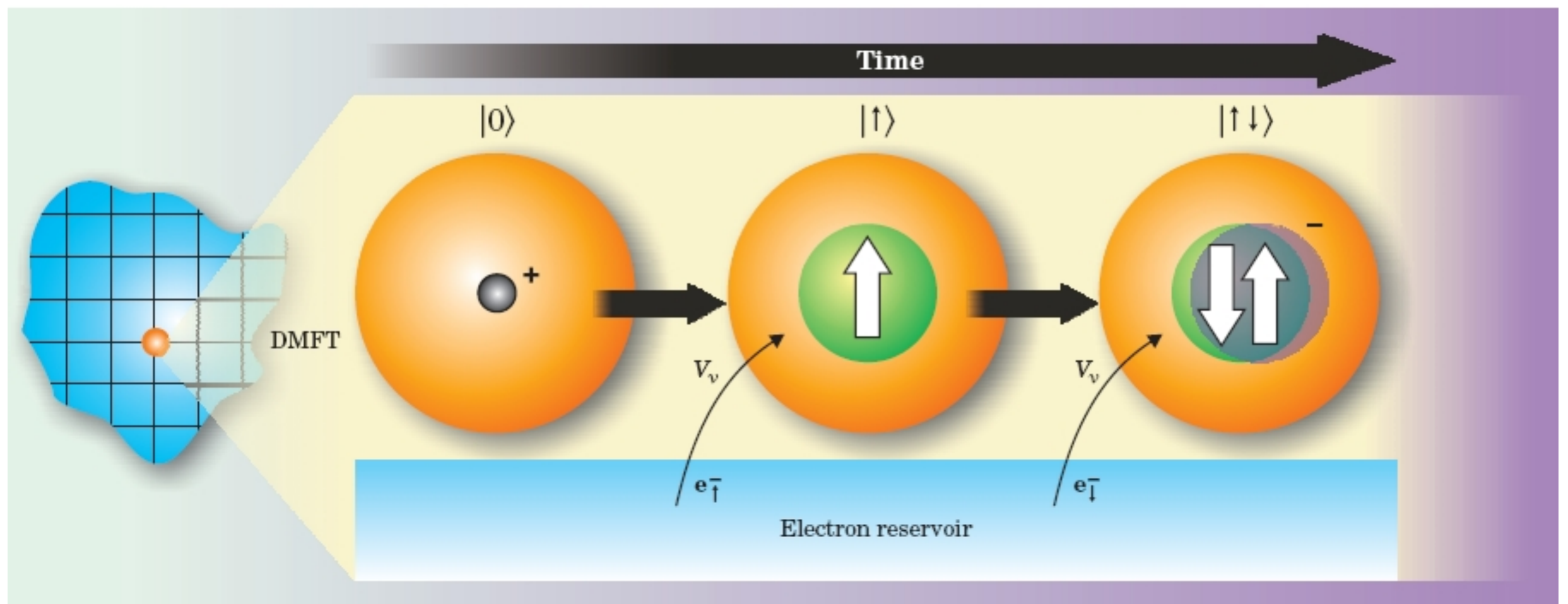
$$H_{\text{loc}} = \tilde{h} S$$

$$G_{ii}(\omega) = \sum_k \frac{1}{\omega + \mu - \varepsilon_k - \Sigma(\omega)}$$

$$\tilde{h} = \sum_i J_{0i} s_i + h$$

Dynamical Mean-Field Theory (DMFT)

- Single out a site from the lattice
- Replace the rest of the lattice by an effective medium
- Solve the impurity many-body problem
- Reconstruct lattice quantities



Impurity problem

Hamiltonian formulation

$$H_{\text{loc}} = (\varepsilon - \mu) \sum_{\sigma} c_{\sigma}^{\dagger} c_{\sigma} + U n_{\uparrow} n_{\downarrow} + \sum_{r\sigma} \eta_r b_{r\sigma}^{\dagger} b_{r\sigma} + \sum_{r\sigma} V_r c_{\sigma}^{\dagger} b_{r\sigma} + c.c.$$

$$\Delta(\omega) = \sum_r \frac{|V_r|^2}{\omega - \eta_r}$$

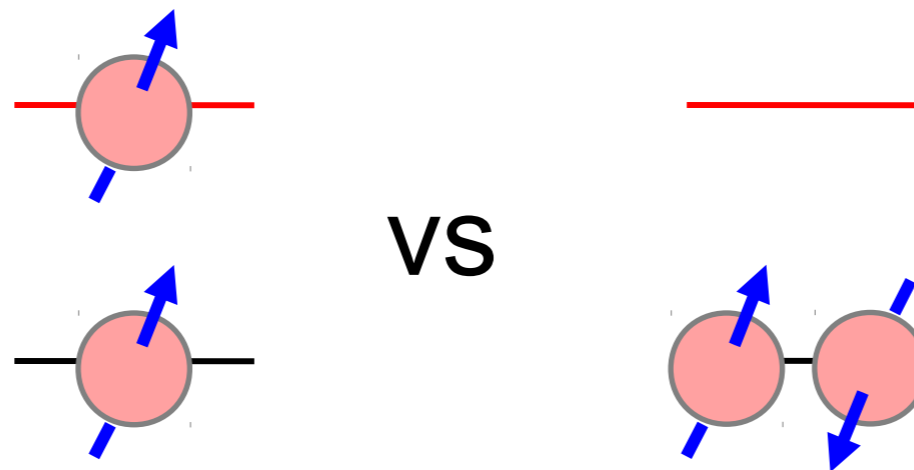
Action formulation

$$S_{\text{imp}} = - \int_0^{\beta} d\tau \sum_{\sigma} \bar{c}_{\sigma}(\tau) \left(\frac{\partial}{\partial \tau} + \mu - \varepsilon \right) c_{\sigma}(\tau) + U n_{\uparrow}(\tau) n_{\downarrow}(\tau) - \int_0^{\beta} d\tau \int_0^{\beta} d\tau' \sum_{\sigma} \bar{c}_{\sigma}(\tau) \Delta(\tau - \tau') c_{\sigma}(\tau')$$

$$G_{ii}(\tau) = - \langle T c_i(\tau) c_i^{\dagger}(0) \rangle$$

Two-band Hubbard model

$$\begin{aligned}
 H = & \sum_{i,\sigma} ((\Delta - \mu)n_{i,\sigma}^a - \mu n_{i,\sigma}^b) + \sum_{\langle ij \rangle, \sigma} (t_{aa} a_{i,\sigma}^\dagger a_{j,\sigma} + t_{bb} b_{i,\sigma}^\dagger b_{i,\sigma}) \\
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 & + (U - 3J) \sum_{i,\sigma} n_{i,\sigma}^a n_{i,\sigma}^b
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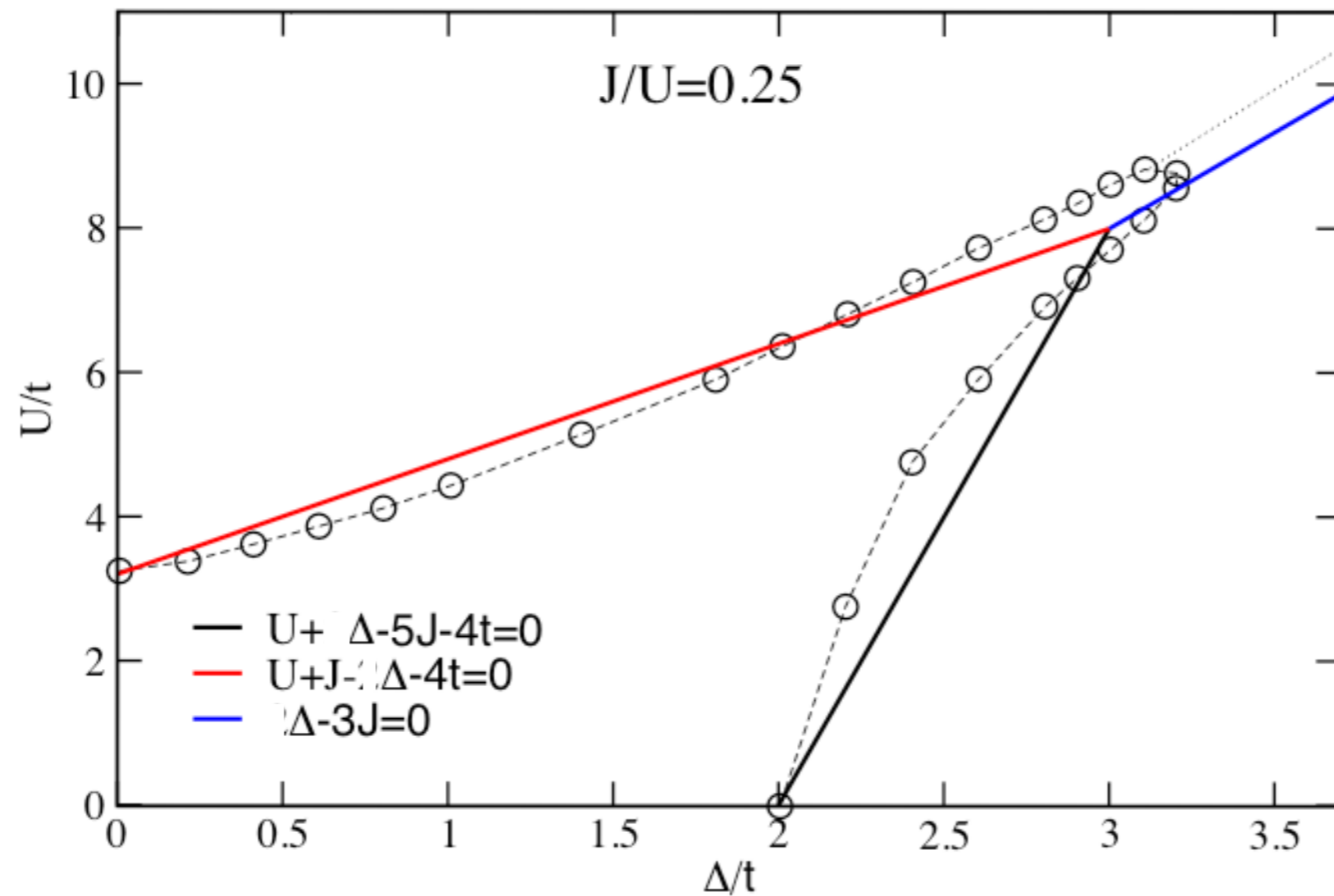


U- Δ phase diagram

Δ - crystal field

J/U - fixed

Uniform phase (arbitrary lattice - DMFT)



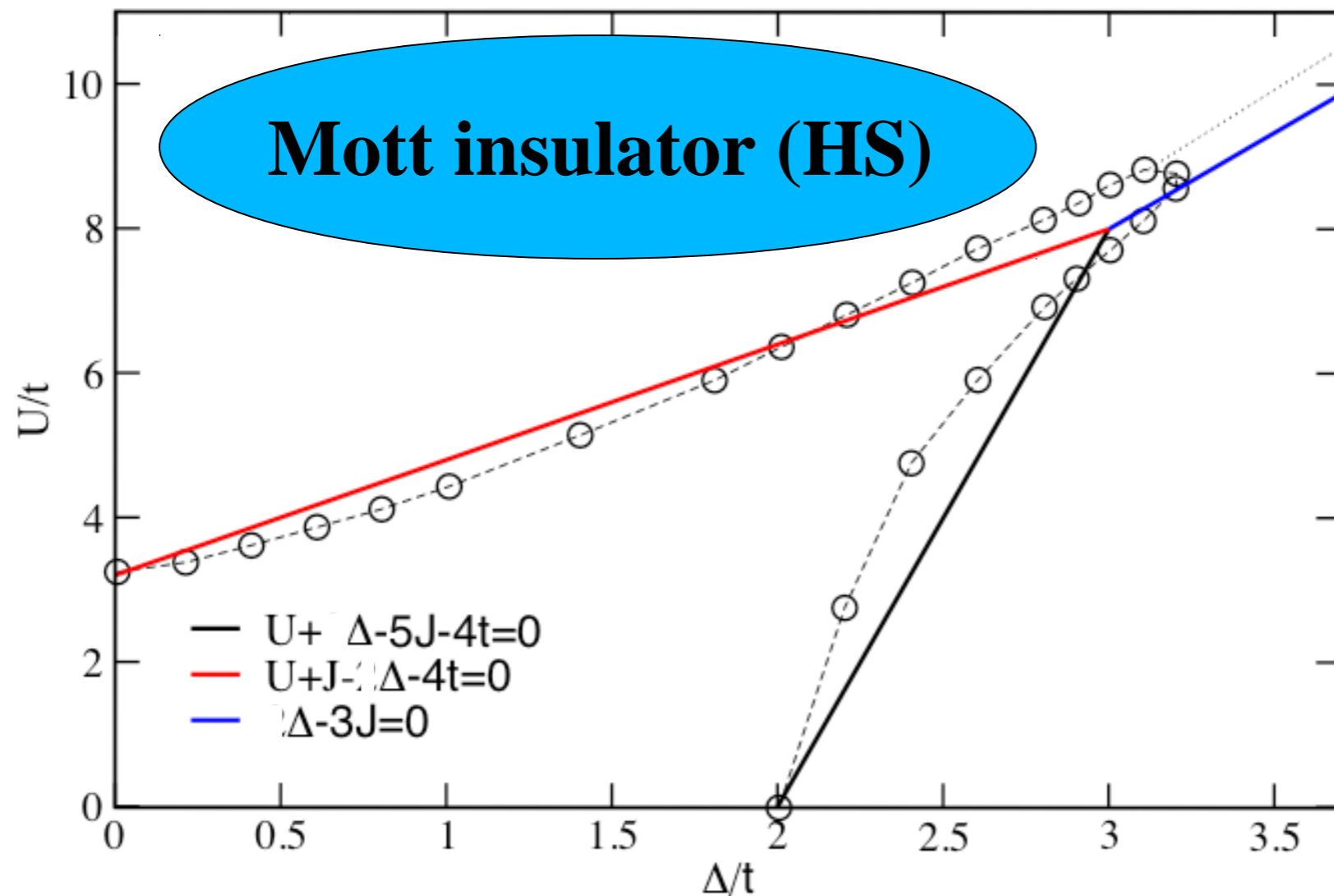
Werner & Millis, *Phys. Rev. Lett.* **99**, 126405 (2007)
JK et al. *Eur. Phys. J. Special Topics* **180**, 5 (2009)

U- Δ phase diagram

Δ - crystal field

J/U - fixed

2D - bipartite lattice (square lattice)



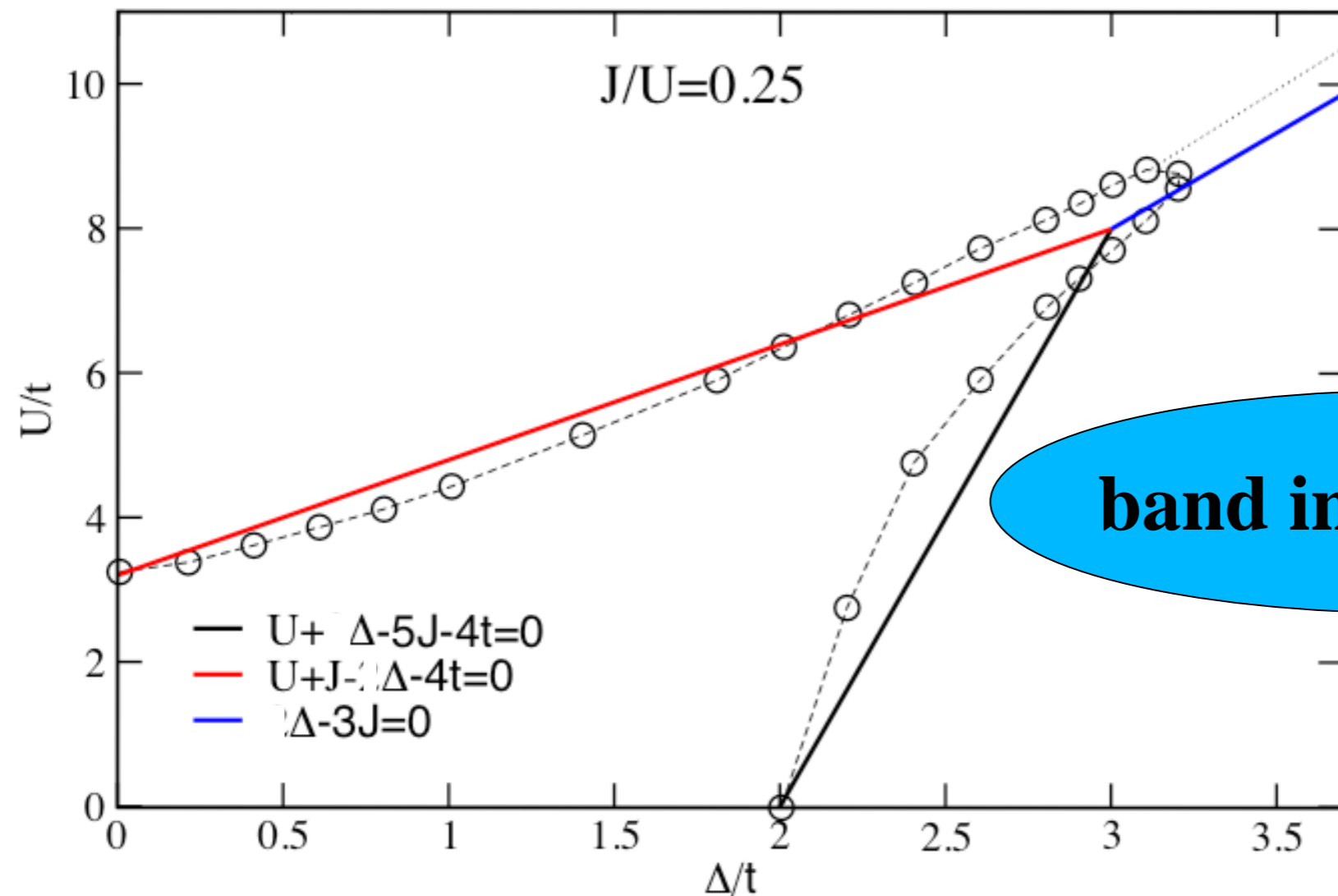
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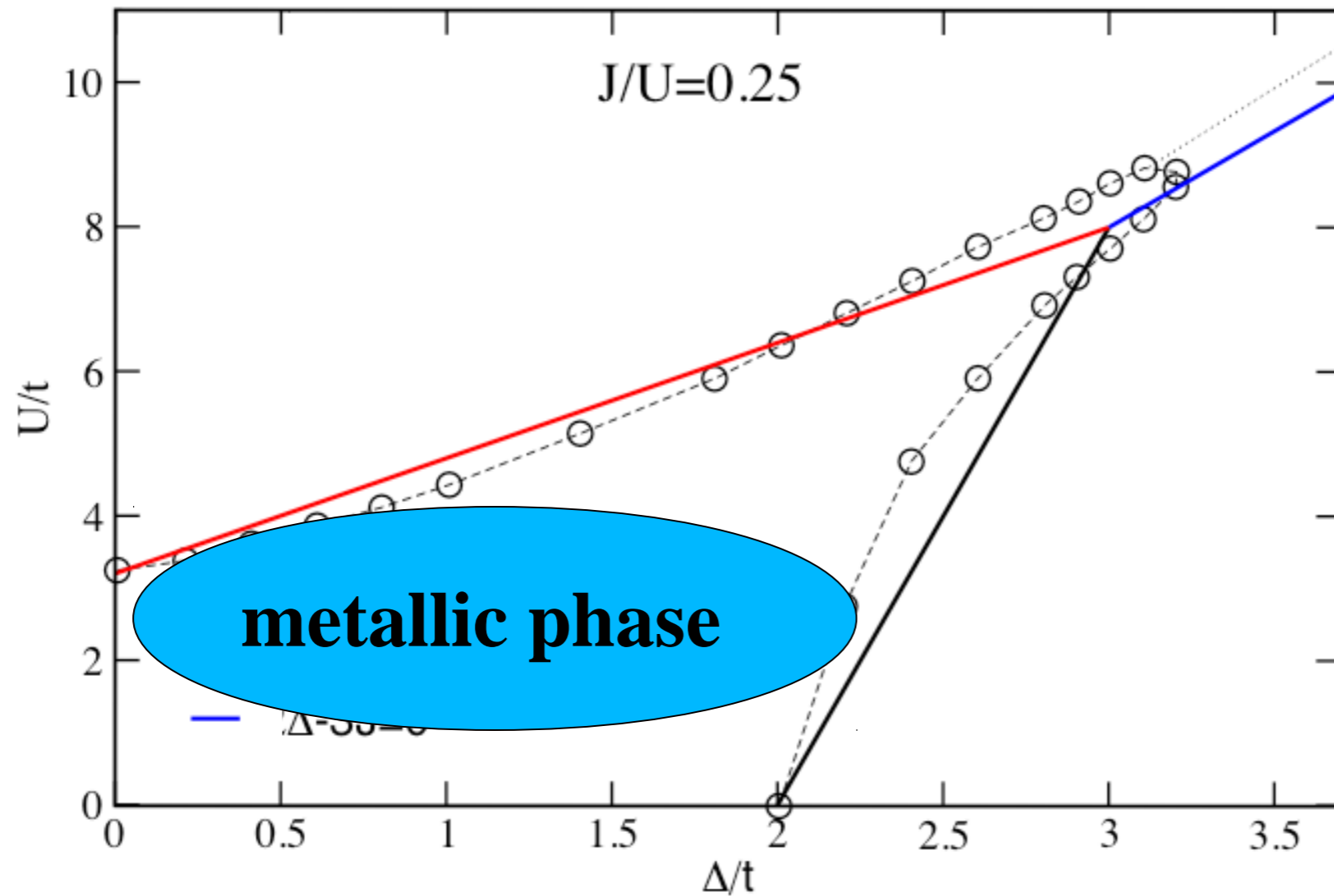
Werner & Millis, *Phys. Rev. Lett.* **99**, 126405 (2007)
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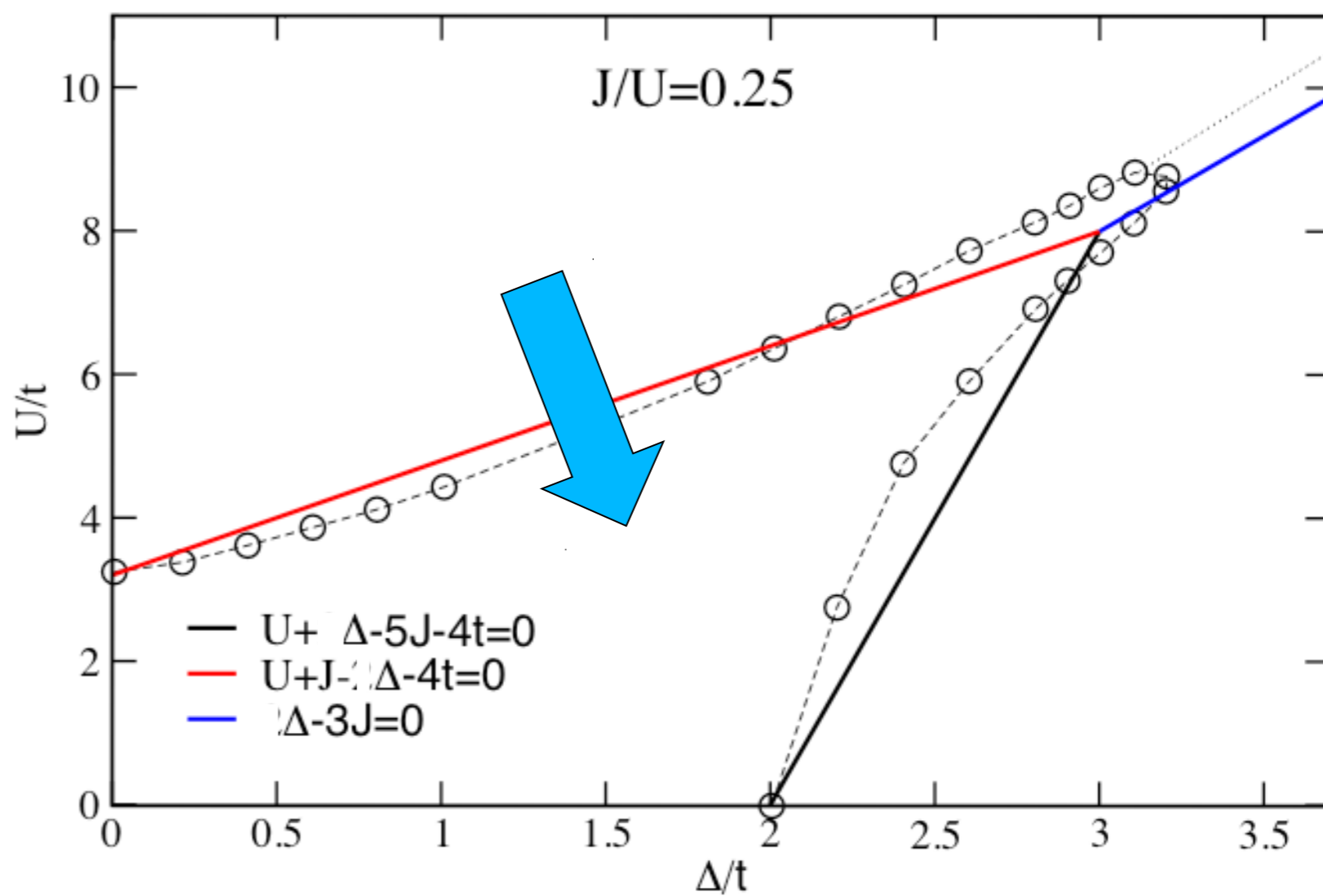
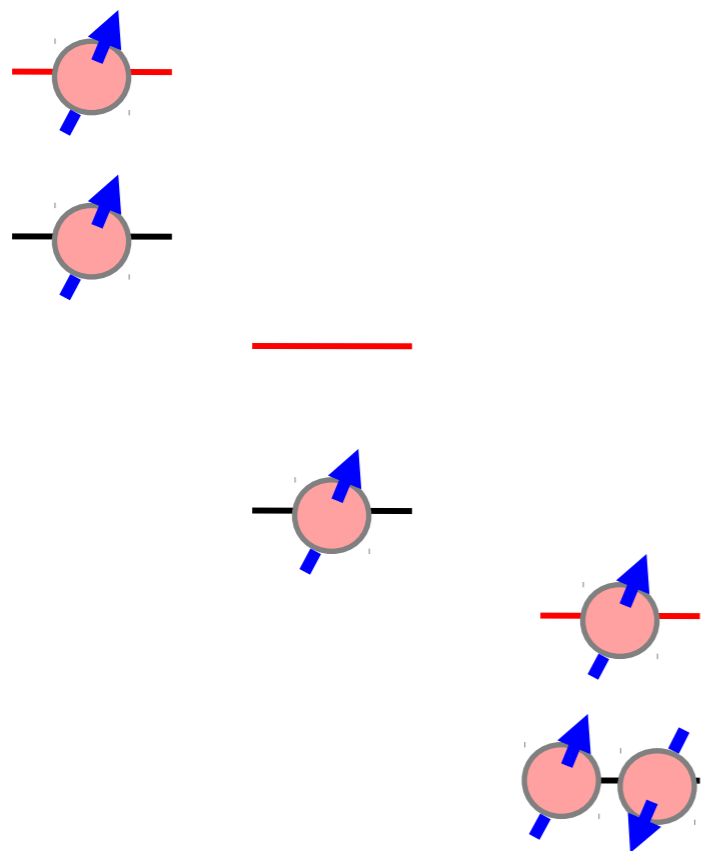


Werner & Millis, *Phys. Rev. Lett.* **99**, 126405 (2007)
JK et al. *Eur. Phys. J. Special Topics* **180**, 5 (2009)

Gap closing

'Mott gap = 0'

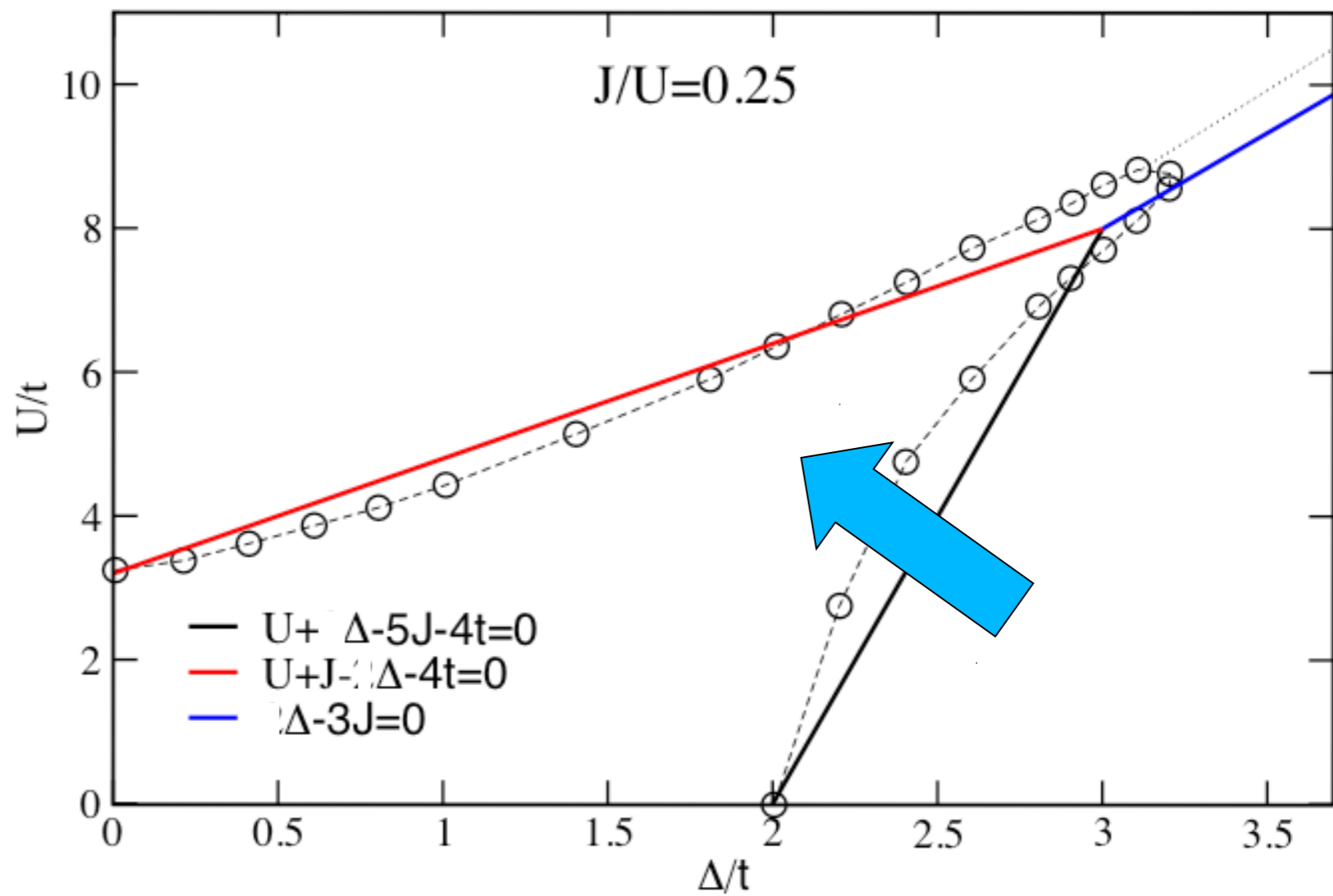
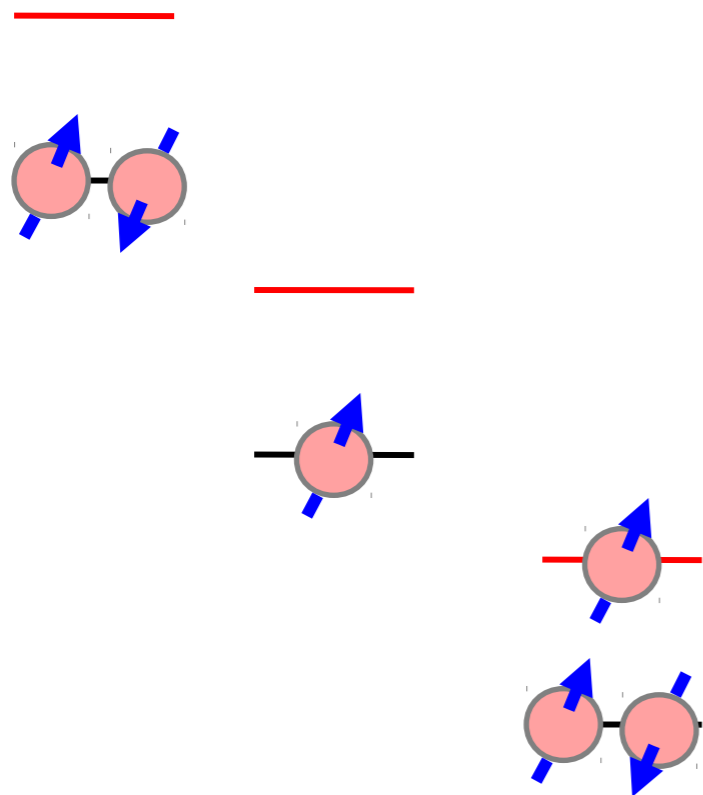
$$E_g = 2E(N) - E(N-1) - E(N+1)$$



Band gap

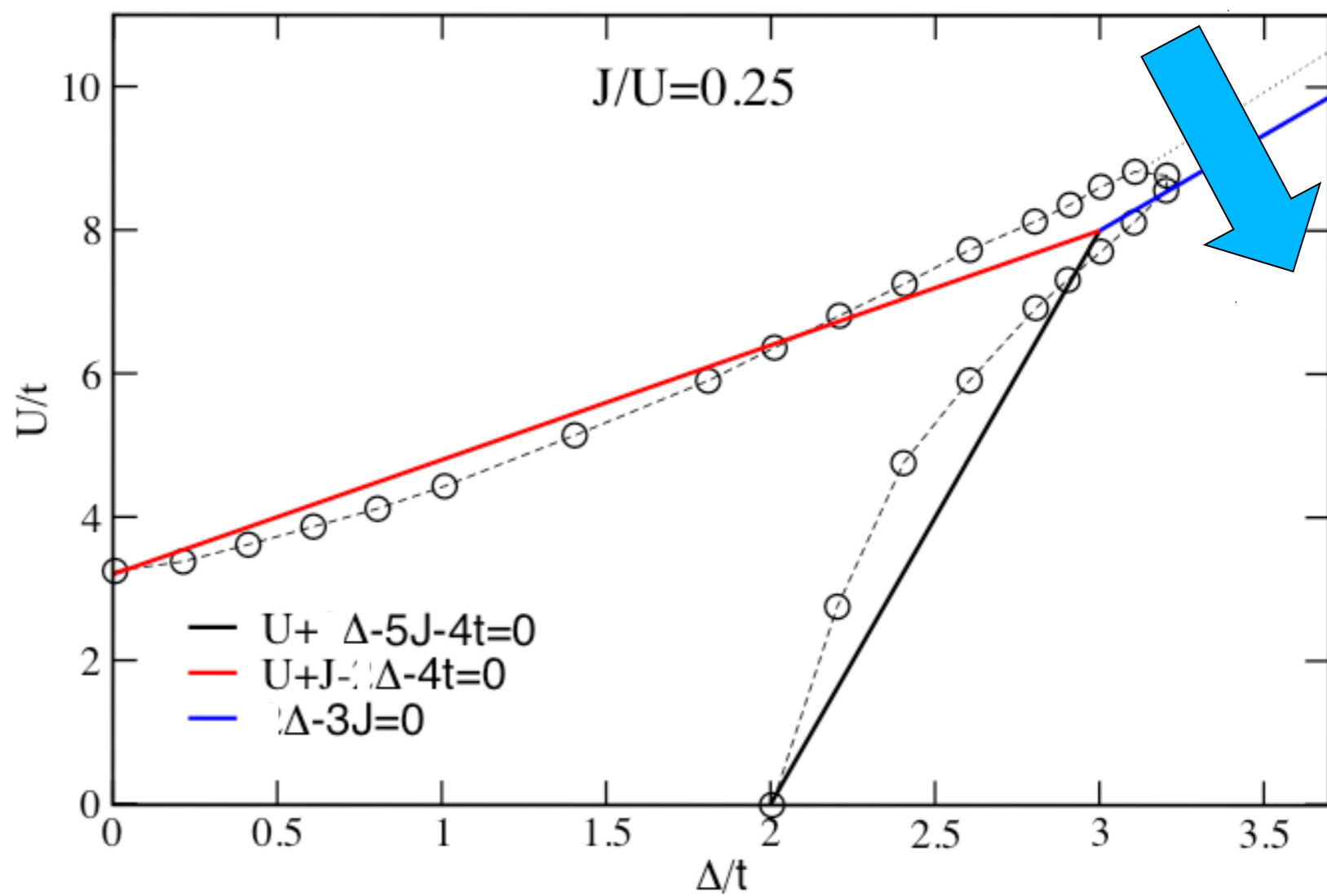
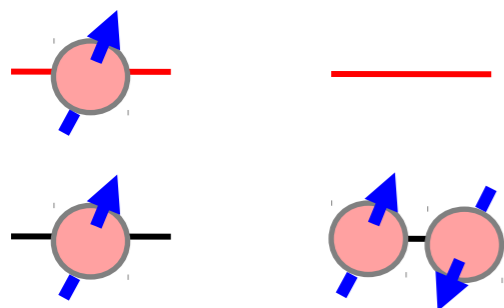
'Band gap = 0'

$$E_g = 2E(N) - E(N-1) - E(N+1)$$



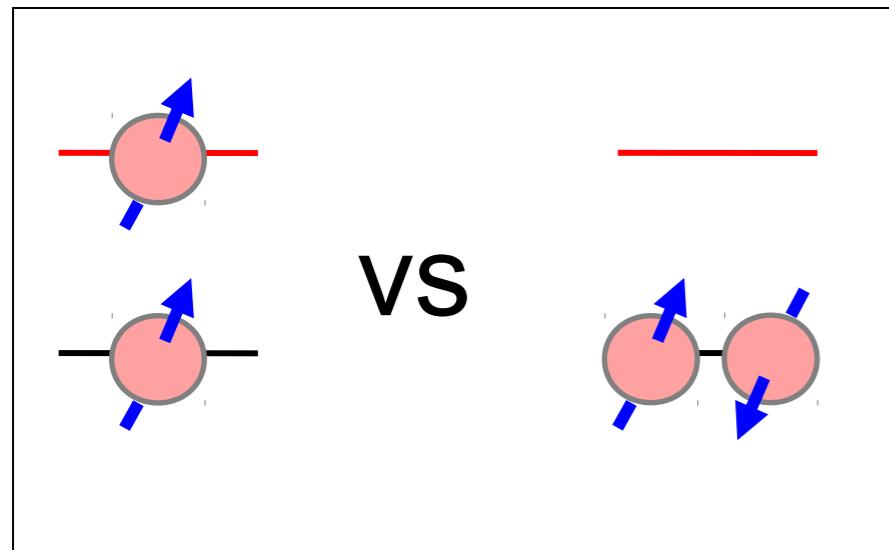
Local state transition

$$'E(\text{HS}) - E(\text{LS}) = 0'$$

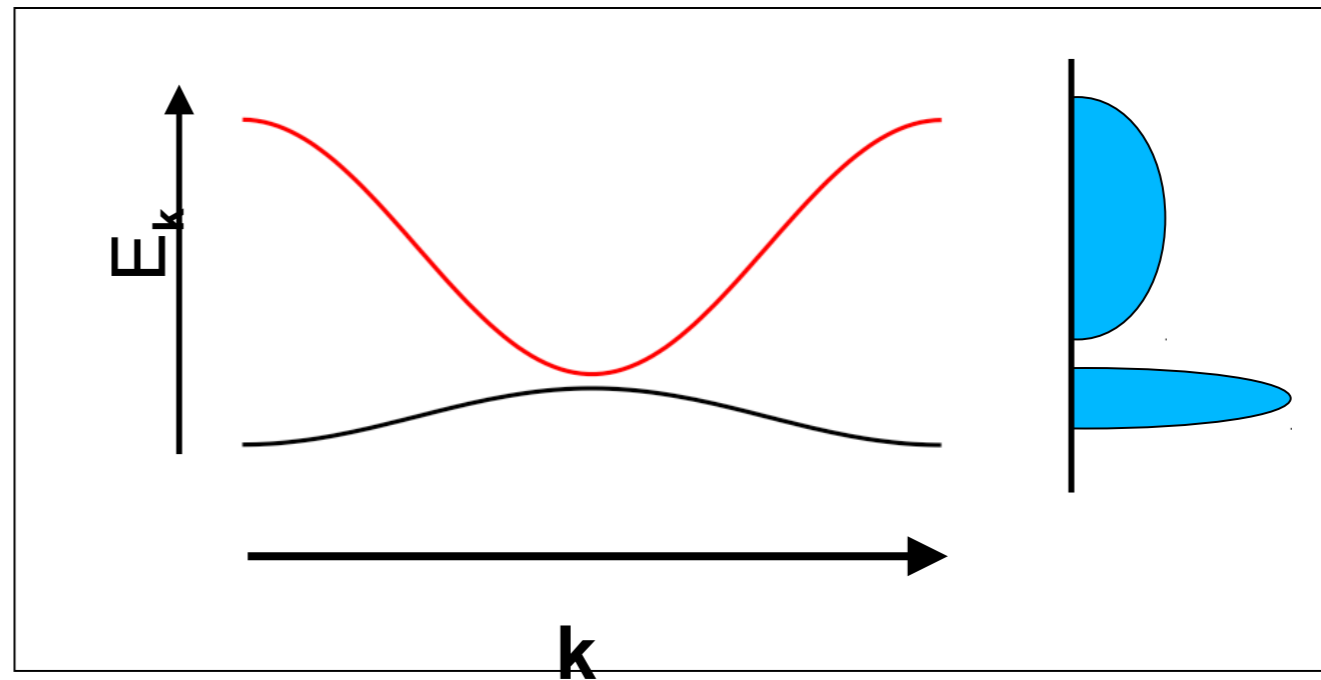


Two-band Hubbard model at half filling

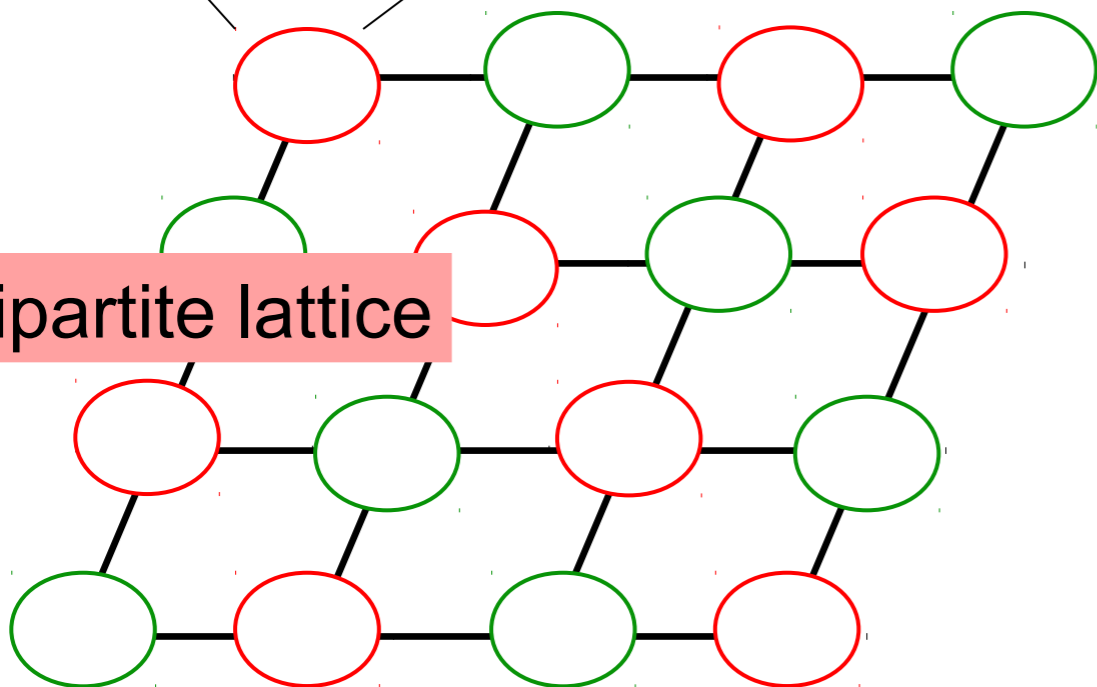
on-site HS-LS competition



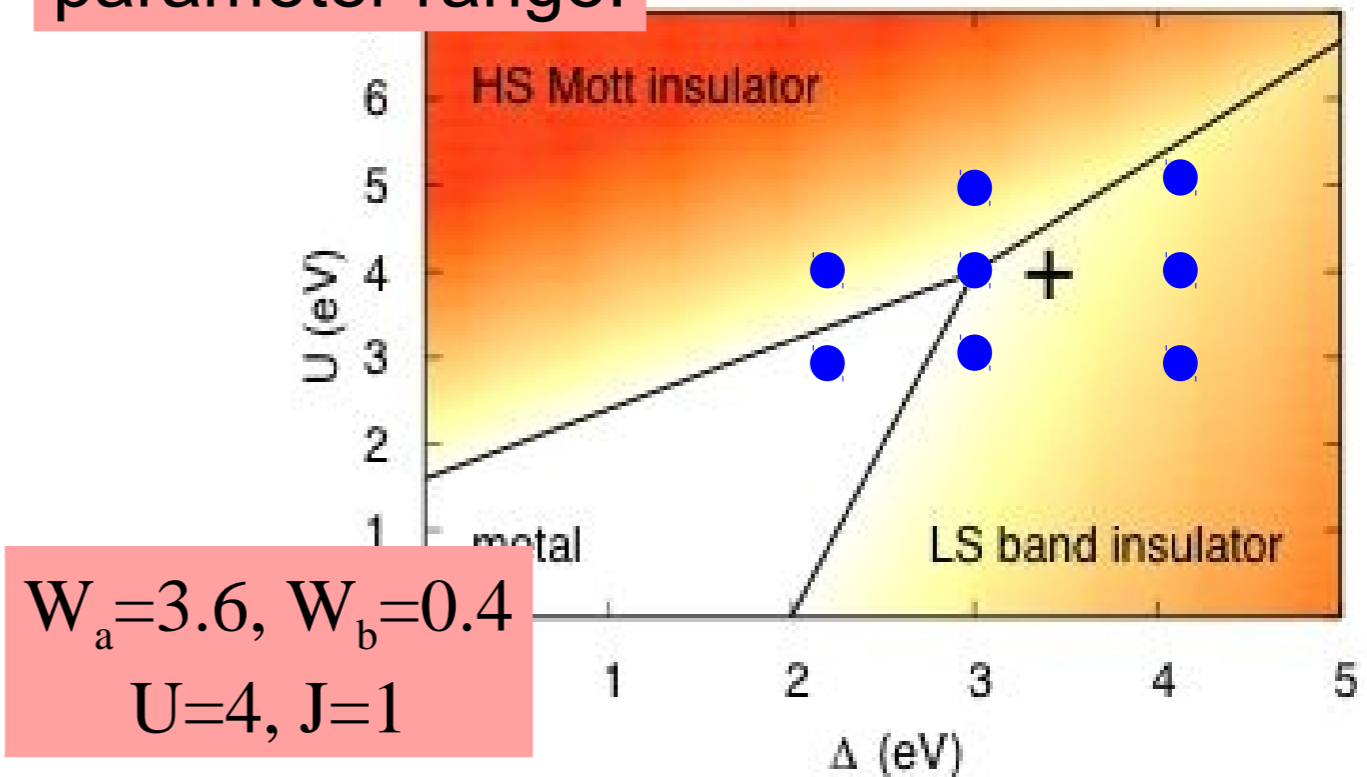
non-interacting band structure



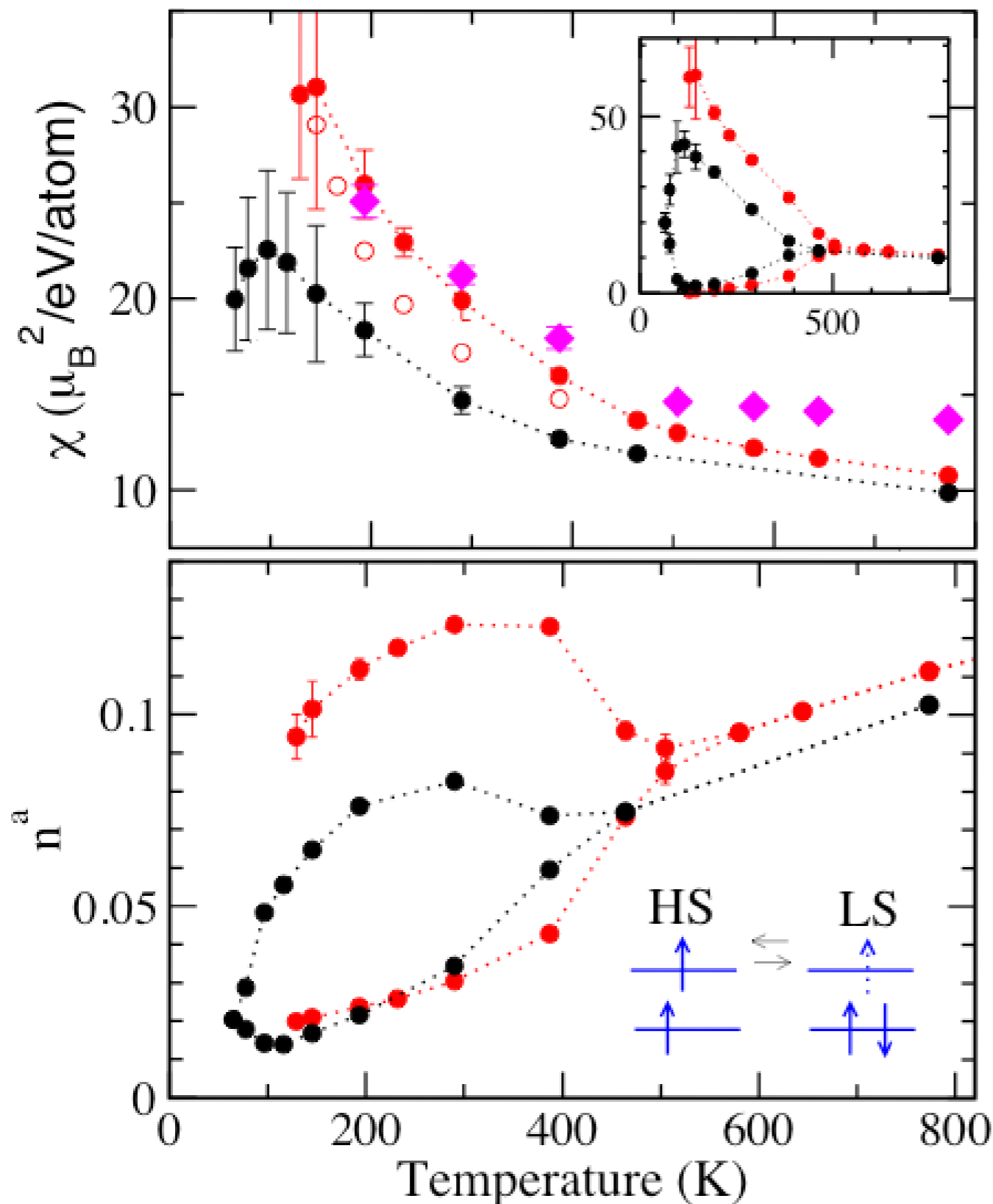
bipartite lattice



parameter range:



Spin susceptibility and disproportionation



$\Delta-3J=0.42$

local susceptibility 


$\Delta-3J=0.40$

local susceptibility 

local susceptibility (homog. ph.) 

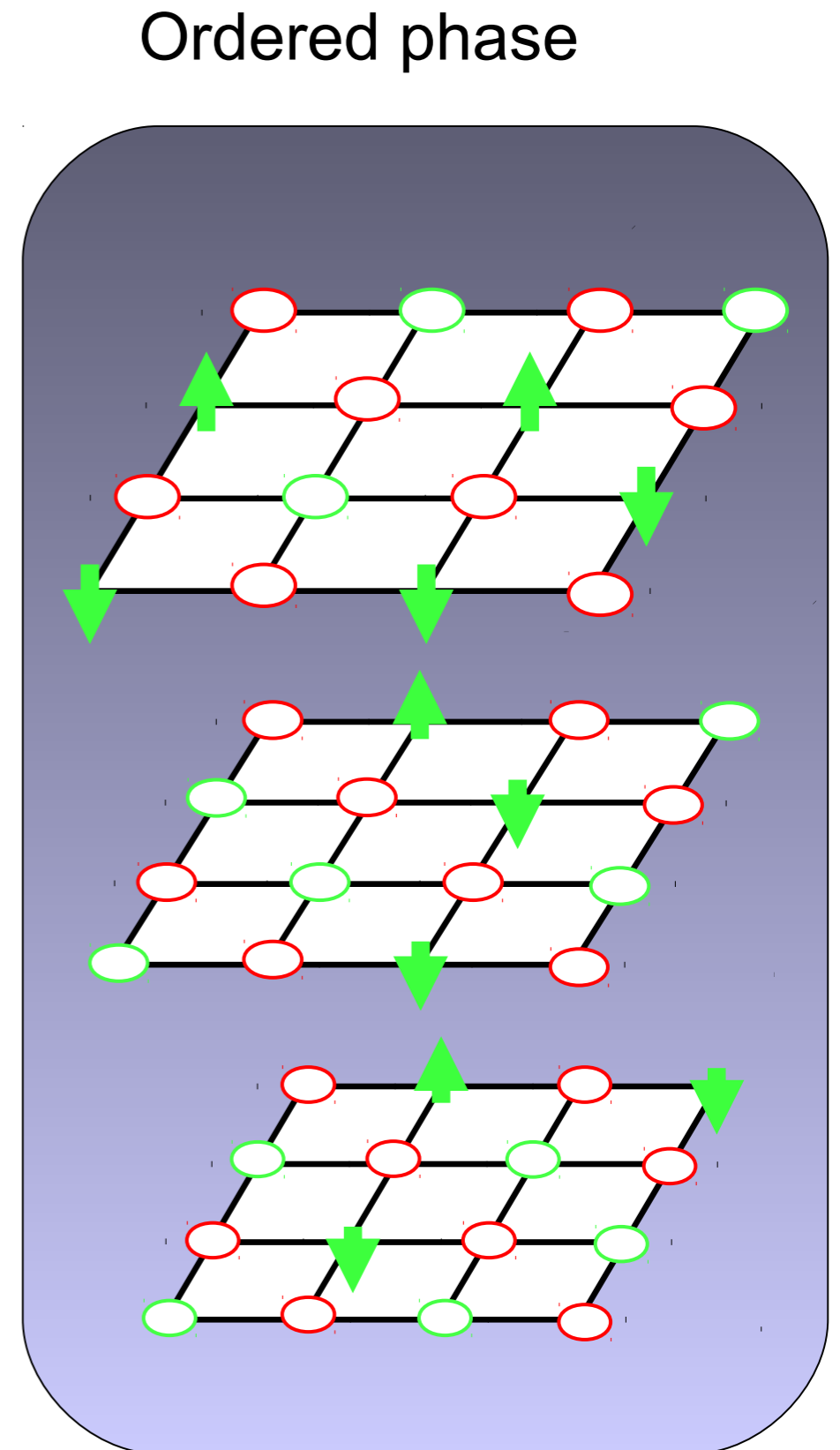
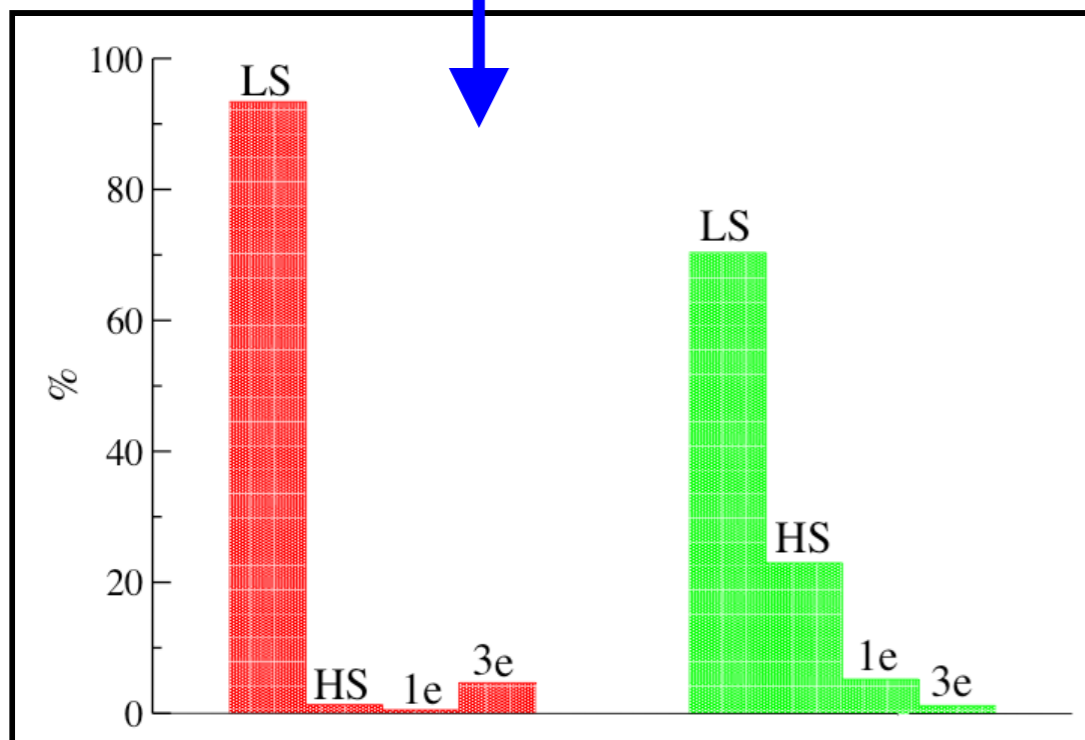
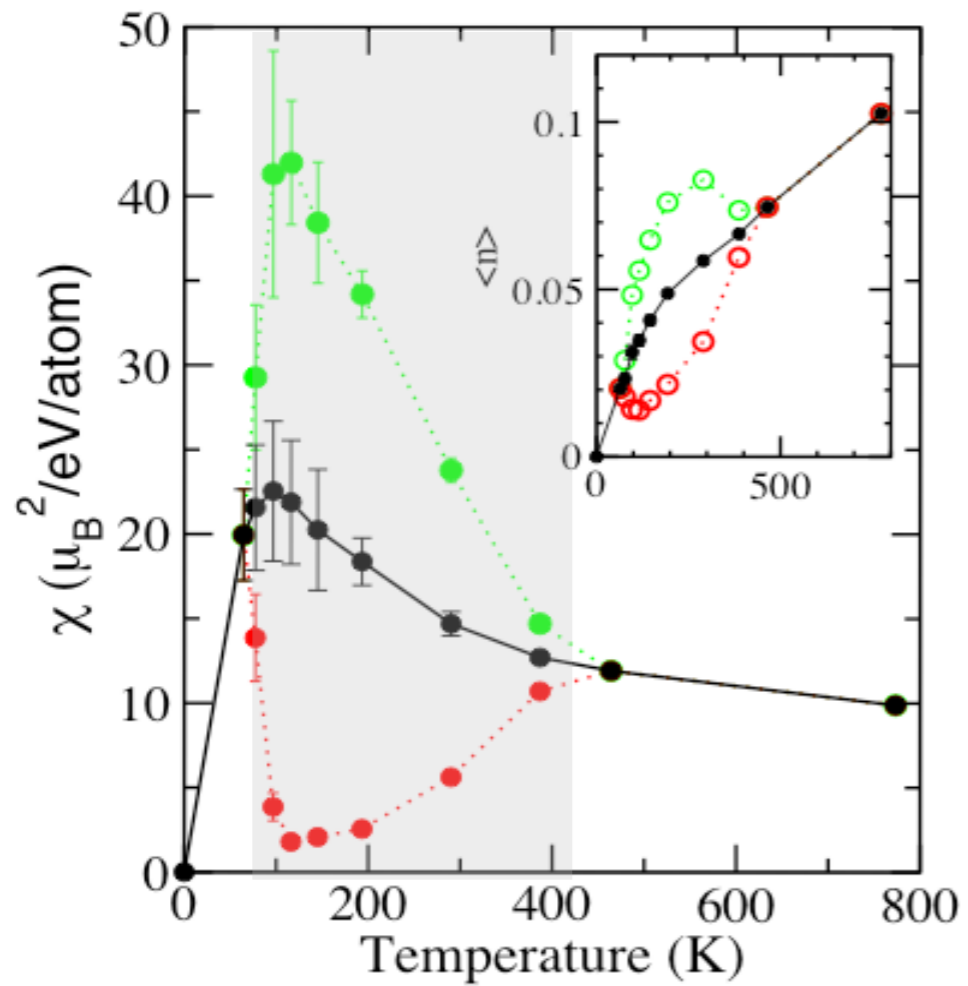
uniform susceptibility 

Site occupancy n^a (upper band)

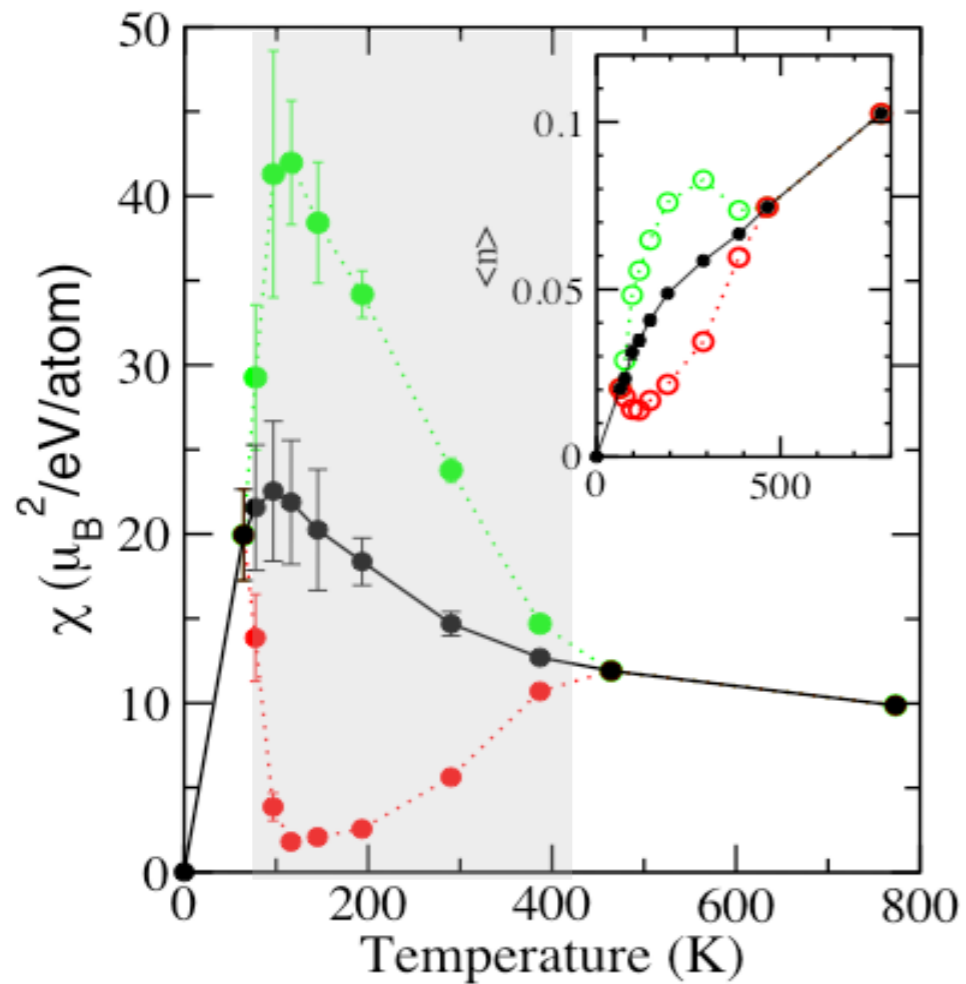
$\Delta-3J=0.42$ 

$\Delta-3J=0.40$ 

Reentrant HS-LS disproportionation

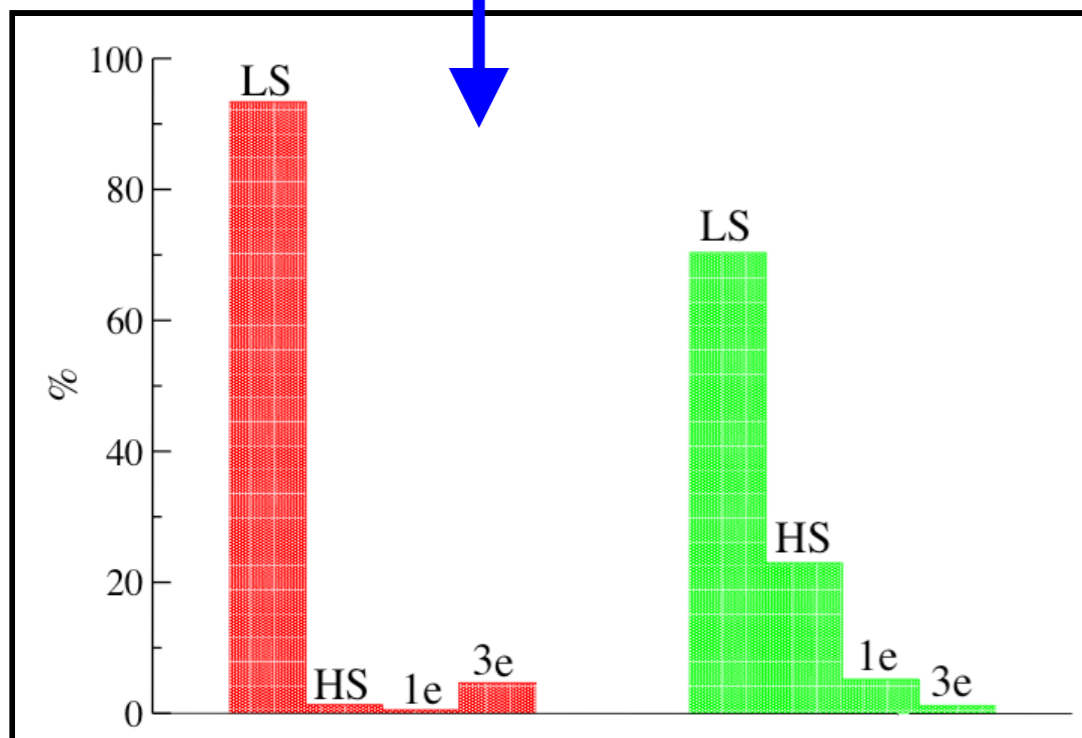
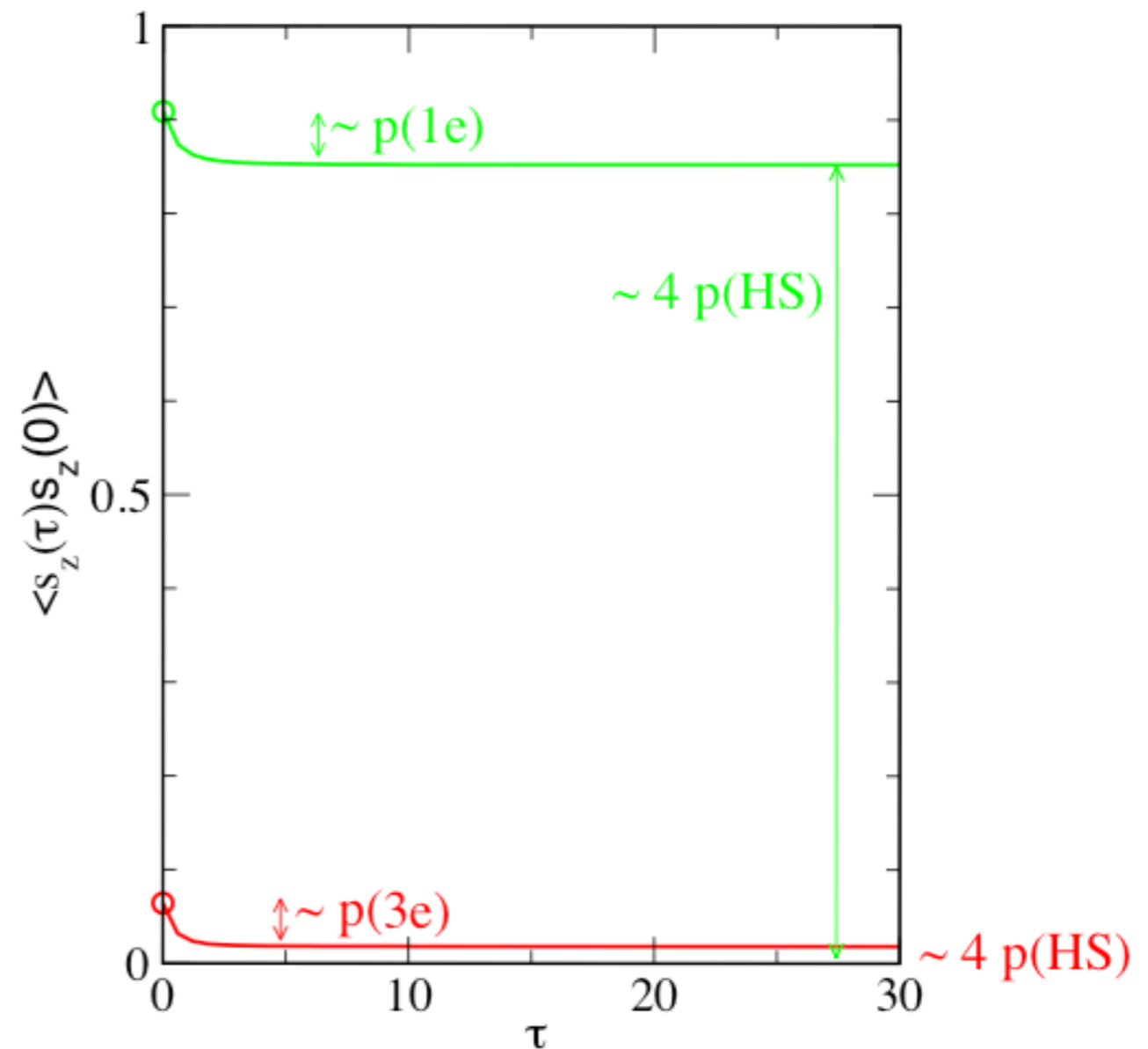


Reentrant HS-LS disproportionation



short excursions
VS
statistical mixture

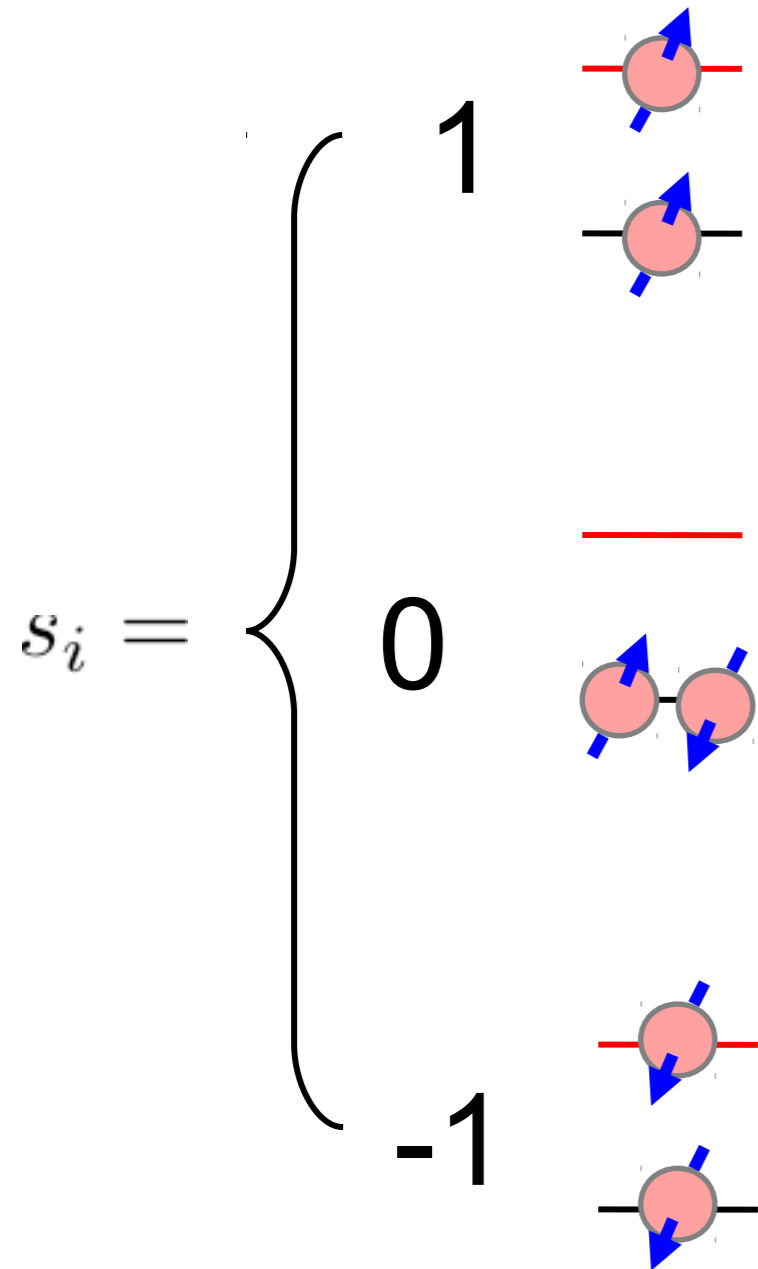
Spin-spin correlations



Blume-Emery-Griffiths model

$$\tilde{H} = D \sum_i s_i^2 + K \sum_{\langle ij \rangle} s_i^2 s_j^2 + I \sum_{\langle ij \rangle} s_i s_j$$

Blume et al., Phys. Rev. A 4, 1071 (1971)

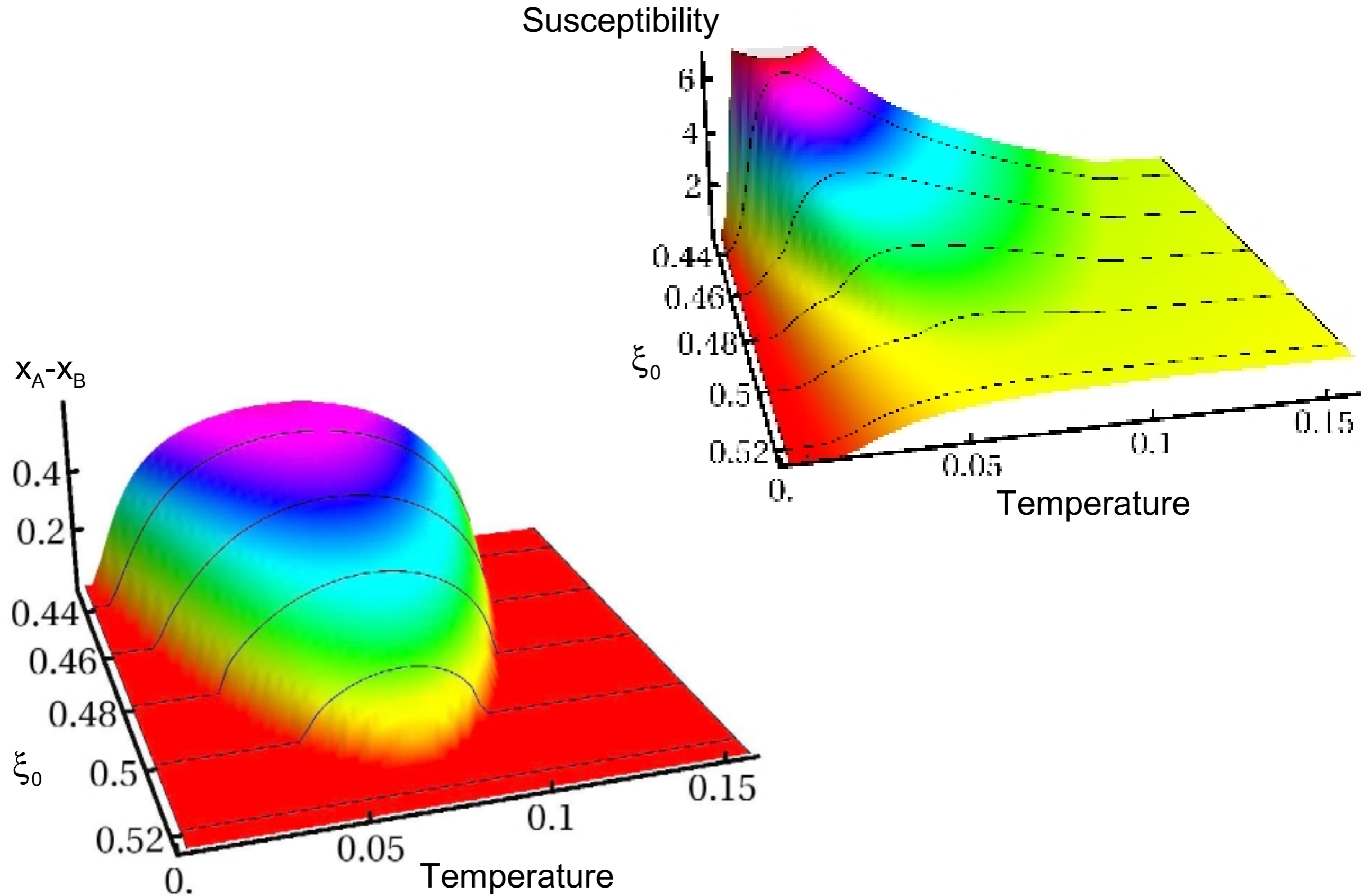


$$D = \Delta - 3J - \frac{Zt^2}{U - 2J}$$

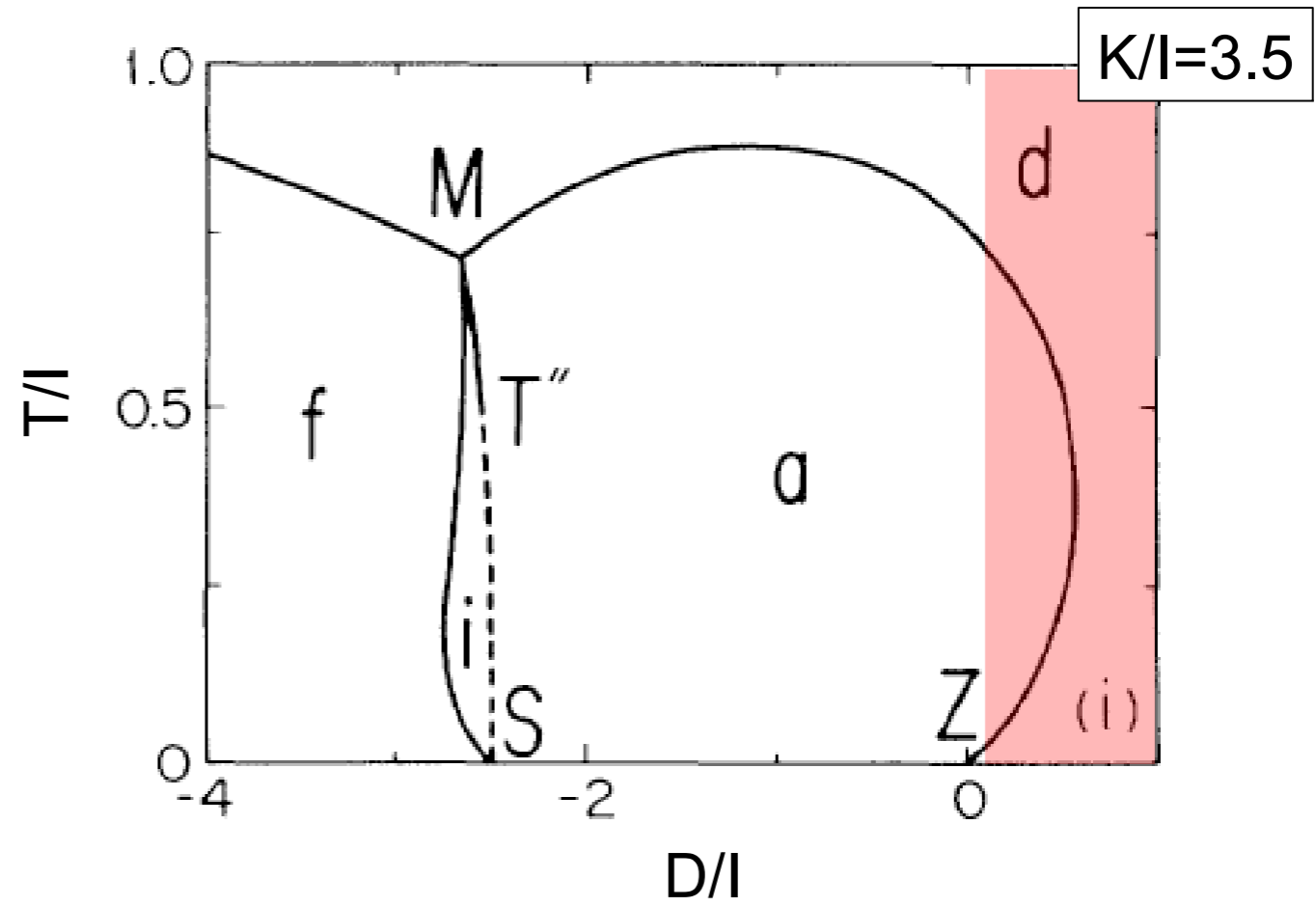
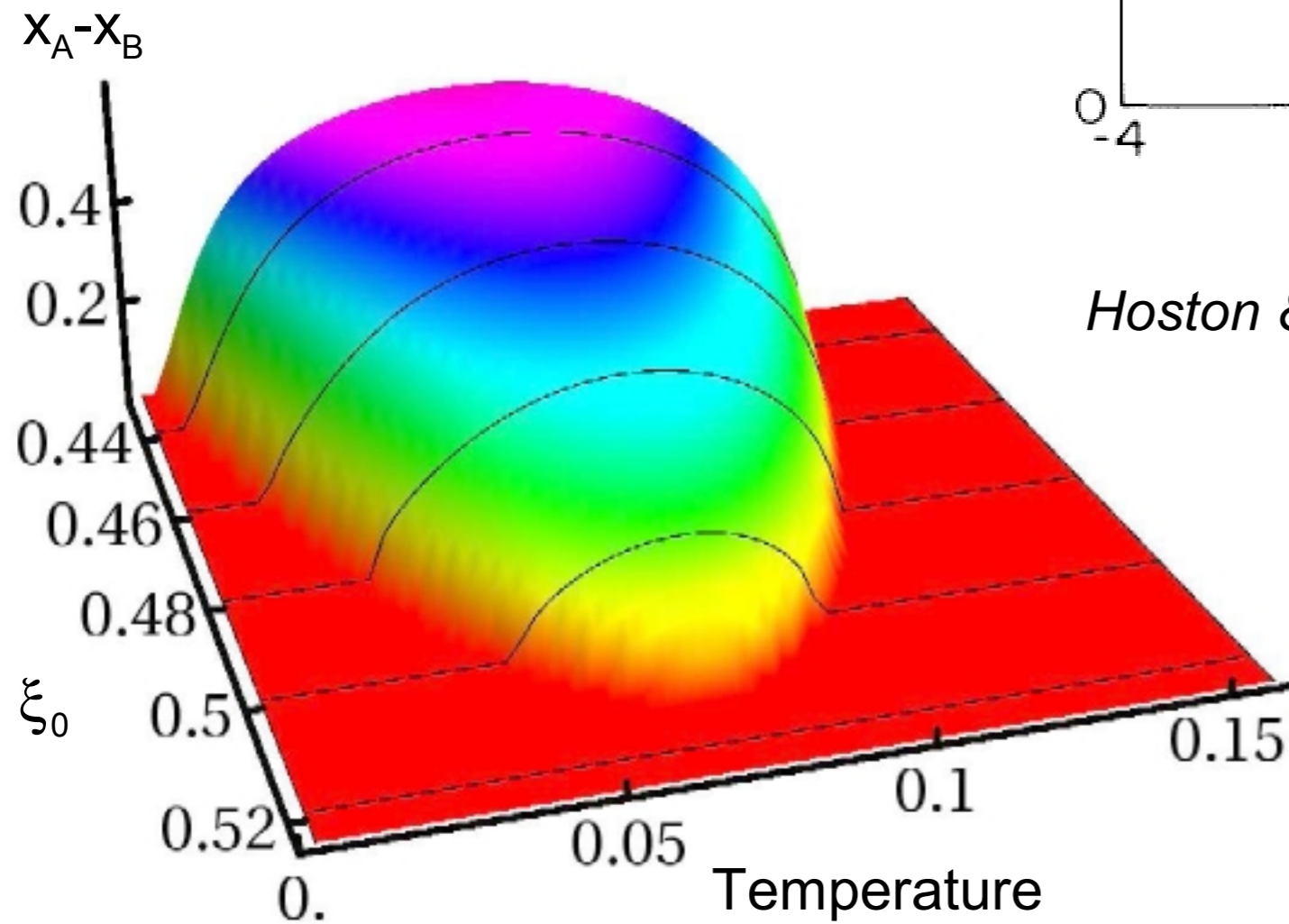
$$K = t^2 \left(\frac{1}{U - 2J} - \frac{1}{U + J} \right)$$

$$I = \frac{t^2}{U + J}$$

Mean-field for the BEG model

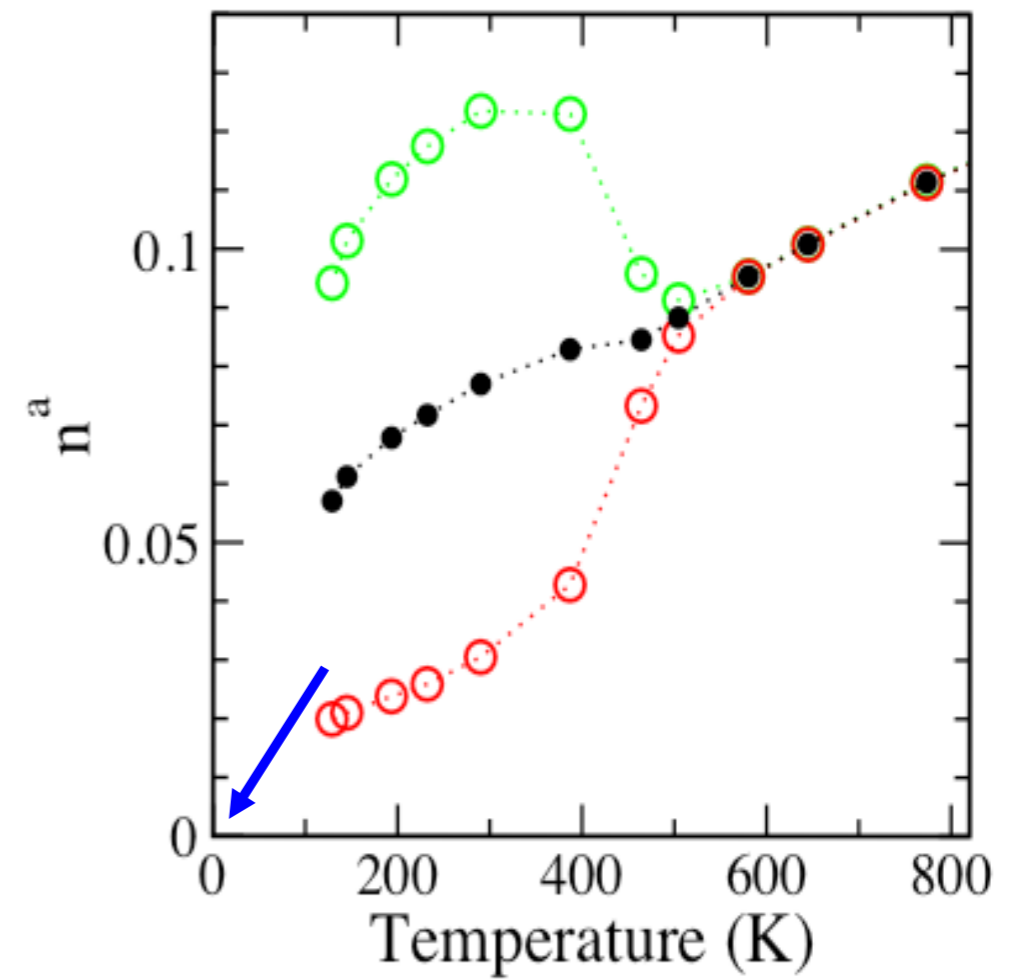
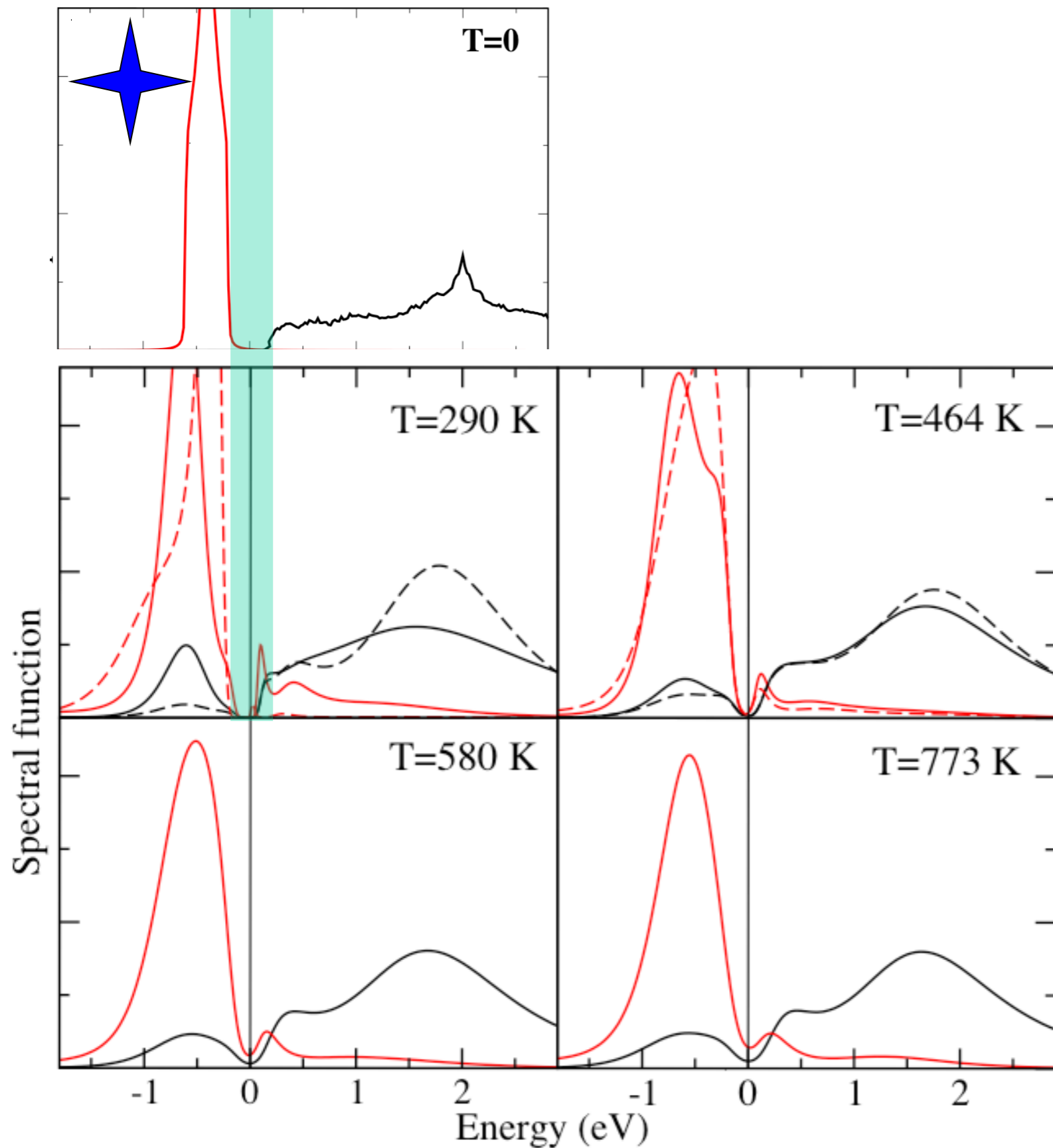


Mean-field for the BEG model



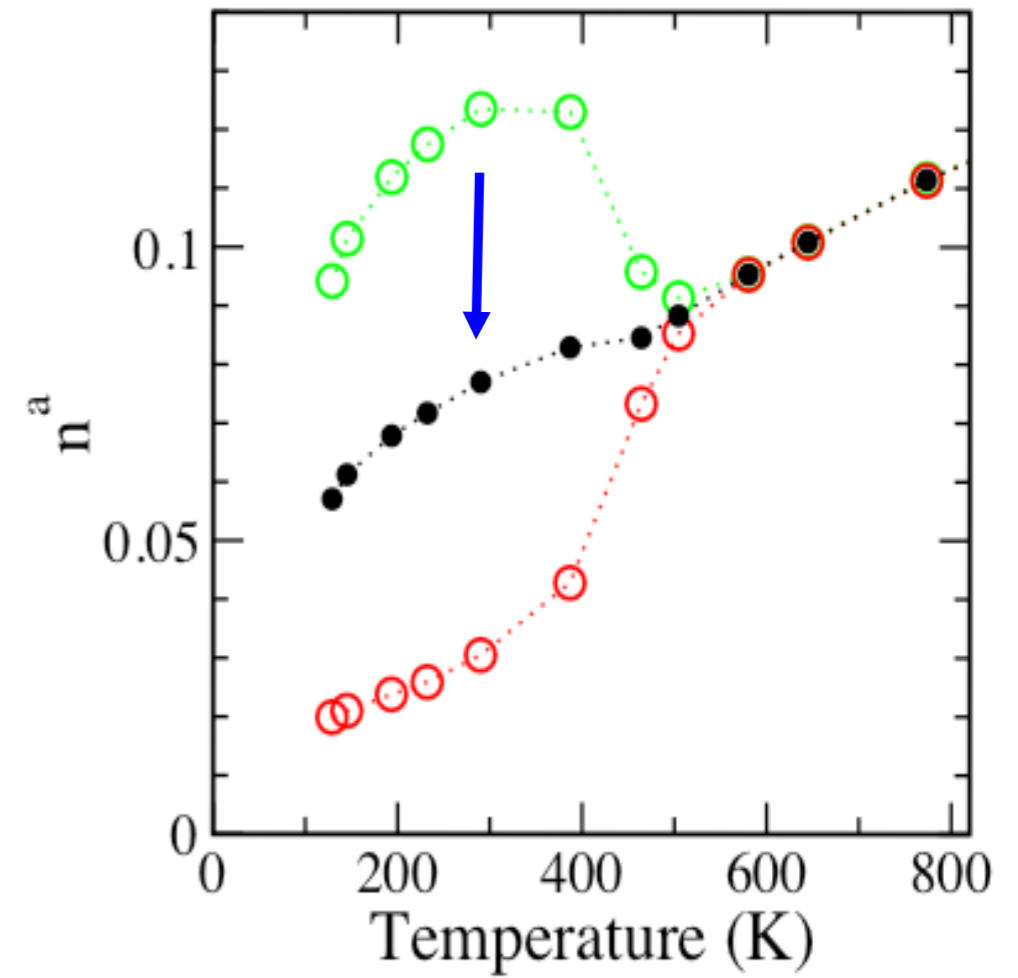
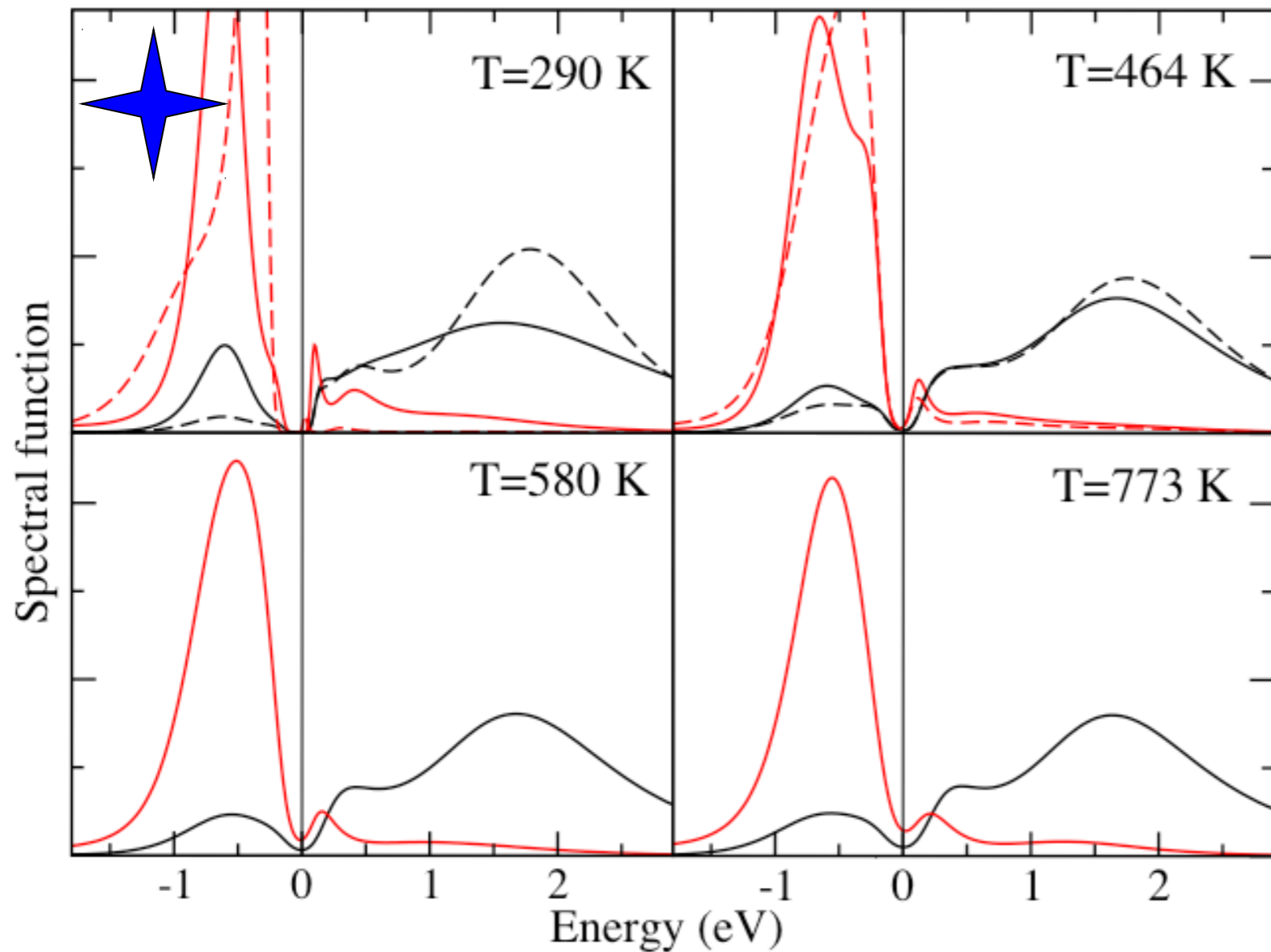
Hoston & Berker, Phys. Rev. Lett. 67, 1027 (1991)

One-particle spectral densities



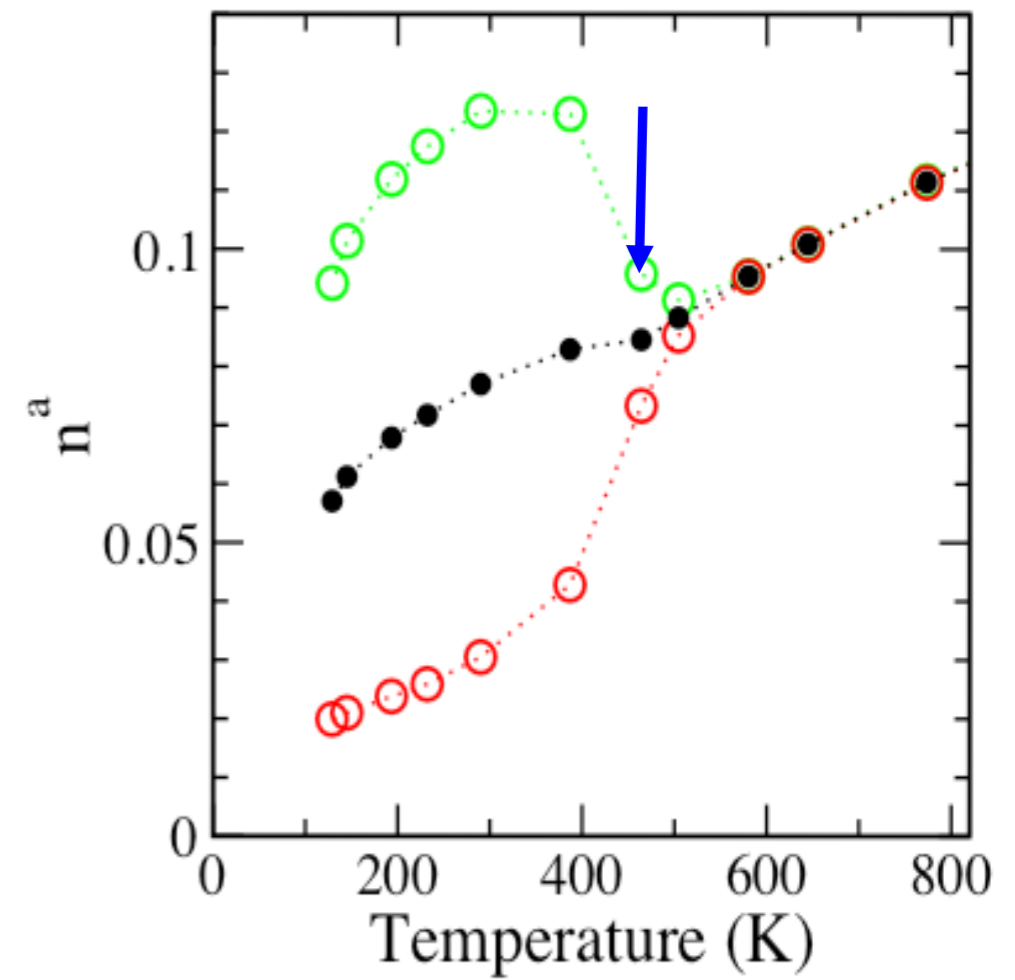
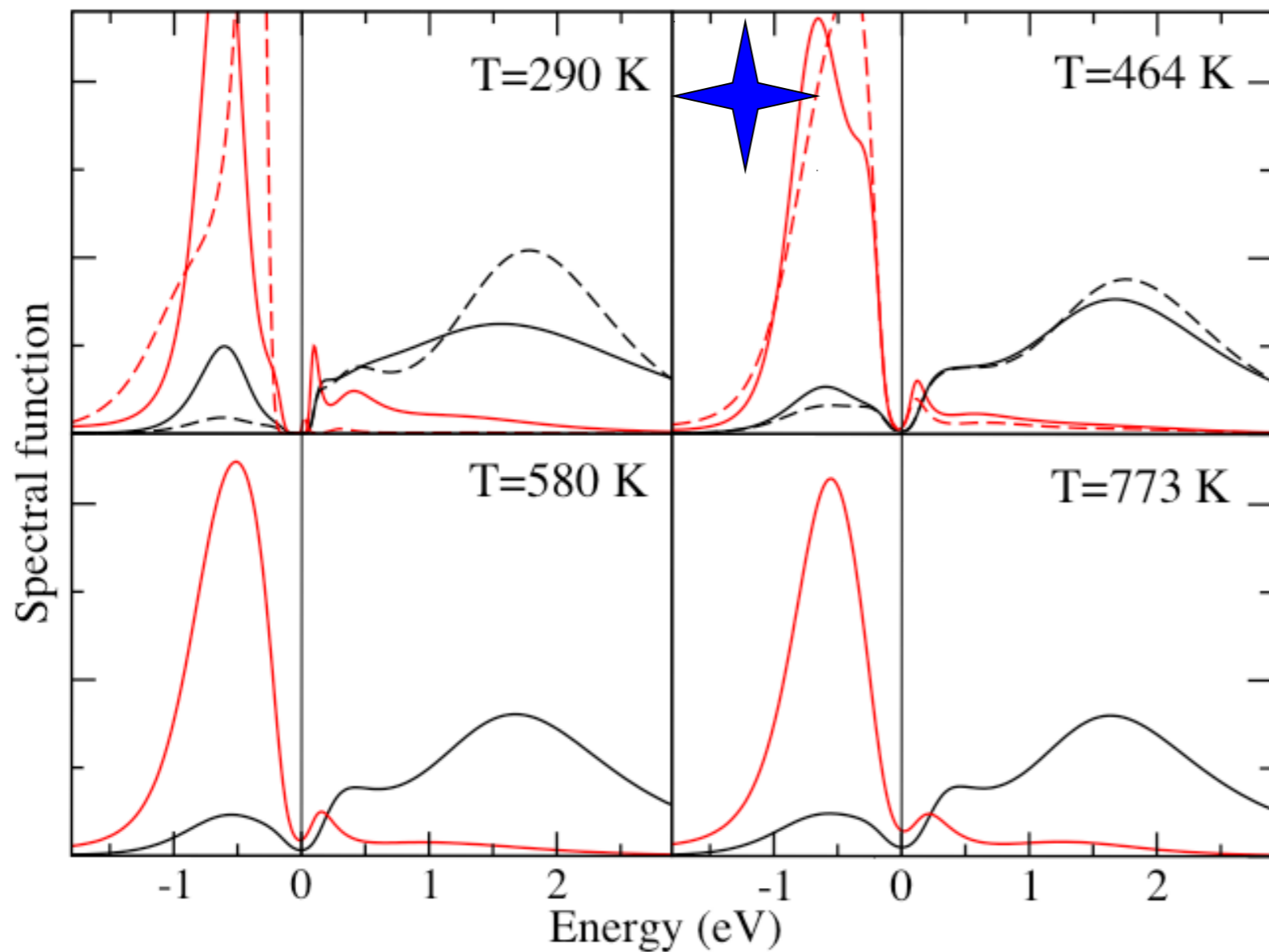
One-particle spectral densities

$A_{aa}(\omega)$ ———
 $A_{bb}(\omega)$ ———



One-particle spectral densities

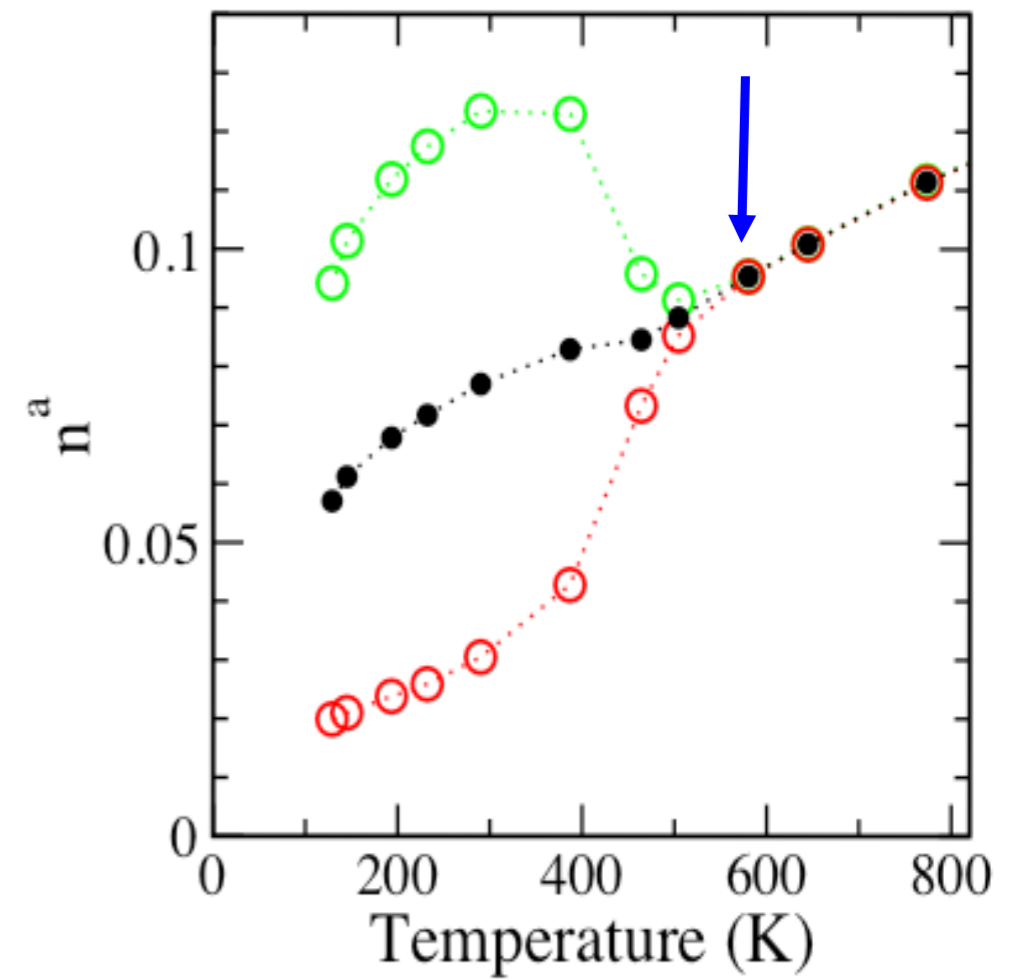
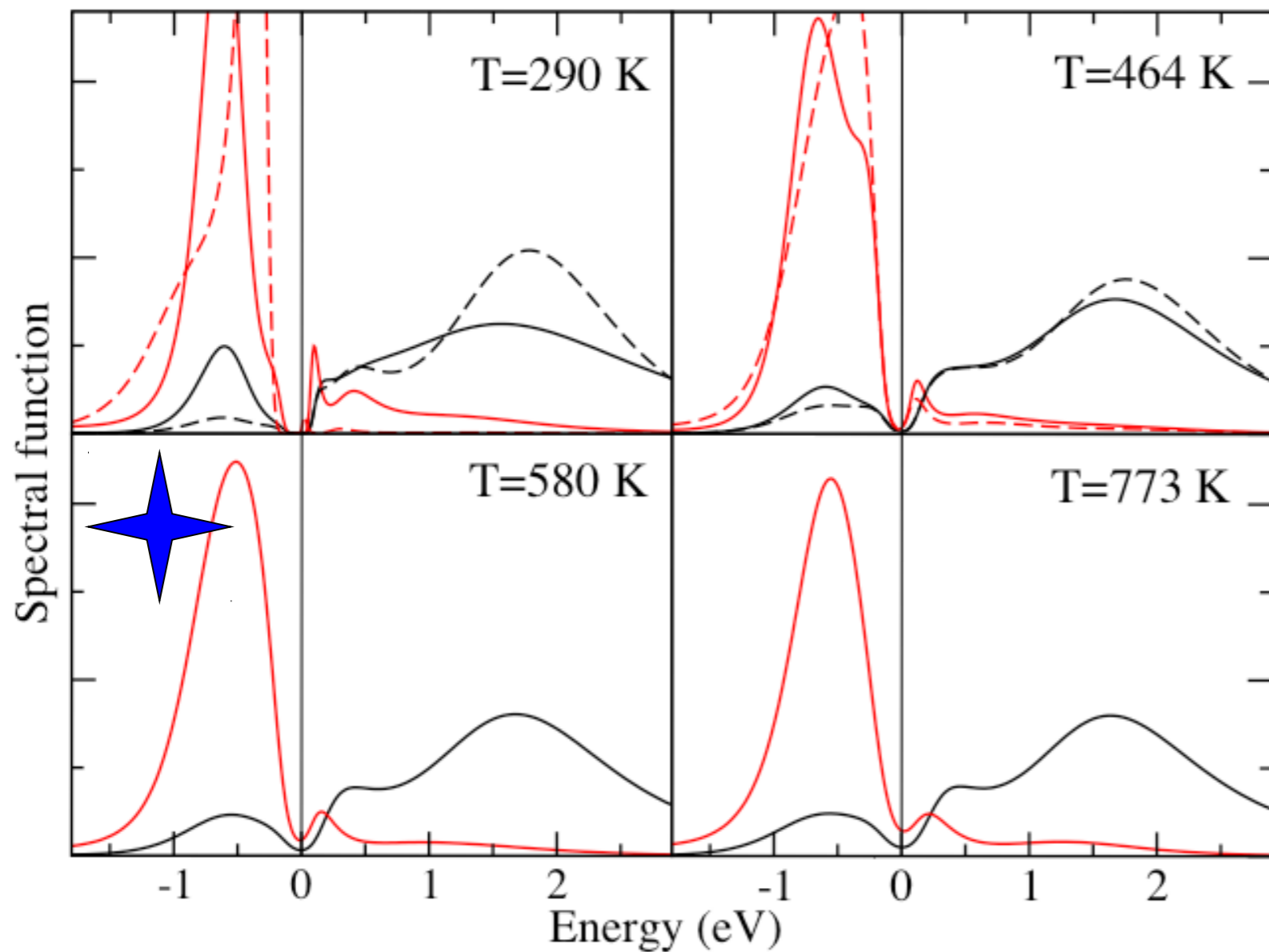
$$A_{aa}(\omega) \quad \text{— (black line)}$$
$$A_{bb}(\omega) \quad \text{— (red line)}$$



One-particle spectral densities

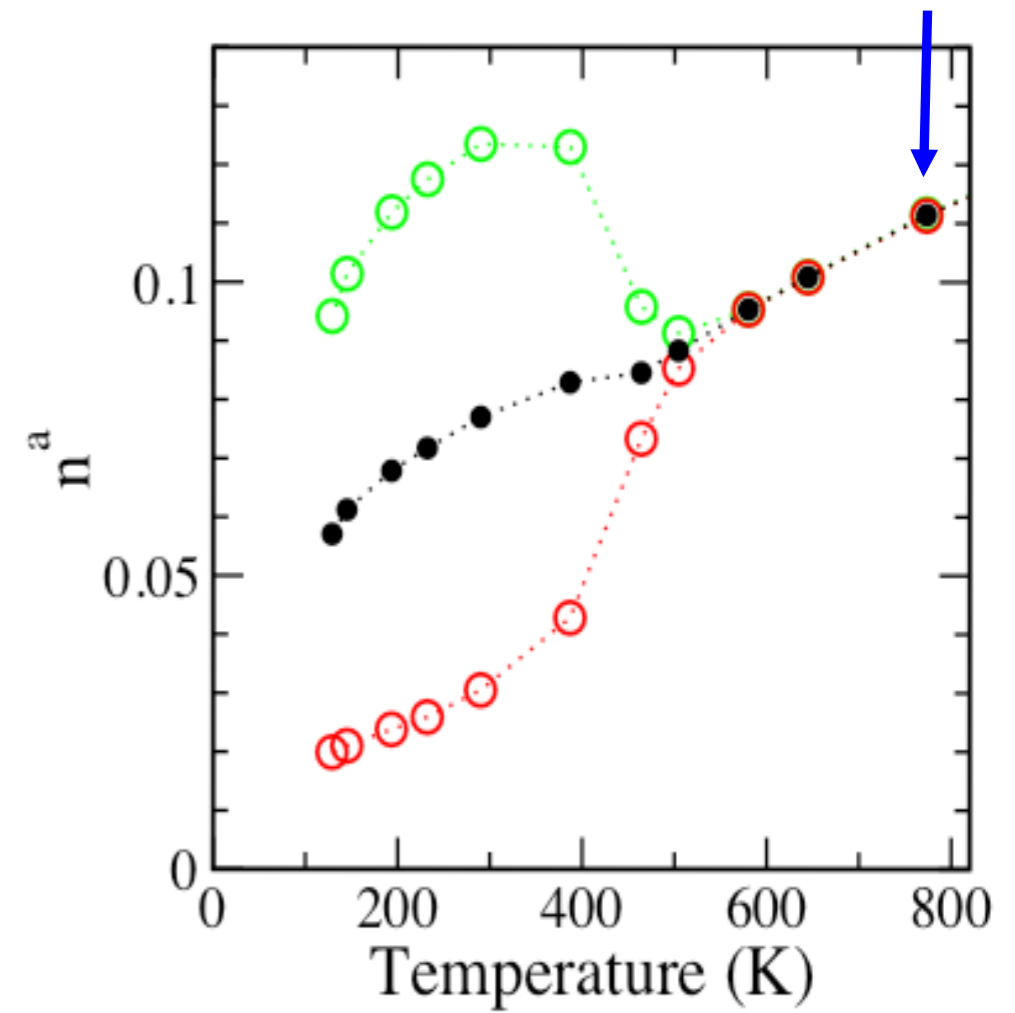
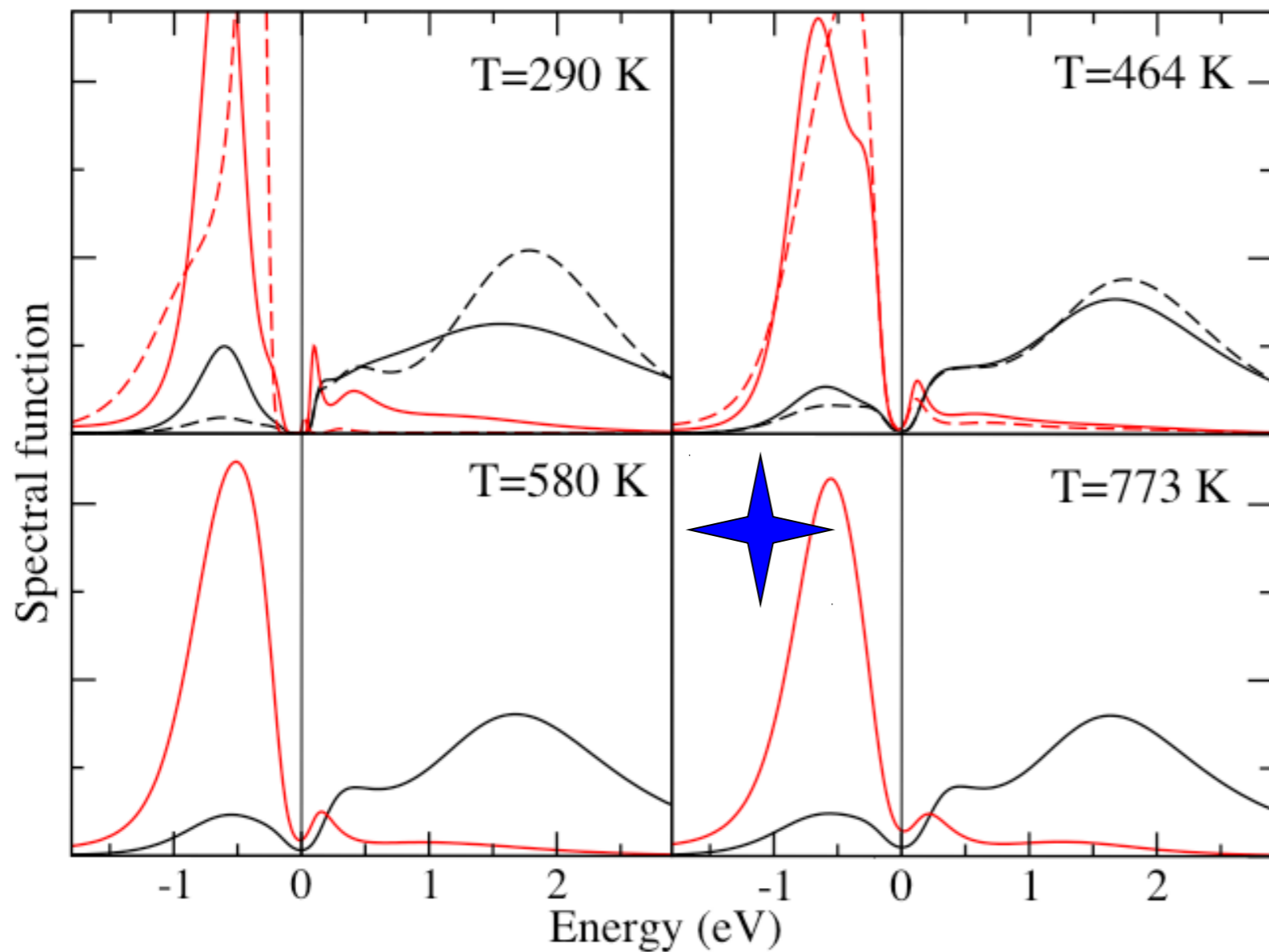
$$A_{aa}(\omega) \quad \text{—}$$

$$A_{bb}(\omega) \quad \text{—}$$

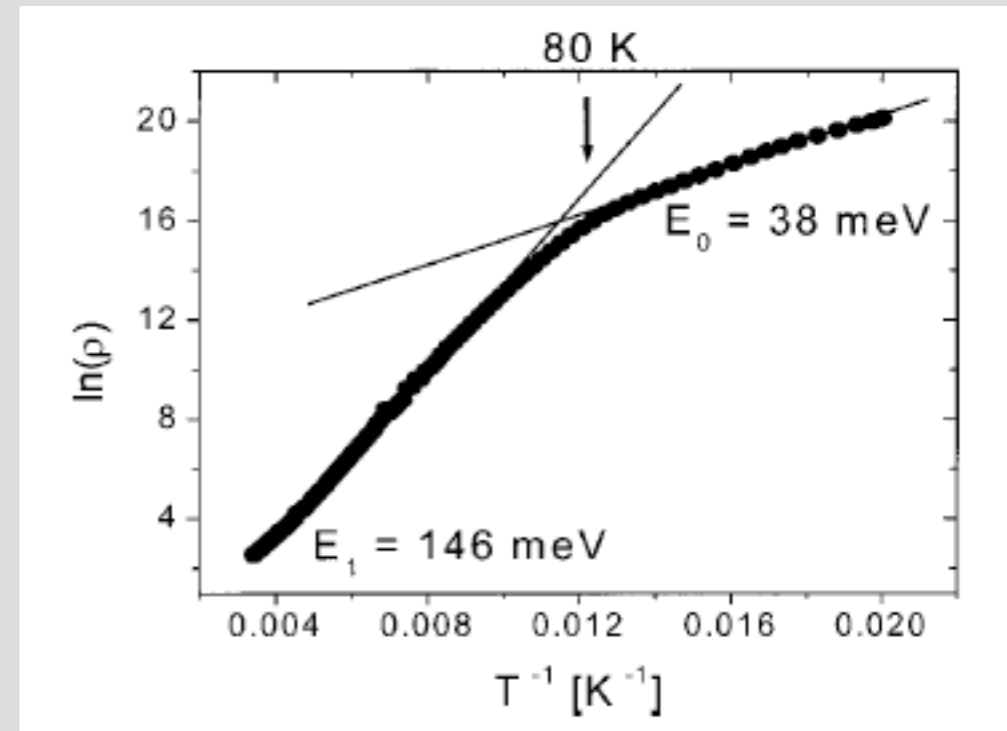
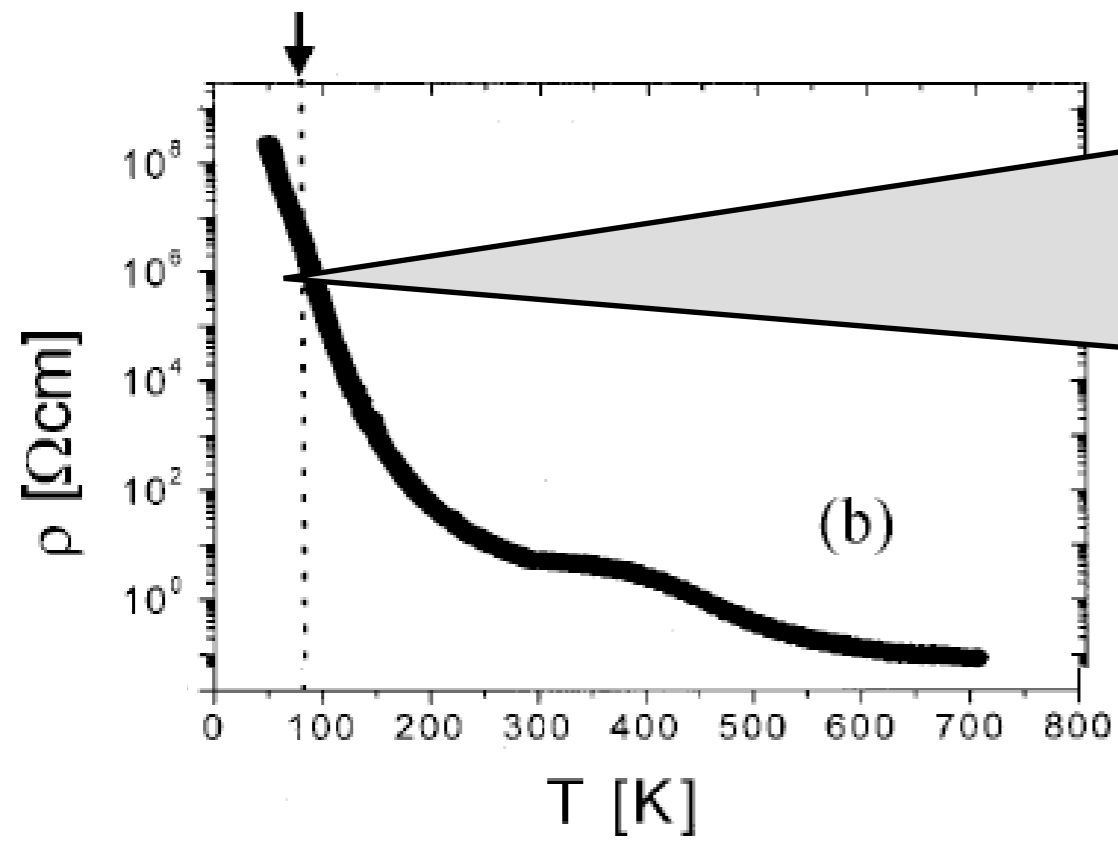


One-particle spectral densities

$A_{aa}(\omega)$ ———
 $A_{bb}(\omega)$ ———



LaCoO₃

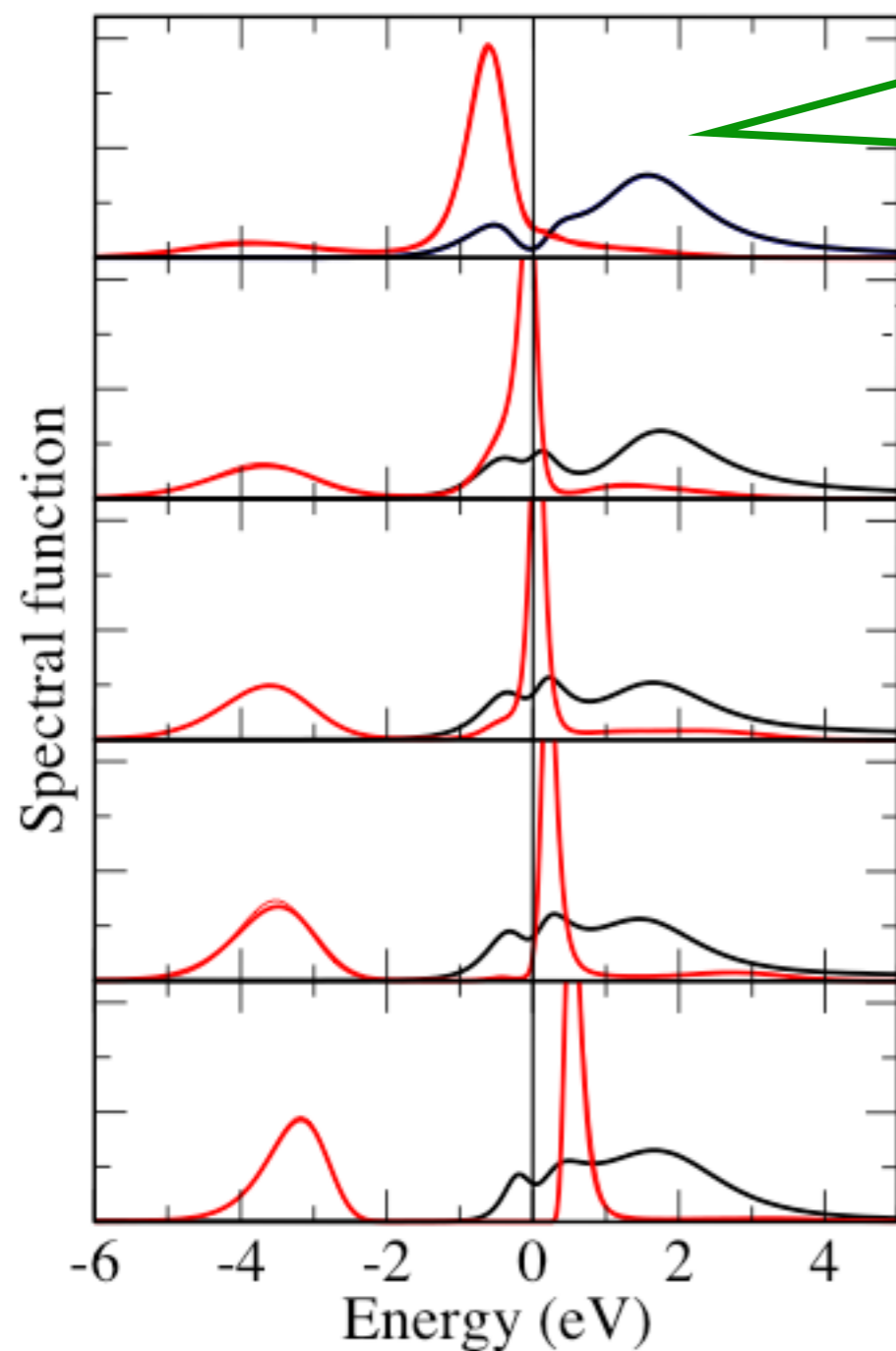


English et al. *Phys. Rev. B* **65**, 220407 (2002)

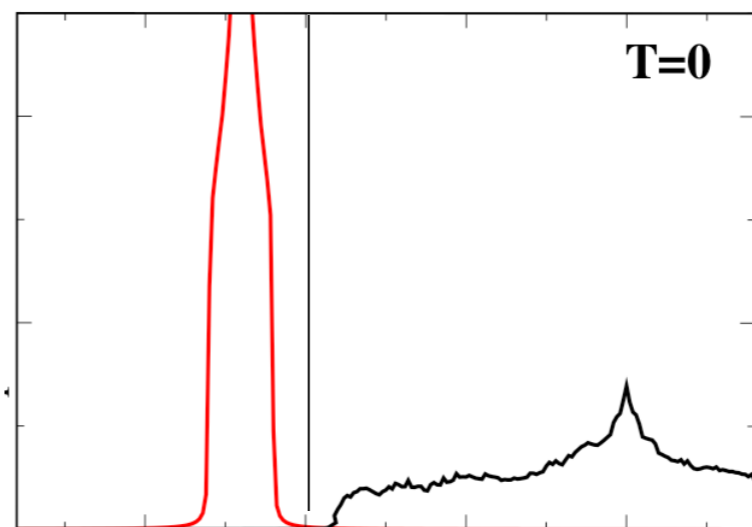
Hole doping

How do we go from localized mo

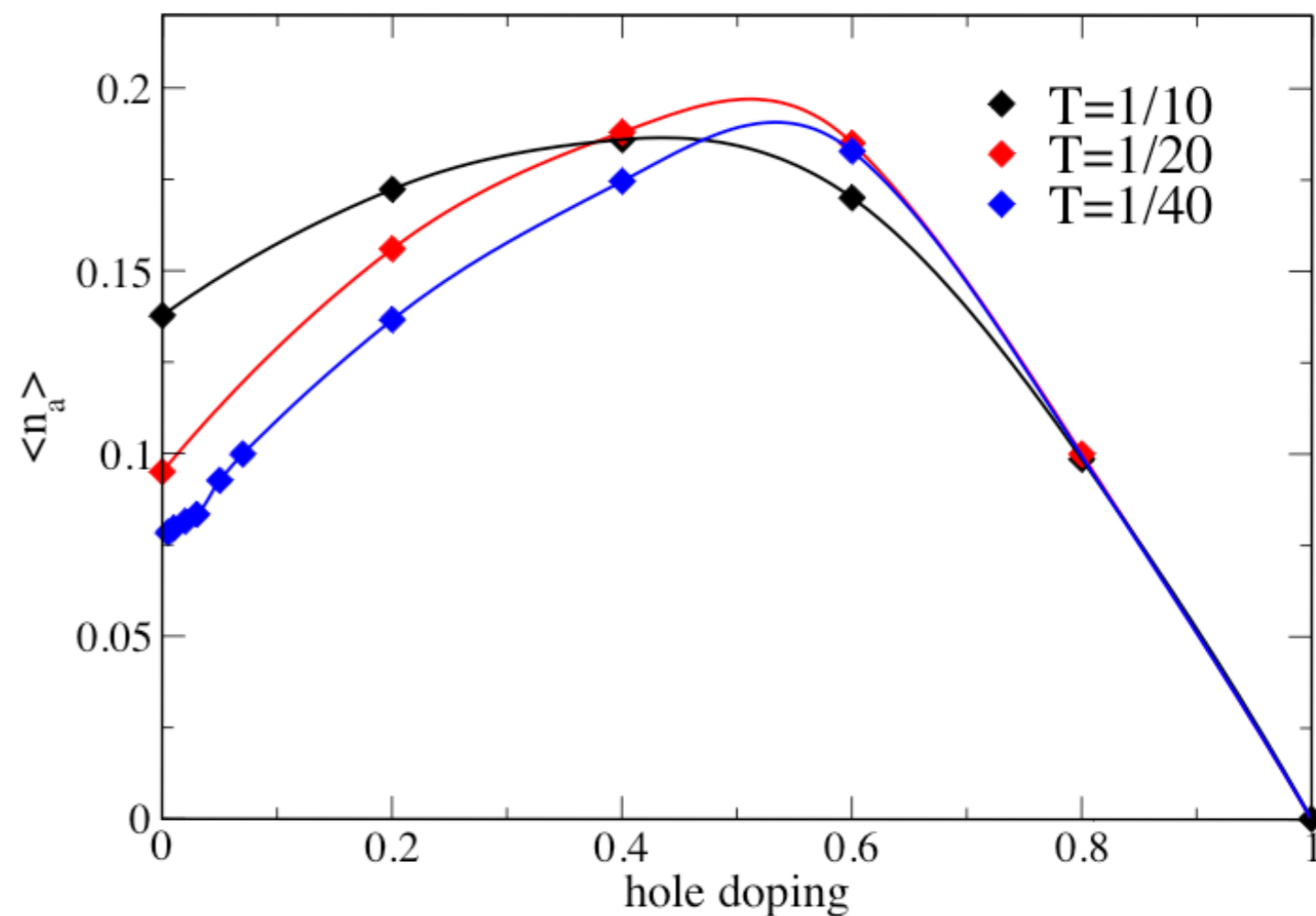
picture?



hole doping

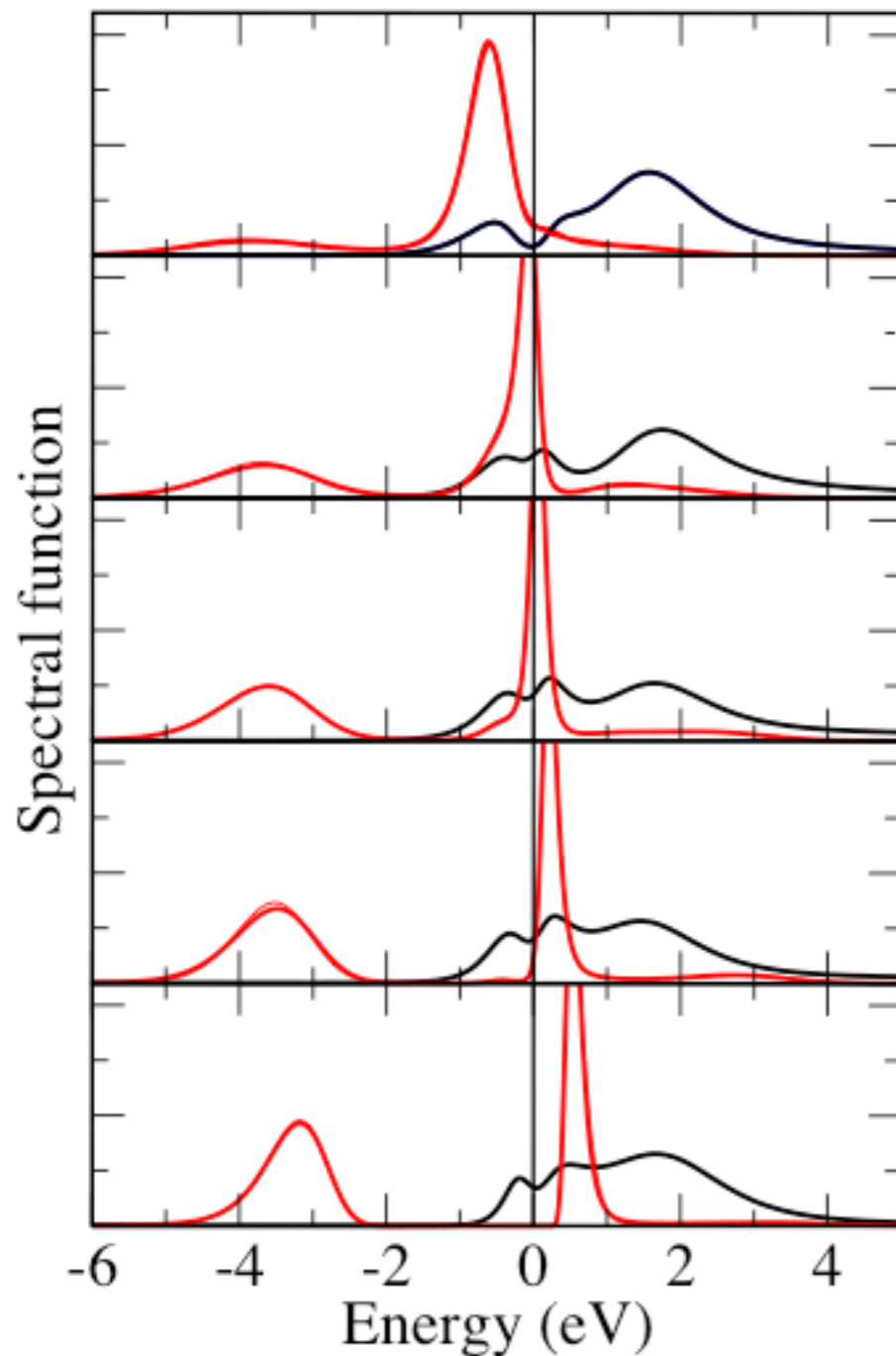


increases !



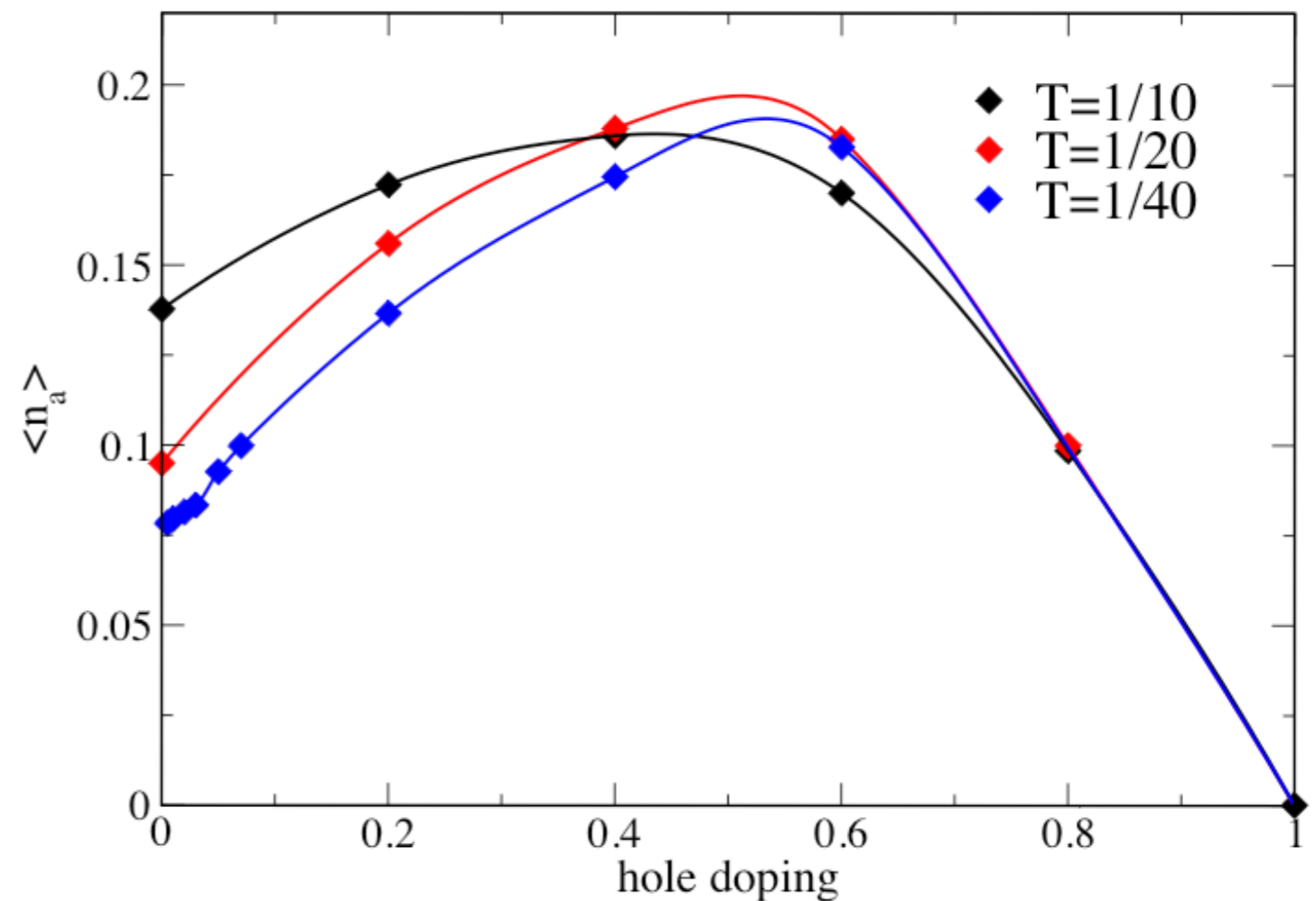
Hole doping

How do we go from localized moments to double exchange picture?

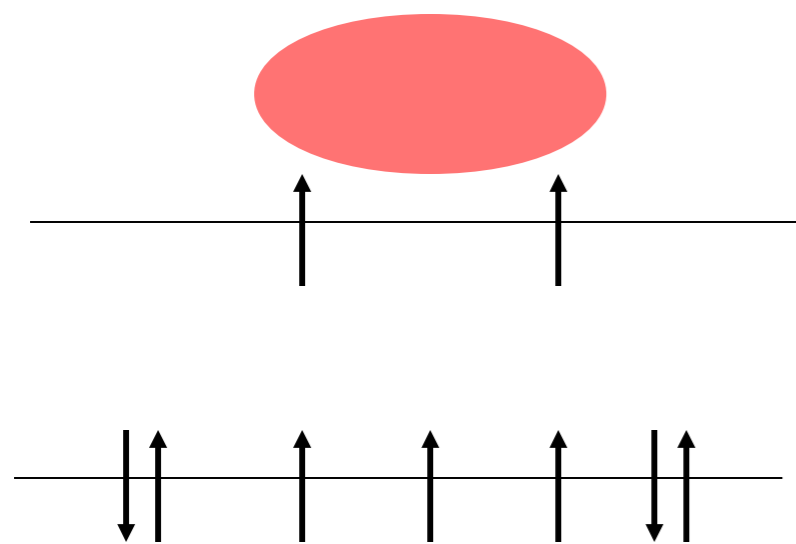
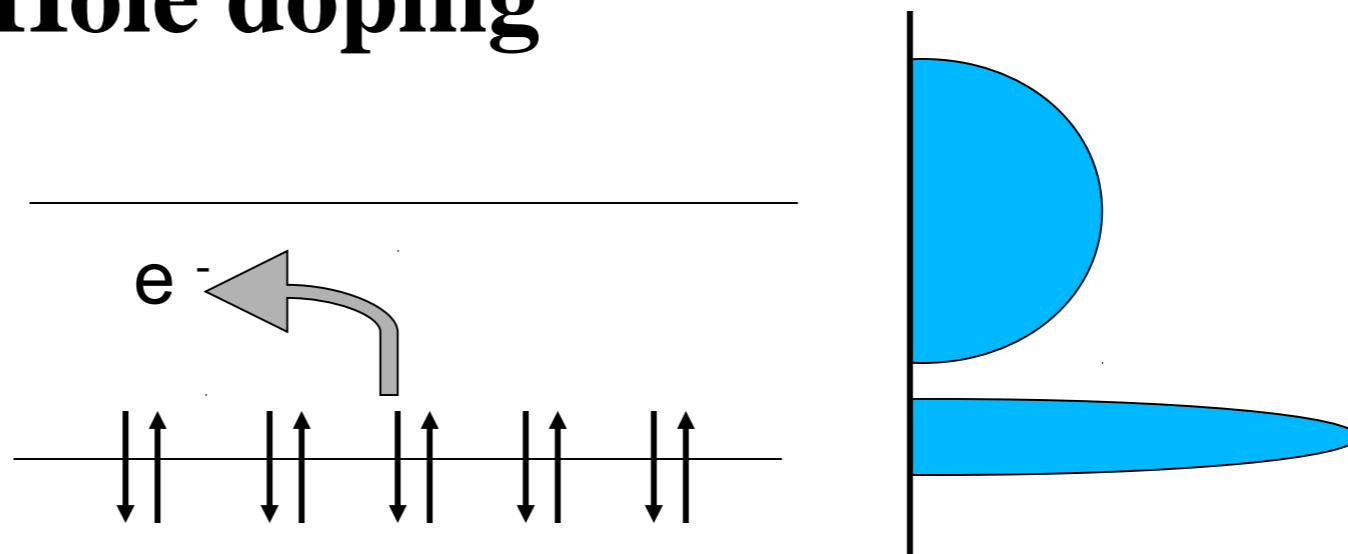


hole doping

Upper band occupancy increases !



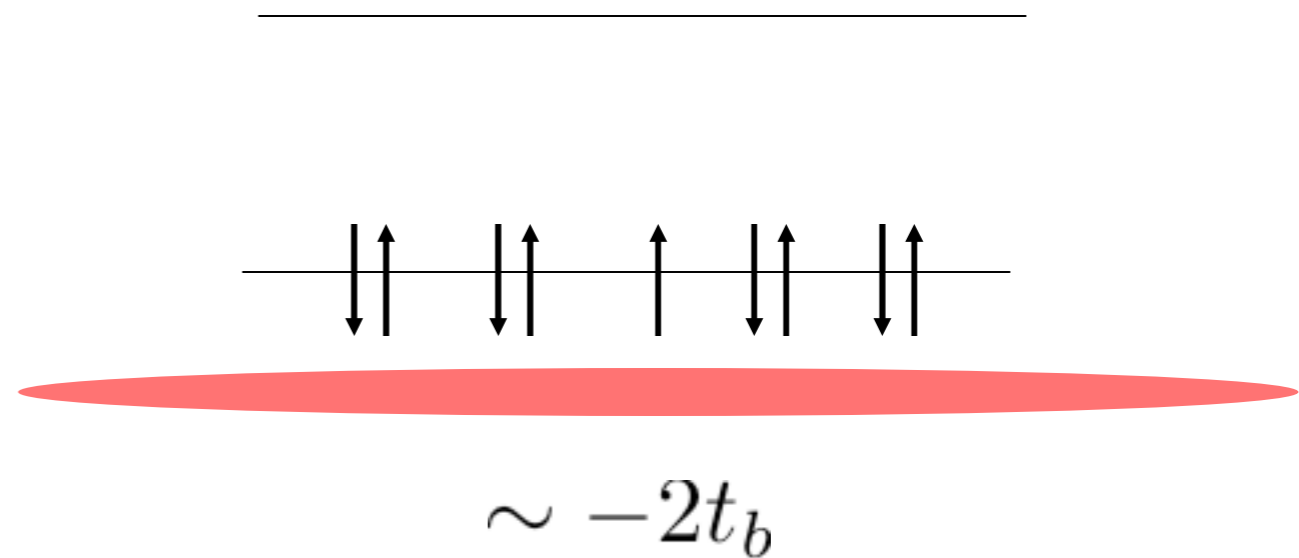
Hole doping



$$\sim -\sqrt{2}t_a + 2\xi_0$$

localized magnetic polaron

OR

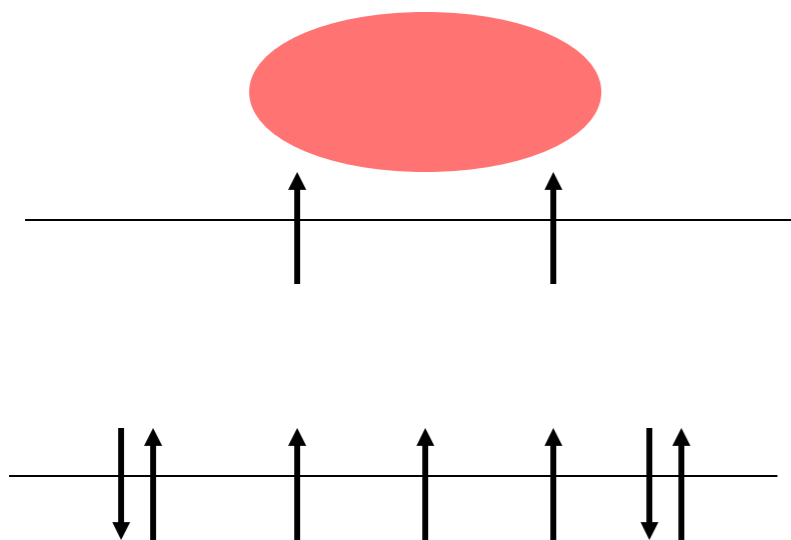


$$\sim -2t_b$$

itinerant hole in lower band

Hole doping

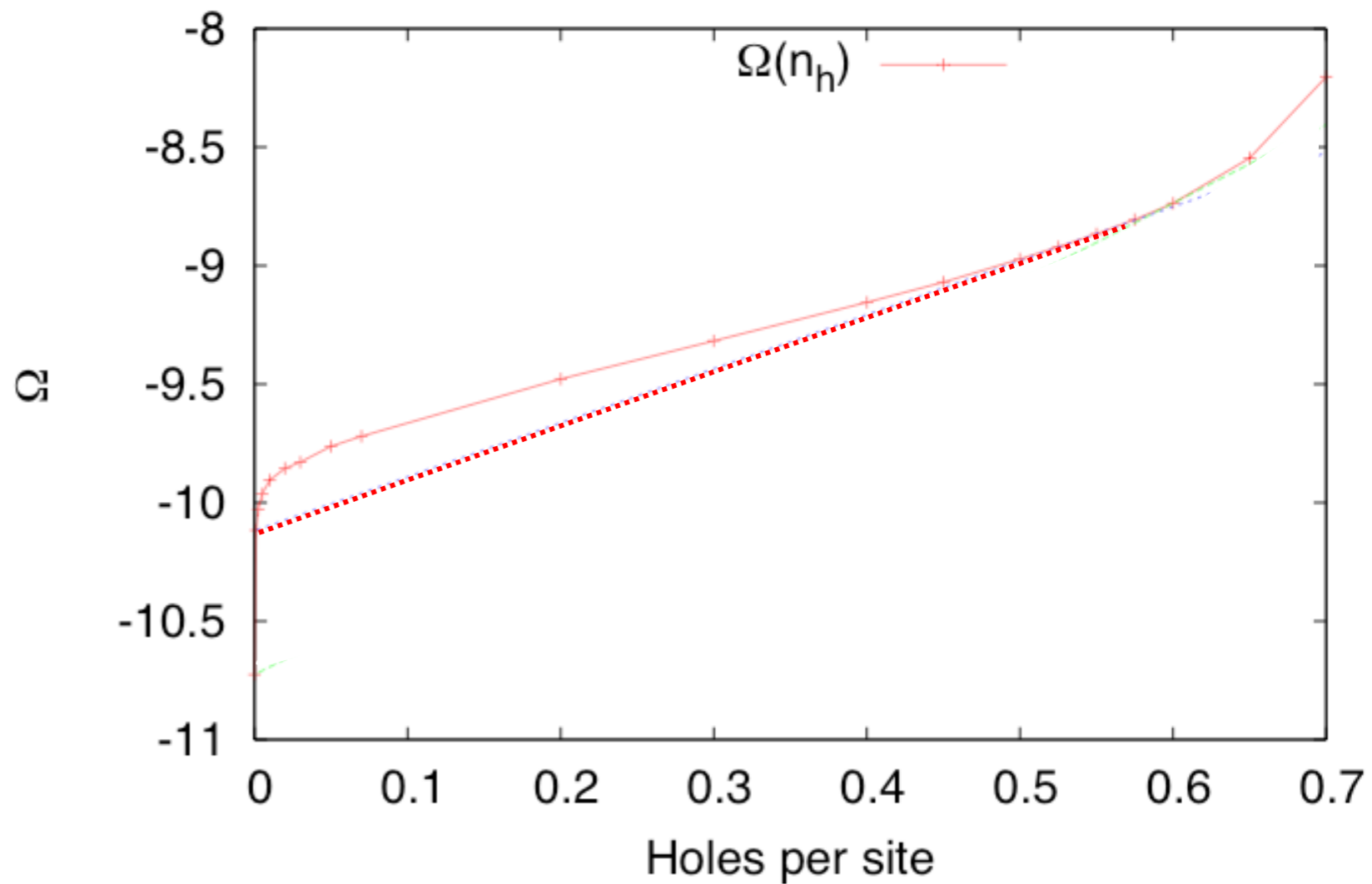
isolated magnetic polarons



OR

phase separation

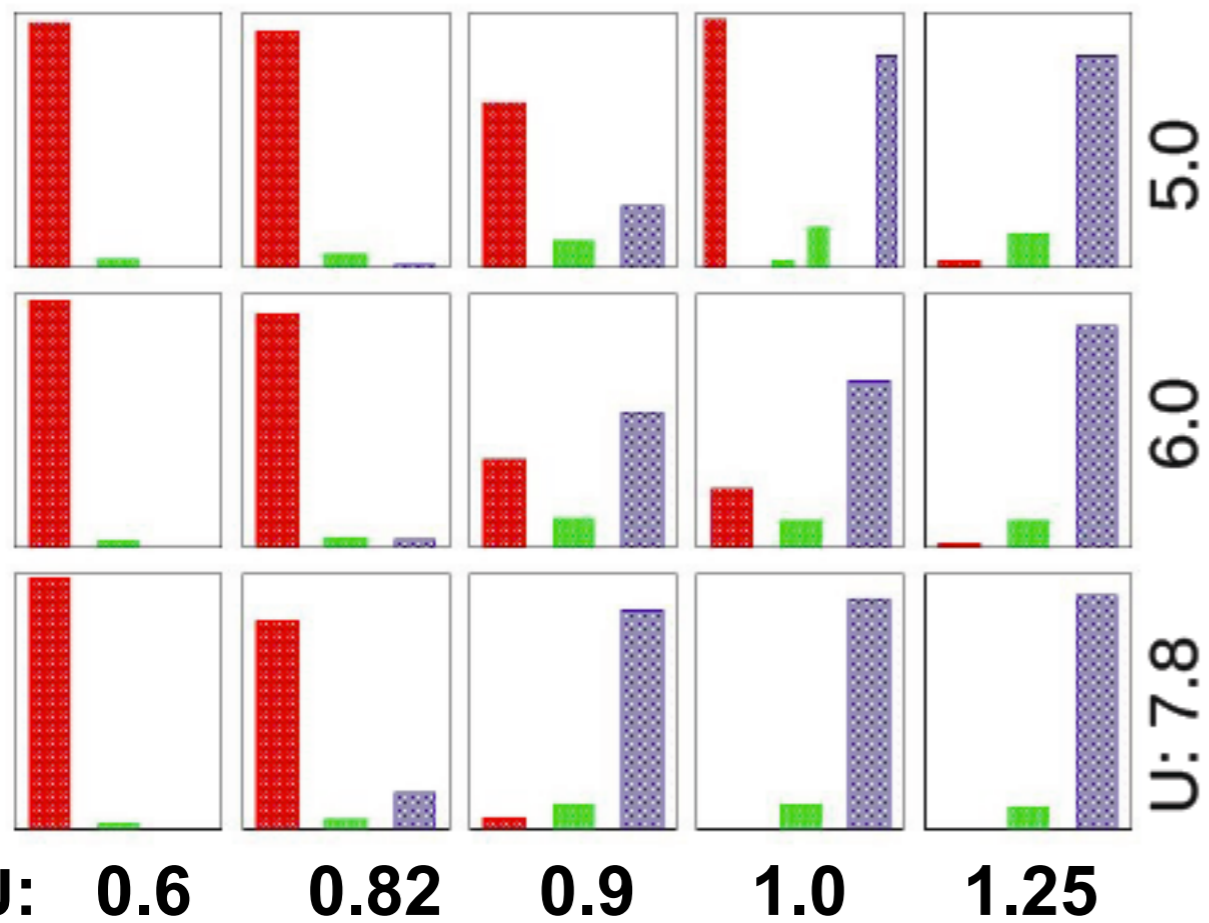
DMFT Grand canonical potential



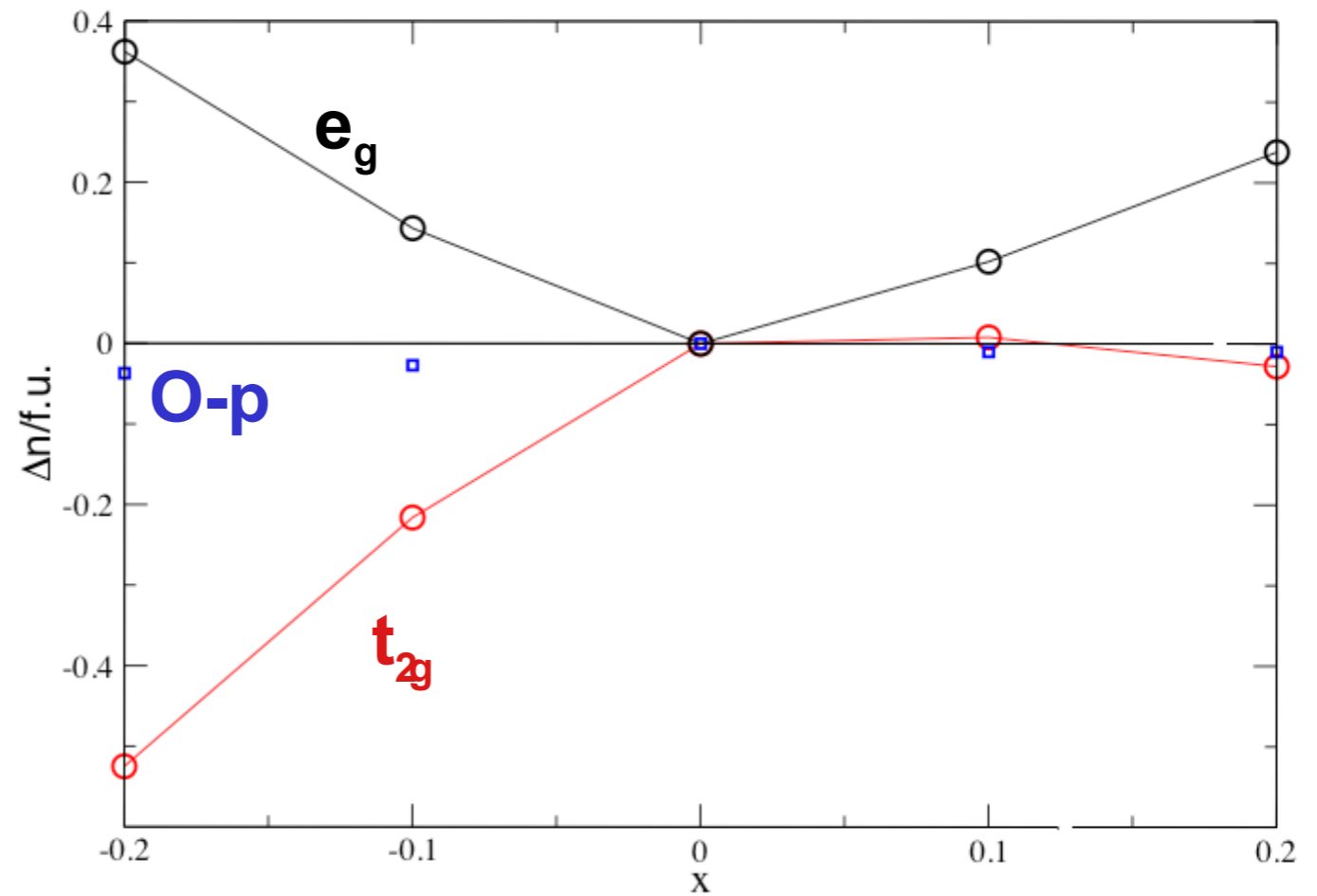
LaCoO₃ - LDA+DMFT results

Magnetic excitations = HS

Hole doping (Sr_xLa_{1-x}CoO₃)



- LS: t^6
- IS: t^5
- HS: t^4



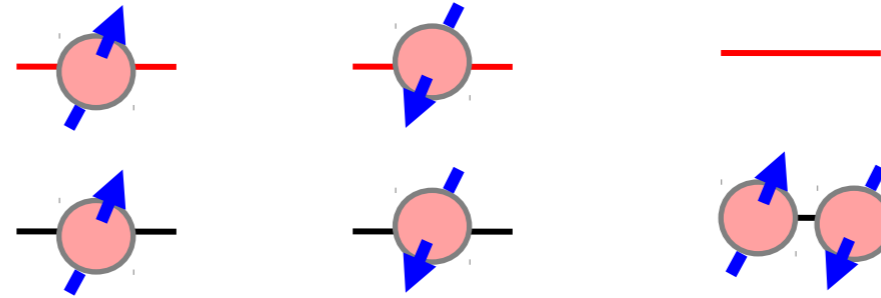
Conclusions

- (Quasi)degeneracy of ionic multiplets leads to rich phase diagrams in strongly correlated systems.
- Effective HS-LS attraction at the HS/LS transitions leads to a ordered state with reduced translational symmetry.
- 2-band Hubbard model with crystal field provides fermionic realization of BEG model and introduces new parameter - doping.
- Under certain circumstances ($W_a \gg W_b$) doping leads to formation of inhomogeneities - magnetic polarons
- We observe similar physics in LDA+DMFT calculations for LaCoO_3 (LS vs HS competition in stoichiometric system, disproportionation, generation of IS by hole doping)

Low-energy model

Integrate out the charge fluctuations:

- keep 3 local states



- treat hopping as perturbation

Hamiltonian

$$\tilde{H} = \xi_0 \sum_{i,\sigma} n_{i,\sigma}^{\text{HS}} + \sum_{\langle ij \rangle, \sigma} (\xi_1 n_i^{\text{LS}} n_{j,\sigma}^{\text{HS}} + \xi_2 n_{i,\sigma}^{\text{HS}} n_{j,-\sigma}^{\text{HS}})$$

$$\xi_0 = \Delta - 3J, \quad \xi_1 = -\frac{t_{aa}^2}{U-2J}, \quad \xi_2 = -\frac{2t_{aa}^2}{U+J}$$

Mean-field free energy

$$\begin{aligned} F(T) = & \frac{\xi_0}{2} (x_A + x_B) + 2\xi_1 (x_A + x_B - 2x_A x_B) - \xi_2 x_A x_B \\ & + \frac{T}{2} (1 - x_A) \ln(1 - x_A) + \frac{T}{2} (1 - x_B) \ln(1 - x_B) \\ & + \frac{T}{2} x_A \ln\left(\frac{x_A}{2}\right) + \frac{T}{2} x_B \ln\left(\frac{x_B}{2}\right), \end{aligned}$$